Lecture Notes on Symmetry Optics

Appendix to Lecture 8

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1 Introduction

Lecture 8 discussed the beam from a symmetry-optical perspective. Most of the concepts in that lecture also have close analogs in the conventional theory of the Gaussian beam. This appendix shows the quantitative similarities and differences between the two theories.

2 Two concepts of gradual change

2.1 Gaussian beam and Fractional Fourier Transform

The Fractional Fourier transform[1,2,3] is a conventional theory describing how patterns of light gradually change as the light propagates away from the flat. For a Gaussian beam, the order of the Fractional Fourier transform (FracFT) is equal to the value of the *Gouy phase*.

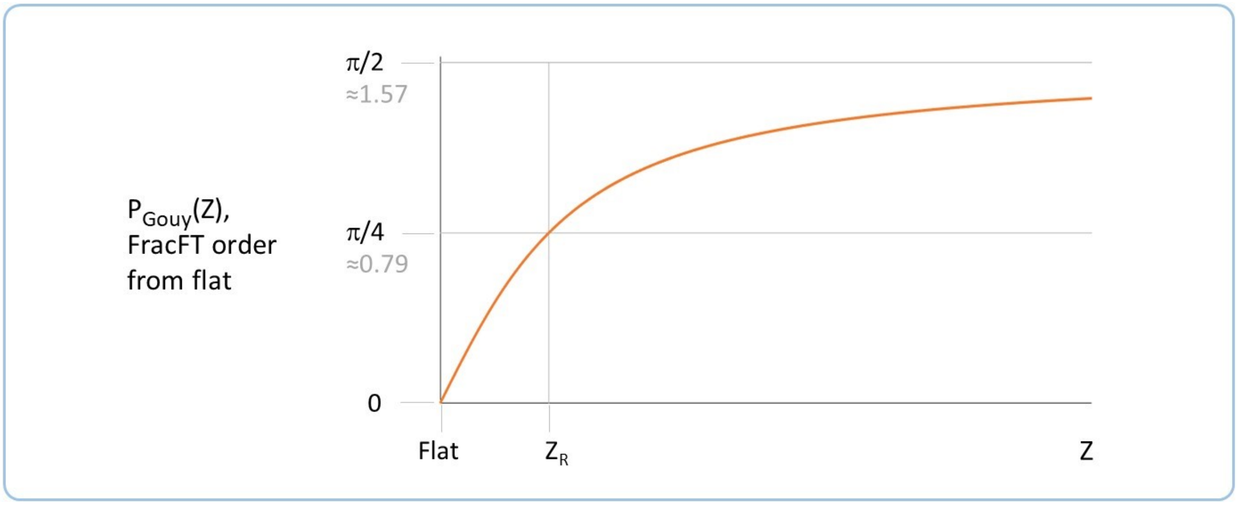
The wavefronts of an ideal plane wave are spaced exactly λ apart in Z. However, the wavefronts of a Gaussian beam are displaced axially from their ideal planes; this discrepancy is the Gouy phase. When the beam waist (flat) is located at Z = 0, and with Z_R denoting the Rayleigh range (elbow),

$$P_{Gouy}(Z) = \arctan(Z / Z_R)$$

The FracFT deals with the pattern of the light in a transverse (X-Y) plane, rather than any axial (Z) displacement; so, the definition of Gouy phase given above does not give a clear physical interpretation of the FracFT. Nonetheless, that is how FracFT is quantified in the Gaussian beam.

Figure 2.1 shows the Gouy phase as a function of Z. At the flat, $P_{Gouy} = 0$. At the elbow (Rayleigh range), $P_{Gouy} = \pi/4$. The full FT occurs when $P_{Gouy} = \pi/2$; however, the beam approaches this level only asymptotically.

