Stochastic Cosmology and the MOND Paradigm

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Abstract

This brief note points out that, if early cosmology drives gravitational dynamics out of equilibrium, a straightforward random walk model of orbital motion can explain away the basis of Modified Newtonian Dynamics (MOND).

Key words: stochastic cosmology, random walk, non-extensive statistical mechanics, MOND.

There are well-founded reasons to believe that early stages of Universe evolution drive gravitational dynamics out of thermodynamic equilibrium. Consider this early regime and the classical Kepler problem involving a body gravitating about a central mass. The fluctuating behavior of the orbital motion may be described by a simple *random walk model*, whereby the gravitating body drifts away from the central mass according to the standard diffusion equation. The grow over time of the mean radial distance is given by

$$\langle r \rangle \propto \sqrt{t} \tag{1}$$

and reflects the evolution towards states with maximal entropy. If the problem has spherical symmetry, the number of configurations Ω in a shell of thickness dr is proportional to r^2. In this case, using the Boltzmann entropy

$$S = k_B \log \Omega \tag{2}$$

the expression of the mean radial force associated with the drift (1) can be shown to take the form

$$\langle F_r \rangle = T \left\langle \frac{dS}{dr} \right\rangle = \frac{2k_B T}{\langle r \rangle} \tag{3}$$

where T denotes the temperature [1-2]. Assume next that the gravitating body consists of a couple of components, each defined by entropies S_1 and

S_2. Because the system is out of equilibrium, its overall entropy contains a non-extensive term, and it reads

$$S = S_1 + S_2 + |q-1| S_1 S_2 \tag{4}$$

where $|q| < 1$ is the Tsallis parameter [3]. Relations (2)-(4) hint that that (4) splits in two contributions, one attributed to Newtonian dynamics (S_N) and the other to MOND (S_{MOND}), namely,

$$S = S_N + S_{MOND} \tag{5}$$

in which

$$S_N = S_1 + S_2 = k_B \log(\Omega_1 \Omega_2) \tag{6a}$$

$$S_{MOND} = |1-q| k_B^2 \log \Omega_1 \log \Omega_2 \tag{6b}$$

A plausible hypothesis is that the system temperature is non-uniform and it drops as reciprocal of the radial scale as in

$$T \propto \langle r \rangle^{-1} \tag{7}$$

It follows from (6a) and (7) that the Newtonian radial force (3) obeys an *inverse square law*, i.e.

$$\boxed{\langle F_r \rangle_N \propto \frac{1}{\langle r \rangle^2}} \tag{8}$$

whereas the correction to Newtonian force generated by (6b) takes the form

$$\boxed{\langle F_r \rangle_{MOND} \propto \frac{1}{\langle r \rangle} \left\langle \frac{dS_{MOND}}{dr} \right\rangle} \tag{9}$$

We close with several key observations:

1) Follow-up analysis must show that the sign of radial force (3) and (8) is *negative*, in sync with the attractive nature of gravitational interaction.
2) Dynamics induced by (8) and (9) is likely to carry over as "memory-like" effects to the current state of the Universe.
3) Our approach matches the idea that the value of the vacuum energy parameter and of the Hubble parameter follow from the stochastic dynamics of the early Universe [4-5]. It is also compatible with the

proposal that MOND is rooted in the *non-integrable* nature of gravitational interaction [6-8].

References

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