Space of Quantum Mechanics

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Quantum mechanics is revealing a profound and remarkable property of space - where the quantum phenomena happen.

It's well-known that the Newtonian classical mechanics implies the Euclidean space and vice versa, i.e.,

$$\left\{\begin{array}{c}\text{Classical}\\\text{Mechanics}\end{array}\right\} \Longleftrightarrow \left\{\begin{array}{c}\text{Euclidean}\\\text{Space}\end{array}\right\}. \tag{1}$$

Einstein replaced classical mechanics by special theory of relativity. In other words, the Euclidean space is replaced by the Minkowski's spacetime, but all the locally defined measurable physical parameters, like mass, momentum, energy etc., remained intact.

$$\left\{\begin{array}{c}\text{Classical}\\\text{Mechanics}\end{array}\right\} \rightarrow \left\{\begin{array}{c}\text{Special Theory}\\\text{of Relativity}\end{array}\right\} \Longrightarrow \left\{\begin{array}{c}\text{Euclidean}\\\text{Space}\end{array}\right\} \rightarrow \left\{\begin{array}{c}\text{Minkowski's}\\\text{Spacetime}\end{array}\right\} \tag{2}$$

Special theory of relativity is not disregarded as a classical theory, although the Euclidean space is replaced by the Minkowski's spacetime. Still it's a classical, but relativistic theory. Also, there are no major issues in accepting the Minkowski's spacetime as a relativistic generalization of the Euclidean space and in interpreting the physical reality revealed by the special theory of relativity.

Let's pause for a brief moment and hypothetically assume the high energy accelerator physics as if happened much before the Maxwell's electromagnetic theory and hence, the special theory of relativity. Obviously, in such a situation, the observed experimental data of the relativistically moving particles has to be analyzed only by using the Newtonian mechanics in 3-dimensional Euclidean space. Then the scientists might have claimed the classical mechanics as strange, weird and counter-intuitive. Because, howsoever the particles are accelerated, strangely they never seem to be moving beyond a particular speed, though, in principle, infinite speed is allowed by the Newtonian mechanics. Weirdly, that particular speed happens to be the speed of light! What does the speed of light have to do anything with the speeds of material particles is counter-intuitively unclear. Hence, scientists might have concluded that the classical behavior of all material particles - during their accelerations - is really strange, weird and counter-intuitive.

But by the time the high-energy particle physics is happening, we already had the special theory of relativity. Therefore, there was never a chance to claim the classical mechanics as strange, weird and counter-intuitive, because, we already know the underlying space (Minkowski's spacetime) where it happens.

The following is similar to Eq. (2):

$$\left\{\begin{array}{c}\text{Special}\\ \text{Theory of}\\ \text{Relativity}\end{array}\right\} \to \left\{\begin{array}{c}\text{General}\\ \text{Theory of}\\ \text{Relativity}\end{array}\right\} \Longrightarrow \left\{\begin{array}{c}\text{Minkowski's}\\ \text{Flat}\\ \text{Spacetime}\end{array}\right\} \to \left\{\begin{array}{c}\text{Riemannian}\\ \text{Curved}\\ \text{Spacetime}\end{array}\right\} \tag{3}$$

One doesn't have to travel at the speed close to the speed of light or make a direct visit to the neighborhood of an event horizon of a black-hole or both, in order to arrive at the required mathematics to describe the theory of relativity. Also, one doesn't have to become a microscopic entity like an atom or an elementary particle in order to discover the mathematics required to describe the quantum mechanics. All the mathematics are discovered in the the Newtonian/Classical regime and there only, all the measurable physical properties of matter, like mass, momentum, energy etc., are also defined. Some other properties, known as the internal properties like spin, isospin, etc., are also expressed in terms of physical parameters measurable in the same classical regime. Otherwise, all these properties become inaccessible to the local experimental observations.

Therefore, it doesn't matter whether a given physical phenomenon belongs to classical, relativistic, quantum mechanical or relativistic quantum mechanical regimes. All these different regimes are characterized using the physical properties defined classically and are described using mathematics found locally. But each one's respective space decides to which respective regime it belongs. Therefore, the classical regime must have a definite overlap with all the different regimes. In other words, all the physical phenomena happening in these regimes are essentially identical to the classical mechanical phenomena when viewed in their respective spaces (spacetimes). Hence, not only the theory of relativity, but also the quantum mechanics can't be strange, weird and counter-intuitive.

