Chiral Symmetry of Neutrino

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Abstract

In this paper (i) is given a new representation for gamma matrices in which is confirmed theoretically the absence of positive helicity neutrino and respectively negative helicity antineutrino, (ii) is proved the equivalence of Dirac equation for mass m with Proca equation for mass $2m$, and (iii) is proposed a discrete symmetry group for weak and strong interactions built with 4 unitary and 4 nilpotent operators. The cosmological constant predicted by theory is $\Lambda = 2\pi G(c/\hbar)^3 m^4$, where m is neutrino mass.

1 Introduction

Let $\mathscr{A}_1$, $\mathscr{A}_2$, $\mathscr{B}_1$ and $\mathscr{B}_2$ be four anticommuting complex Lorentz transformations, two being Hermitian and two anti-Hermitian

$$\begin{aligned} \mathscr{A}_1{}^\dagger &= \mathscr{A}_1 \quad , & \mathscr{B}_1{}^\dagger &= -\mathscr{B}_1 \quad , \\ \mathscr{A}_2{}^\dagger &= \mathscr{A}_2 \quad , & \mathscr{B}_2{}^\dagger &= -\mathscr{B}_2 \quad , \end{aligned} \tag{1}$$

and

$$\mathscr{A}_1{}^2 = \mathscr{A}_2{}^2 = 1 \quad , \qquad \mathscr{B}_1{}^2 = \mathscr{B}_2{}^2 = -1 \quad . \tag{2}$$

The four complex Lorentz transformations are explicitly given by

$$\begin{aligned} \mathscr{A}_1 &= \begin{pmatrix} 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad , & \mathscr{B}_1 &= \begin{pmatrix} 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \quad , \\ \mathscr{A}_2 &= \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 1 \\ i & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad , & \mathscr{B}_2 &= \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & -1 \\ i & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad , \end{aligned} \tag{3}$$

and used to define four contravariant gamma matrices of our representation

$$\gamma^0 = \mathscr{A}_1 \quad , \qquad \gamma^1 = i\ \mathscr{A}_2 \quad , \qquad \gamma^2 = \mathscr{B}_2 \quad , \qquad \gamma^3 = \mathscr{B}_1 \quad . \tag{4}$$

The positive energy plane wave solutions $\Phi_1(p)$ and $\Phi_2(p)$ of Dirac equations are

$$\Phi_1(p) = \frac{1}{[2\varepsilon(\varepsilon + p_3)]^{1/2}} \begin{pmatrix} m \\ -i(\varepsilon + p_3) \\ p_1 - ip_2 \\ 0 \end{pmatrix} e^{-ipx} \quad , \qquad \Phi_2(p) = \frac{1}{[2\varepsilon(\varepsilon - p_3)]^{1/2}} \begin{pmatrix} 0 \\ p_2 - ip_1 \\ \varepsilon - p_3 \\ m \end{pmatrix} e^{-ipx} \quad , \tag{5}$$

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with

$$\mathbf{H}\Phi_1(p) = +\varepsilon\,\Phi_1(p) \quad , \qquad \mathbf{H}\Phi_2(p) = +\varepsilon\,\Phi_2(p) \quad , \tag{6}$$

and the negative energy plane wave solutions $\Psi_1(p)$ and $\Psi_2(p)$ are

$$\Psi_1(p) = \frac{1}{[2\varepsilon(\varepsilon+p_3)]^{1/2}} \begin{pmatrix} p_1+ip_2 \\ 0 \\ m \\ -\varepsilon-p_3 \end{pmatrix} e^{+ipx} \quad , \qquad \Psi_2(p) = \frac{1}{[2\varepsilon(\varepsilon-p_3)]^{1/2}} \begin{pmatrix} -i(\varepsilon-p_3) \\ m \\ 0 \\ p_2+ip_1 \end{pmatrix} e^{+ipx} \quad , \tag{7}$$

with

$$\mathbf{H}\Psi_1(p) = -\varepsilon\,\Psi_1(p) \quad , \qquad \mathbf{H}\Psi_2(p) = -\varepsilon\,\Psi_2(p) \quad , \tag{8}$$

where $\mathbf{H} = \mathscr{A}_1\mathscr{A}_2\,\partial_1 - i\mathscr{A}_1\mathscr{B}_2\,\partial_2 - i\mathscr{A}_1\mathscr{B}_1\,\partial_3 + m\mathscr{A}_1$ denotes the Hamiltonian, and $\varepsilon = +\sqrt{\mathbf{p}^2+m^2}$.