Figure 2.1, Gouy phase and FracFT order in the Gaussian beam

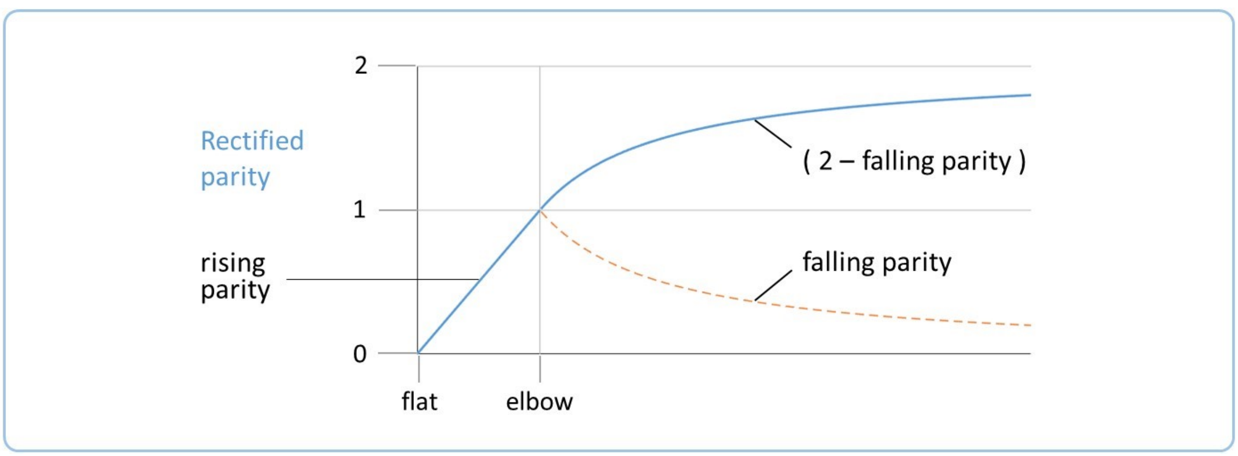


2.2 Provolution

Provolution is quantified by *parity* between roundness and width. Parity is a piecewise function, consisting of a rising stage and a falling stage.

To facilitate a comparison with Gouy phase, we introduce the *rectified parity*, which is shown in Figure 2.2. It too is formed piecewise from two segments. The first segment is simply the rising parity, and in this region the rectified parity and ordinary parity are identical.

Figure 2.2, Rectified parity, formed from parity



For the second segment, the sense of change is flipped so that rectified parity *rises* as ordinary imparity *falls*. Rectified parity after the elbow resembles the falling parity, reflected across a horizontal line passing through 1. As falling parity asymptotically approaches 0 at infinite Z, the rectified parity asymptotically approaches its final value of 2, which represents a complete FT from the flat.