Consider the equivalent situation in quantum mechanics to those given in Eqs. (2) and (3): By replacing the commuting classical variables, position and momentum (x and p), by the corresponding non-commuting position and momentum operators ($\hat{x}$ and $\hat{p}$), respectively, using Dirac's prescription,

$$\{x\ ,\ p\}_{\mathrm{PB}} = \frac{[\hat{x}\ ,\ \hat{p}]}{i\hbar} \implies [x\ ,\ p] = 0 \longrightarrow [\hat{x}\ ,\ \hat{p}] = i\hbar, \tag{4}$$

the quantum mechanical description of any given classical Hamiltonian system can be obtained; here, $\{\ ,\ \}_{\mathrm{PB}}$ stands for the classical Poisson bracket, $i = \sqrt{-1}$, $\hbar = h/(2\pi)$, h is the Planck's constant. Dirac's prescription actually implies the following mapping,

$$\begin{aligned}
[x,p] = 0 \to [\hat{x},\hat{p}] = i\hbar \implies & \left\{\begin{array}{c}\text{Euclidean Space}\\ \text{of}\\ \text{Classical Mechanics}\end{array}\right\} \to \left\{\begin{array}{c}\text{Hilbert Space}\\ \text{of}\\ \text{Quantum Mechanics}\end{array}\right\} \\
= & \left\{\begin{array}{c}\text{Commuting}\\ \text{Phase Space of}\\ \text{Classical Mechanics}\end{array}\right\} \to \left\{\begin{array}{c}\text{Noncommuting}\\ \text{Phase Space of}\\ \text{Quantum Mechanics}\end{array}\right\},
\end{aligned} \tag{5}$$

and hence, the mapping of classical Hamiltonian into time-independent quantum mechanical Hamiltonian,

$$H = \frac{p^2}{2m} + V(x) = E_T \longrightarrow \hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{x}) = E, \tag{6}$$

where, H, $p^2/(2m)$, $V(x)$ and E_T are the classical Hamiltonian, kinetic energy, potential energy and the total energy of a particle of mass m, respectively; $\hat{H}$, $\hat{p}^2/(2m)$ and $V(\hat{x})$ are the quantum mechanical Hamiltonian, kinetic energy and potential energy operators, respectively and E is the energy eigenvalue associated with the particle of mass m [1], such that,

$$\hat{H}|\psi> = E|\psi> \quad \Longleftrightarrow \quad \left\{\frac{d^2}{dx^2} + \frac{2m}{\hbar^2}[E - V(x)]\right\} < x|\psi> = 0\,. \tag{7}$$

The state vector $|\psi>$ is the energy eigenstate whose position basis representation is the Schrödinger's wave function, $< x|\psi >$.

In Eq. (2), the Euclidean space is replaced by the Minkowski's spacetime, but notice that the former is still the space where local experimental observations happen. Any relativistic physical phenomenon is captured by some suitable experimental apparatus living in the Euclidean space. This doesn't mean that the observed relativistic phenomenon happened in the Euclidean space. If we try to explain the observed data without invoking the Minkowski's spacetime, then the behavior of the relativistic system appears to be strange, weird and counter-intuitive as already pointed out earlier with an example.

Similarly in Eq. (6), the Euclidean space is replaced by the quantum mechanical Hilbert space. However, if the Hilbert space is not recognized as the underlying physical space akin to the Minkowski spacetime in Eq. (2), then the observed quantum phenomena surely force to infer the quantum mechanics as strange, weird and counter-intuitive.

As mentioned earlier, accepting Minkowski's space as the underlying physical space - where the relativistic phenomena happen - does not rule out the existence of Euclidean space where the experimental observations happen. Similarly, accepting Hilbert space as the underlying physical space - where the quantum phenomena happen - does not rule out the existence of Euclidean space where the experimental observations of eigenvalues happen. This can be easily seen from Eq. (7), where the set of eigenstates $\{|\psi>\}$ spans the Hilbert space whereas the set of eigenvalues $\{E\}$ is detected in the Euclidean space.

In conclusion, any given physical phenomenon, depending on the range of physical parameters, may differ from itself occuring in the classical regime. This is not because the locally defined classical properties describing the physical phenomenon are becoming invalid, but the space, where the physical phenomenon is happening, is no more the same as in the classical regime.

The true nature of physical space is not accessible to the direct experimental observations independent of the material phenomena happening in it. It's very important to identify the appropriate physical space where a given physical phenomenon happens [1–10].

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