2 Spinors under Charge Conjugation

We introduce two charge conjugation operators $\mathbf{C}_1$ and $\widetilde{\mathbf{C}}_2$ which transform positive energy spinors $\Phi_1(p)$ and $\Phi_2(p)$ into negative energy spinors $\Psi_1(p)$ and respectively $\Psi_2(p)$, and another two charge conjugation operators $\widetilde{\mathbf{C}}_1$ and $\mathbf{C}_2$ which transform negative energy spinors $\Psi_1(p)$ and $\Psi_2(p)$ into positive energy spinors $\Phi_1(p)$ and respectively $\Phi_2(p)$.

$$\begin{aligned} \Psi_1(p) &= \mathbf{C}_1\Phi_1(p) \quad , & \Phi_2(p) &= \mathbf{C}_2\Psi_2(p) \quad , \\ \Phi_1(p) &= \widetilde{\mathbf{C}}_1\Psi_1(p) \quad , & \Psi_2(p) &= \widetilde{\mathbf{C}}_2\Phi_2(p) \quad , \end{aligned} \tag{9}$$

where

$$\begin{aligned} \mathbf{C}_1 &= i\mathscr{A}_1\mathscr{B}_1\mathscr{B}_2\,\mathscr{C} \quad , & \mathbf{C}_2 &= -\mathscr{A}_1\mathscr{A}_2\mathscr{B}_1\,\mathscr{C} \quad , \\ \widetilde{\mathbf{C}}_1 &= -i\mathscr{B}_2\,\mathscr{C} \quad , & \widetilde{\mathbf{C}}_2 &= \mathscr{A}_2\,\mathscr{C} \quad , \end{aligned} \tag{10}$$

and $\mathscr{C}$ denotes the complex conjugation operator. Every charge conjugation operator is antilinear, acts on a wave function first by complex conjugation operator $\mathscr{C}$ and then by a unitary matrix. Under charge conjugation the absolute value of the inner product remains invariant, therefore it represent a symmetry operation. The charge conjugation operators all commute and the following relations holds

$$\begin{aligned} \mathbf{C}_1\,\mathbf{C}_1 &= \widetilde{\mathbf{C}}_1\widetilde{\mathbf{C}}_1 = -\mathscr{A}_1\mathscr{B}_1 \quad , & \mathbf{C}_2\,\mathbf{C}_2 &= \widetilde{\mathbf{C}}_2\widetilde{\mathbf{C}}_2 = +\mathscr{A}_1\mathscr{B}_1 \quad , \\ \mathbf{C}_1\widetilde{\mathbf{C}}_1 &= \widetilde{\mathbf{C}}_1\,\mathbf{C}_1 = 1 \quad , & \mathbf{C}_2\widetilde{\mathbf{C}}_2 &= \widetilde{\mathbf{C}}_2\,\mathbf{C}_2 = 1 \quad , \\ \mathbf{C}_1\widetilde{\mathbf{C}}_2 &= \widetilde{\mathbf{C}}_2\,\mathbf{C}_1 = -i\,\mathscr{A}_2\,\mathscr{B}_2 \quad , & \mathbf{C}_2\,\mathbf{C}_1 &= \mathbf{C}_1\,\mathbf{C}_2 = +i\,\mathscr{A}_1\,\mathscr{A}_2\,\mathscr{B}_1\,\mathscr{B}_2 \quad , \\ \widetilde{\mathbf{C}}_1\widetilde{\mathbf{C}}_2 &= \widetilde{\mathbf{C}}_2\widetilde{\mathbf{C}}_1 = -i\,\mathscr{A}_1\,\mathscr{A}_2\,\mathscr{B}_1\,\mathscr{B}_2 \quad , & \mathbf{C}_2\widetilde{\mathbf{C}}_1 &= \widetilde{\mathbf{C}}_1\,\mathbf{C}_2 = i\,\mathscr{A}_2\,\mathscr{B}_2 \quad . \end{aligned} \tag{11}$$

