The Undecidable Charge Gap and the Oil Drop Experiment

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Abstract

Decision problems in physics have been an active field of research for quite a few decades resulting in some interesting findings in recent years. However, such research investigations are based on a priori knowledge of theoretical computer science and the technical jargon of set theory. Here, I discuss a particular, but a significant, instance of how decision problems in physics can be realized without such specific prerequisites. I expose a hitherto unnoticed contradiction, that can be posed as a decision problem, concerning the oil drop experiment and thereby resolve it by refining the notion of "existence" in physics. This consequently leads to the undecidability of the charge spectral gap through the notion of "undecidable charges" which is in tandem with the completeness condition of a theory as was stated by Einstein, Podolsky and Rosen in their seminal work. Decision problems can now be realized in connection to basic physics, in general, rather than quantum physics, in particular, as per some recent claims.

1 Introduction

Considering Einstein's viewpoint[1], "*Physics constitutes a logical system of thought.... The justification (truth content) of the system rests in the proof of usefulness of the resulting theorems on the basis of sense experiences, where the relations of the latter to the former can only be comprehended intuitively.*", where "sense experiences" means "human experience" that occur through "experiment and measurement"[2]. Hilbert, in his sixth problem, called for *Mathematical Treatment of the Axioms of Physics*[3][1] in search of rigour and precision of reasoning in physics from the stand point of mathematical logic[4] by considering physics, like Einstein, as "*a logical system of thought*". Now, it is quite well known today, due to Goedel[12], that any logical system contains *undecidable* statements i.e. statements which can not be decided to be true or false within that concerned logical system – also termed as *decision problem* as per Hilbert and Ackermann[4]. Naturally one can wonder whether there are undecidable statements or decision problems in physics.

As a matter of fact, decision problems in physics have been a matter of investigation since a few decades and continues to be an active field of research[13, 14, 15, 16, 18, 17, 19, 20, 21, 22, 23]. However, these works are mostly rooted to the notion of Turing's computability and *halting problem* associated with his "automatic machines" (that excluded "choice machines") that was indeed an application of a decision problem, but restricted to the issue of computability of numbers[24, 25]. Therefore, the research investigations of refs.[13, 14, 15, 16, 18, 17, 19, 20, 21, 22, 23] are based upon the knowledge of computer science and the technical jargon of set theory which itself is fraught with inherent inconsistencies of reasoning[26].

As I plan to discuss here, decision problems in physics can be realized, without requiring any knowledge of computer science and the intricacy of set theory, in a much more elementary way by investigating the language in which experimental experience is expressed to tally with the theory. I demonstrate my viewpoint by pointing out a hitherto unnoticed contradiction, which can be posed as a decision problem, in the oil drop

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[1] In modern days one generally terms Hilbert's call as "*axiomatization of physics*", which necessarily involves investigations concerning the "*semantics of physics*"[5] or, in simple words, the logic and language of physics[6, 7]. Significance of such investigations has always been realized[8, 9, 10], which has given birth to serious problems in mathematical science[11].

experiment and thereby resolving it by *intuitively* refining the notion of "existence", in physics in general, and of charge in particular, leading to the undecidability of the charge spectral gap. Here, the word "intuition" means the intellect with which we construct reason, take decisions, make choices and therefore, that remains at the foundation of our thought and goes beyond formalism[28]. The process of reasoning is in tandem with the completeness condition of a theory as was stated by Einstein, Podolsky and Rosen (EPR)[2].

The structure of this article can be debriefed as follows. In Section(2), I focus on the relevant literature to expose the difference in approach, to deal with decision problems in physics, with the present work. Consequently, this provides the motivation for the present discussion. In Section(3), I revisit the definition of electric field from a logical point of view and, in view of this, I point out how the conclusion of the oil drop experiment contradicts with its premise so as to give rise to a decision problem. In Section(4), I discuss how such a decision problem can be resolved by refining the notion of "existence" in physics, which nevertheless leads to the notion of "undecidable charges". Also, I point out how such a resolution of the decision problem is in tandem with the completeness condition, of a theory, that was stated by EPR in their seminal article[2]. Finally, in Section(5), I conclude with some comments.

2 Salient features of and recent trends concerning decision problems in physics

Decision problems or undecidability in classical physics (to mention, classical mechanics and chaotic systems) had been discussed earlier in refs.[14, 15, 16, 17, 13], which however require a fairly detailed knowledge of set theory and mathematical logic, relating to the works of Goedel[12] and also to the works of Turing[24, 25] that form the basis of theoretical computer science. Such knowledge has hardly any, a priori and explicit, connection to basic graduate level textbook physics which the physicists study in the usual practice. Thus, such literature, in spite of being novel in its own right, does not effectively connect decision problems to the logical structure of the very basic theoretical physics that comes under everyday practice.