The rectified parity R(z) can be expressed as

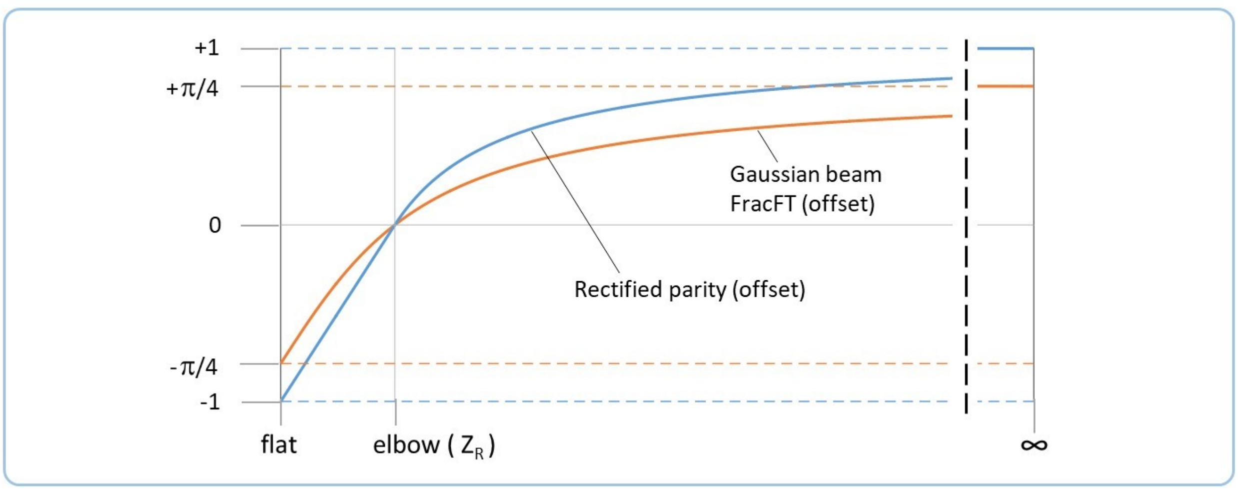
$$R(Z)_{Z \leq Z\_Elbow} = P(Z)$$

$$R(Z)_{Z > Z\_Elbow} = 2 - P(Z)$$

2.3 Comparison

Figure 2.3 shows the fidelity of the agreement between the Gouy phase and rectified parity. To further facilitate the comparison, both functions are offset vertically to take value 0 at the elbow. Both at Z = 0 and (asymptotically) at Z = ∞, the two functions have the same slope. However, the values of rectified parity at those extremes is greater than the Gouy phase by a correction factor π/4, which appears as a discrepancy in many similar approximations in symmetry optics.

Figure 2.3, Comparing Gaussian and FO models



Note when comparing these two functions that the distance Z may be expressed two different ways: equations describing the Gaussian beam usually use units of length (m, mm, nm), while parity and all other values in symmetry optics computations are always normalized to wavelength, so that values are dimensionless.

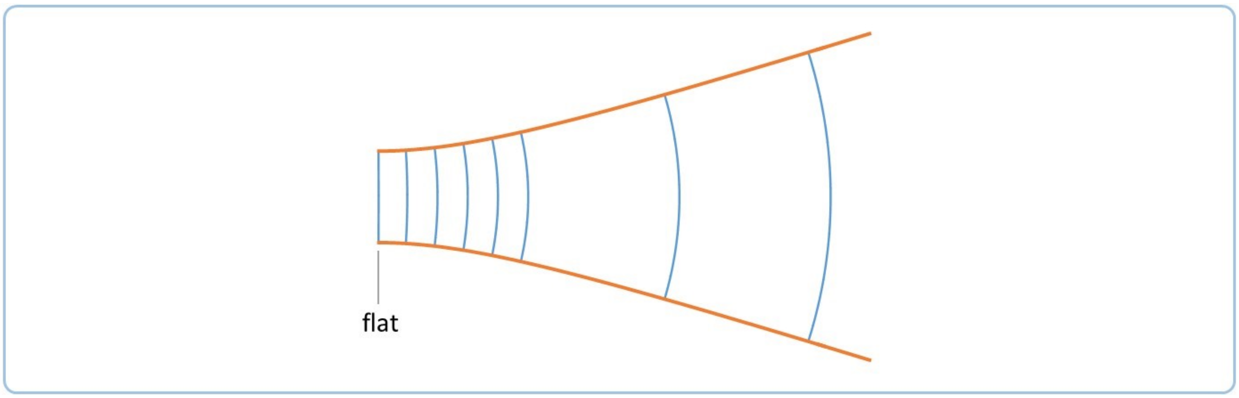
The equation for Gouy phase is simply a scaled form of the arctangent function. Therefore, the concept of rectified parity is actually applicable far beyond the context of symmetry optics; it is simply an approximation to the arctangent function.

3 Curvature

3.1 Gaussian beam curvature

The Gaussian beam is named for its transverse profile, whose intensity has a Gaussian distribution. However, we are mostly concerned with other properties of the beam and how they vary along the propagation axis Z, not along the transverse axis. The beam has a flat wavefront at the waist (flat), but in any other plane the wavefront is curved like a portion of a notional sphere with some radius, as shown in Figure 3.1.

Figure 3.1, Curved wavefronts in the Gaussian beam



This radius of curvature varies with propagation in Z following the equation

$$R_{\text{Gaussian}}(Z) = Z \cdot \left[1 + \left(\frac{Z_R}{Z}\right)^2 \right]$$

The curvature is simply the inverse of the radius.

$$C_{\text{Gaussian}}(z) = \frac{1}{R_{\text{Gaussian}}(Z)}$$

3.2 Symmetry-optical curvature

In symmetry optics, curvature and radius of curvature are not essential concepts. However, they are related to essential symmetry concepts in a simple way.