Such observations also apply to a more recent section of literature, namely refs.[19, 20, 21, 22, 23], which however manifest a special feature of connecting decision problems uniquely to quantum physics as I point out in what follows.

- Refs.[19, 20] convey the message that undecidable problems have some particular connection to quantum physics only. For example, the authors of ref.[20] write in the abstract itself that "*apparently simple problems in quantum measurement theory can be undecidable even if their classical analogues are decidable. Undecidability hence appears as a genuine quantum property here.*"

- Refs.[21, 22, 23], epitomized by the mass gap problem of ref.[11], convey the message that the spectral gap problem is uniquely associated with quantum theory. Thus, the fact that it is an undecidable problem, which is the central point of emphasis of refs.[21, 22, 23], attains its validity only in association with quantum physics.

Such philosophy of connecting decision problems uniquely to quantum mechanics has its roots in a popular book of Penrose[18] which was nevertheless counteracted to certain extent in ref.[15].

While such works are interesting and novel in their own rights, I intend to explicate how decision problems in physics can be realized in a more elementary and direct fashion, without requiring the specific knowledge of set theory and computer science but, through a truthful analysis of the language in which a theory is written by the physicist to express it as a logical system of thought so as to intuitively comprehend what physical reality is in terms of experiment[1] – a process that Brouwer might have called "inner inquiry"[27] or I may call self-inquiry[6, 7, 29]. Such an investigation is expected to provide a different view point towards the question of undecidable problems in physics.

Following Hilbert and Ackermann, from p.112 of ref.[4], I may call a "*decision problem*" as a "*problem of the universal validity*" of some formal statement i.e. I investigate whether we can keep a formal statement to be universally valid throughout a process of reasoning while constructing a theory of physics and drawing experimental conclusions from that. Therefore, if I consider the notion of a "decision problem" associated with Turing's computability as algorithmic and artificial in nature, then my consideration of the notion of "decision problem" is founded on intuitive refinement of the epistemological rigour of the axioms themselves which are stated by us (humans). While the former is outward inquiry whose purpose is to fit the input of a machine like that of Turing, the latter is inward inquiry that is performed by a human so as to choose and refine the axioms through the execution of intellect and intuition. If the works of refs.[14, 15, 16, 17, 18, 19, 20, 21, 22, 23] concern what is computable or not in physics from the computer science point of view, then the present discussion concerns how logical the foundations of physics are as far as its language (i.e. the axioms, postulates, definitions, etc.) is concerned. With such clarifications, I intend to discuss that the spectral gap problem is undecidable in its own right. Neither the spectral gap problem nor the problem of undecidable statements in physics has a priori any unique connection to quantum physics; rather decision problems are part of human reasoning and therefore, plague science as a whole e.g. see refs.[37, 38, 39] apart from ref.[4] for the general scenario and see refs.[40, 41, 42] for particular elementary examples. I expose my viewpoints through an analysis of the principles that underlie the oil drop experiment[35, 36], which is expected to impact the reasoning of basic physics that is a concern of everyday practice of the physicist.

3 The oil drop experiment and a decision problem

To elucidate my concern, I consider the example of the famous oil drop experiment that, was performed by Millikan[35] and Fletcher[36] and, can be considered as one of the most important experimental epitomes of modern physics. I discuss how the interpretation of the oil drop experiment is founded on a hitherto unnoticed contradiction, which can nevertheless be posed as a decision problem.

3.1 Definition of electric field

I begin by revisiting the standard definition of electric field so as to highlight the constituent logical elements. The definition of electric field is given by considering the Coulomb's law as a premise that is stated as follows. The magnitude of the force (F) between two objects having charges (q_1, q_2), which are at rest with respect to each other, is proportional to the product of the two charges and inversely proportional to the square of the distance between those two point charges. This statement is expressed as

$$F = k\frac{q_s q_t}{r^2} \tag{1}$$

where k is a proportionality constant that needs to be fixed by experiment and r is the distance between the two charges, each considered as a point. Now, the electric field due to q_s (source charge), is *defined* as

$$E(q_s) := \lim_{q_t \to 0} \frac{F}{q_t} \quad \ni F = k\frac{q_s q_t}{r^2}. \tag{2}$$

q_t is called the test charge (e.g. see page no. 16 of ref. [31], page no. 29 of ref.[32], page no. 25 of ref.[33]). The symbolic statement "$\lim_{q_t \to 0}$" conveys the meaning that "*the test charge q_t is chosen to be arbitrarily small*". This requirement of the limiting condition is necessary for the definition of an electric field due to any arbitrary charge configuration or distribution as well[2]. Millikan used the concept of electric field explicitly in ref.[35] – he called it "electric field strength" and denoted it by the symbol "F", rather than the symbol "E" that I use here according to the modern convention.

3.2 A contradiction or a decision problem

Now, having revisited the definition of electric field, I consider the following statement whose logical truth is to be investigated.

M: *Arbitrarily small charge does, hence a charge gap does not, exist.*

The decision problem that concerns the oil drop experiment is posed through the following question:

Q: *Can it be decided whether* **M** *is true or false, considering the interpretation of the oil drop experiment to be a logical truth?*

[2] Here, I do not digress to a discussion regarding the reason behind using this limiting condition on the test charge, for which one may consult ref.[6], and rather treat the condition as a logical truth that plays a role in the definition of electric field.

To explain that the answer to **Q** is in the negative, I point out the following two possibilities, of which the first one concerns the conclusion and the second one concerns the premise of the oil drop experiment.

- **M** *is false. It is a conclusion of the oil drop experiment that a smallest charge, hence a charge gap, exists.*
- **M** *is true. Electric field can be defined if and only if arbitrarily small charge does, hence a charge gap does not, exist.*

Since the theoretical analysis of the oil drop experiment in ref.[35] is based on the use of the concept of "electric field" (i.e. truth of **M**) and the respective interpretation of the experimental observations lead to the conclusion of the existence of a smallest ("elementary") charge (i.e. falsehood of **M**), hence, the whole process of reasoning associated with the oil drop experiment does not let us decide whether a charge gap exists or not because **M** is both true and false in the same process of reasoning that underlies the interpretation of the oil drop experiment. Therefore, a hitherto unnoticed contradiction underlies the interpretation of the oil drop experiment. The situation can be viewed in an alternative way. That is, the following scenario arises if one decides to consider any one of the above two answers to be universally valid throughout the whole process of reasoning.

- **Decision 1 (D1): M** is false. Therefore, the conclusion of the oil drop experiment is logically true, but the premise i.e. the definition of electric field, that leads to such a conclusion, is logically false.
- **Decision 2 (D2): M** is true. The conclusion of the oil drop experiment is logically false.

D1 renders the notion of "electric field" to be undefined and, hence, the symbol "E" meaningless. **D2** jeopardizes the logical validity of the conclusion of the oil drop experiment. Thus, in totality, the logic underlying the interpretation of the oil drop experiment is plagued by a decision problem and this explains why the answer to **Q** is in the negative.

4 Resolution of the contradiction or the decision problem

Now, the matter of investigation is to find a way to resolve the contradiction or, equivalently, to find a solution to the decision problem. I propose a possible way to do so by refining the statement **M** itself. I begin by dismissing the validity of the question **Q** by claiming the *incompleteness* of **M**. To be more specific, the notion of "*existence*" is not completely specified in **M**. Therefore, **M** *is neither true nor false*. This renders **Q** to be a logically invalid question and then there is no decision to be made i.e. no decision problem. So, the necessary task is to *complete* the statement **M** by specifying the *type* of *existence*. To do that, at first, I need to consider the following postulate.

Postulate on Existence ($\mathbf{P}_{EX}$): *There are two types of "existence" in physics. One is "theoretical existence" which is what we assume to exist in terms of language and expressions to write a theory. The other one is "experimental existence" which is what we can experimentally verify considering the theory to be a mean of interpretation of the experiment.*

From $\mathbf{P}_{EX}$ it is now evident how **M** is a logically incomplete statement and that **M** can be completed in two ways, leading to two distinct statements as follows.

- $\mathbf{M}_T$**:** *Arbitrarily small charge does, hence a charge gap does not, exist theoretically.*
- $\mathbf{M}_E$**:** *Arbitrarily small charge does, hence a charge gap does not, exist experimentally.*

Thus, **M** loses any meaning in itself and it is now refined into two different statements $\mathbf{M}_T$ and $\mathbf{M}_E$.

Now, by $\mathbf{P}_{EX}$ and $\mathbf{M}_T$ the test charge for the definition of electric field can be chosen arbitrarily small in theory. When the experiment yields the smallest detectable charge, say e, then $\mathbf{M}_E$ is falsified. Thus, in the whole process of reasoning, that underlies the interpretation of the oil drop experiment, $\mathbf{M}_T$ is true and $\mathbf{M}_E$ is false. There is neither any contradiction nor any decision problem.