First, we take the functions f(Z) and g(Z) to be the two component functions which are combined piecewise to form parity:

$$f(z) = \frac{\text{live}(Z)}{\text{static}(Z)}$$

$$g(z) = \frac{\text{static}(Z)}{\text{live}(Z)}$$

When these functions are added and scaled, they equal the radius of curvature.

$$R_{\text{Symmetry}}(Z) = [\, f(z) + g(z) \,] \cdot Z_{\text{Elbow}} \cdot \lambda$$

Note that $R_{Symmetry}(Z)$ and $R_{Gaussian}(Z)$ are exactly the same values, and one equation can be derived from the other; the only difference is that the symmetry version suggests an interpretation in which ratios of static and live are important.

Note also that symmetry optics fails to account for negative curvature, since static stripe and live stripe are always positive quantities.

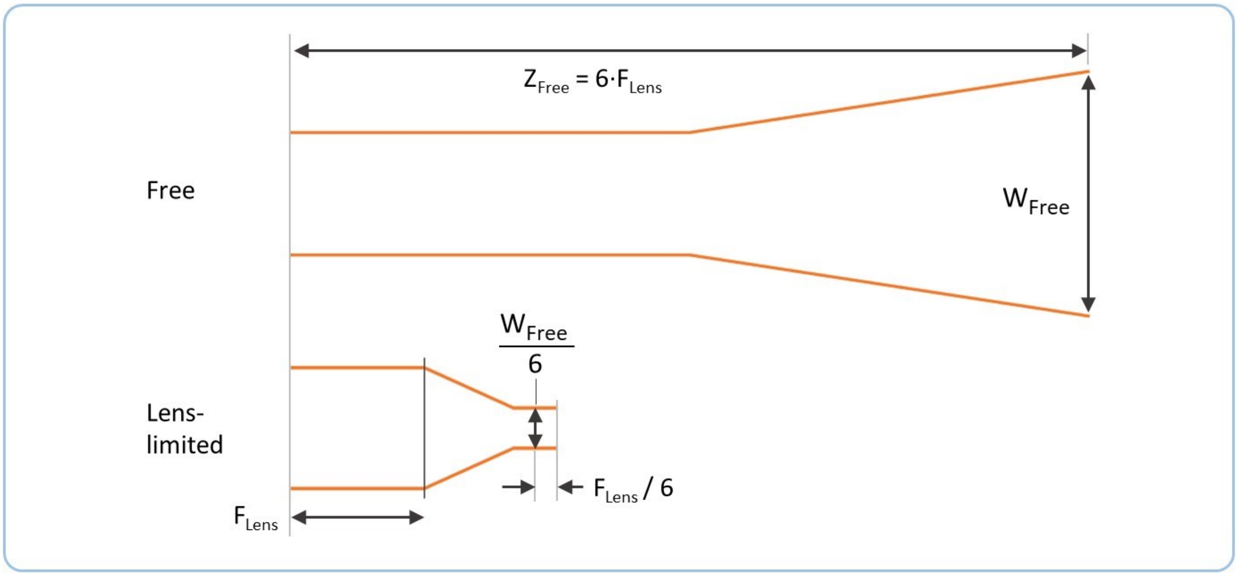
4 Demagnification by the lens

The lecture video showed that for every plane in the free configuration, there always exists a corresponding plane with the same parity in the lens-limited configuration.

In addition, the pattern in the lens-limited configuration is always a demagnified image of the free-configuration pattern. The image is real; the object pattern is virtual. This is true not only for the beam, but also for many-slit interference or any other arbitrary pattern.

The demagnification factor is the inverse of the lens overshoot. As an example, Figure 4.1 shows the width of the free beam W_{Free} in the plane which overshoots the lens plane by a factor of 6. When an identical beam passes through a lens, the corresponding plane is demagnified by a factor of 1/6. Also, as discussed in the lecture, the plane is located at $F_{Lens}/6$ from the rear focal plane.

Figure 4.1, Demagnification in the lens-limited configuration



5 References

1. Ozaktas, H.M. and Mendlovic, David. Fractional Fourier transform as a tool for analyzing beam propagation and spherical mirror resonators. Optics Letters, Vol. 19, No. 21, 1994

2. Ozaktas, H.M. and Mendlovic, David. Fractional Fourier Optics, J. Opt. Soc. Am. A, Vol. 12, No. 4, April 1995

3. Ozaktas, H.M., Zalevsky, Zeev, and Kutay, M. Alper. The Fractional Fourier Transform with Applications in Optics and Signal Processing. Wiley and Sons, 2001