4.1 Undecidable Charges and the EPR Completeness Condition

Now, I may note that theoretically there exist charges (Q_u-s), by $\mathbf{M}_T$, such that for any Q_u the following condition holds: $Q_u < e$, or equivalently, $Q_u = \epsilon_u e \ \ni 0 < \epsilon_u < 1$.

Since e represents the charge such that any charge smaller than e can not be experimentally detectable and hence, the experimental existence of Q_u-s can neither be proven true nor be proven false i.e. existence of Q_u-s is experimentally undecidable.

Now, there remains the question of theoretical undecidability i.e. considering $\mathbf{M}_T$, or theoretical existence of Q_u-s, as a "hypothesis" whether it can be proven to be true or false within the theory. This question can be answered as follows. Using $\mathbf{M}_T$, Q_u-s are "hypothesized" to define electric field, founded on which is the theoretical construction. Therefore, the theoretical construction is built upon the "hypothesis" of Q_u-s i.e. $\mathbf{M}_T$. Now, if one wonders whether there is a theoretical proof or disproof of this "hypothesis", the answer is obviously negative in either cases. This is because the "hypothesis" is the only reason that let us write down the theory itself by avoiding any logical fallacy. Any attempt either to prove or to disprove the "hypothesis" theoretically is like using the premise to either prove or disprove itself. A successful proof of the "hypothesis" leads to a useless tautology:

"*Q_u exists theoretically, if the theoretical existence of Q_u is hypothesized to construct the theory.*"

And, a successful disproof of the same leads to a useless contradiction:

"*Q_u can not exist theoretically, if the theoretical existence of Q_u is hypothesized to construct the theory.*"

Therefore, the "hypothesis" is theoretically neither provable nor unprovable i.e. undecidable. This is why the theoretical existence of the undecidable charges (Q_u-s) is a undecidable hypothesis i.e. a postulate. It is only such a postulate that lets us logically define the electric field and construct the theory which in turn provides the underlying framework to analyze the experimental data of the oil drop experiment that leads to the experimental existence of e.

Now, let me explain how $\mathbf{M}_T$ and $\mathbf{M}_E$, and hence the postulate of Q_u-s (undecidable charges) is actually a different way of stating the completeness condition of a "*physical*"[3] theory as was written by EPR[2], which I may call EPR's Completeness Condition (ECC):

"*every element of the physical reality must have a counterpart in the physical theory*".

It is quite obvious that ECC is a logical implication rather than a logical equivalence i.e. *every element of the theory may NOT have a counterpart in the physical reality.* This can be realized as the fact that the undecidable charges are not parts of the physical reality that can be realized through experiment and measurement. However, these undecidable charges are required to define the concept of "electric field" and hence, to construct the theory that underlies the explanation of the experimental data of the oil drop experiment leading to the conclusion about what physical reality is.

5 Conclusion

Decision problems are not only part of mathematical logic[4], but also part of human reasoning in general[37, 38, 39]. While the mathematical treatment of the axioms of physics is required for clarity of reasoning[3], intuitive refinement of the elements of logic is also required for precision of expressions that directly correspond to human "sense experiences"[1] – a process that may be termed as "inner inquiry"[27] or self-inquiry[6, 7, 29]. The resolution of the hitherto unnoticed decision problem, which plagues the underlying explanation of the oil drop experiment, showcases one such example where a decision problem is resolved through intuitive refinement of a postulate regarding the existence of quantity by categorizing the notion of "existence" into two *types*. Also, such refinement is in tandem with the completeness condition of a theory that was stated by Einstein, Podolsky and Rosen[2]. In view of this it becomes strongly manifest that decision problems in physics and the undecidability of the spectral gap have a priori no connection to quantum mechanics unlike what has been the underlying message of refs.[19, 20, 21, 22, 23] (although the other aspects of such

[3] Unlike EPR, I remain more conservative by not judging, a priori, any theory to be physical or unphysical and let it remain open to be decided by the experimental verification. Therefore, in contrast to EPR, I drop the adjective "physical" and consider the word "theory" instead of "physical theory".

literature are novel in their own rights). While decision problems in general physics have also been discussed earlier in ref.[14, 15, 16, 17], alongside the more recent refs.[19, 20, 21, 22, 23], but the present work differs by its demonstration of the fact that decision problems in physics can be understood without the prerequisites of theoretical computer science and the technical jargon of set theory (which itself is fraught with inherent logical inconsistencies[26]). While this work showcases a decision problem that directly relates to basic physics, on the other hand, the refinement of the notion of "existence" in physics now can potentially open the door for new questions concerning the definition of "quantity" as far as the foundations of physics and, in particular, metrology is concerned[43]. I plan to discuss such issues in near future.

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