Using charge conjugation operators we can define, in similar fashion of shift operators of $\mathsf{SU}(3)$, two left and two right chiral conjugation operators

$$\begin{aligned} \mathbf{C}_\mathsf{L} &= \frac{1}{2}(\,\mathbf{C}_1 - i\widetilde{\mathbf{C}}_2\,) \quad , & \mathbf{C}_\mathsf{R} &= \frac{1}{2}(\,\mathbf{C}_1 + i\widetilde{\mathbf{C}}_2\,) \quad , \\ \widetilde{\mathbf{C}}_\mathsf{L} &= \frac{1}{2}(\widetilde{\mathbf{C}}_1 + i\,\mathbf{C}_2\,) \quad , & \widetilde{\mathbf{C}}_\mathsf{R} &= \frac{1}{2}(\widetilde{\mathbf{C}}_1 - i\,\mathbf{C}_2\,) \quad . \end{aligned} \tag{12}$$

The left chiral operators $\mathbf{C}_\mathsf{L}$, $\widetilde{\mathbf{C}}_\mathsf{L}$ together with $\mathscr{A}_1\,\mathscr{A}_2\,\mathscr{B}_1\,\mathscr{B}_2 = \gamma_5$ and $\mathscr{A}_2\,\mathscr{B}_2 = \mathbf{P}$ form a closed set under commutation, and similarly the right chiral operators $\mathbf{C}_\mathsf{R}$, $\widetilde{\mathbf{C}}_\mathsf{R}$ together with γ_5 and $\mathbf{P}$ form a closed set under commutation

$$\begin{aligned} [\,\gamma_5\,,\,\mathbf{C}_\mathsf{L}\,] &= -2\,\mathbf{C}_\mathsf{L} \quad , & [\,\gamma_5\,,\,\mathbf{C}_\mathsf{R}\,] &= +2\,\mathbf{C}_\mathsf{R} \quad , \\ [\,\gamma_5\,,\,\widetilde{\mathbf{C}}_\mathsf{L}\,] &= -2\widetilde{\mathbf{C}}_\mathsf{L} \quad , & [\,\gamma_5\,,\,\widetilde{\mathbf{C}}_\mathsf{R}\,] &= +2\,\mathbf{C}_\mathsf{R} \quad , \\ [\,\mathbf{P}\,,\,\mathbf{C}_\mathsf{L}\,] &= -2\widetilde{\mathbf{C}}_\mathsf{L} \quad , & [\,\mathbf{P}\,,\,\mathbf{C}_\mathsf{R}\,] &= +2\widetilde{\mathbf{C}}_\mathsf{R} \quad , \\ [\,\mathbf{P}\,,\,\widetilde{\mathbf{C}}_\mathsf{L}\,] &= -2\,\mathbf{C}_\mathsf{L} \quad , & [\,\mathbf{P}\,,\,\widetilde{\mathbf{C}}_\mathsf{R}\,] &= +2\,\mathbf{C}_\mathsf{R} \quad , \\ [\,\gamma_5\,,\,\mathbf{P}\,] &= 0 \quad , & [\,\gamma_5\,,\,\mathbf{P}\,] &= 0 \quad , \end{aligned} \tag{13}$$

which will be called the left algebra LA and respectively right algebra RA. The union of left algebra and right algebra is also a closed algebra under commutation, the following relations between left and right operators holds

$$\begin{aligned} &[\widetilde{\mathsf{C}}_{\mathsf{L}}\,,\,\mathsf{C}_{\mathsf{R}}\,] = -\gamma_5 \quad , && [\widetilde{\mathsf{C}}_{\mathsf{R}}\,,\,\mathsf{C}_{\mathsf{L}}\,] = +\gamma_5 \quad , \\ &[\,\mathsf{C}_{\mathsf{L}}\,,\,\mathsf{C}_{\mathsf{R}}\,] = -\mathsf{P} \quad , && [\widetilde{\mathsf{C}}_{\mathsf{R}}\,,\,\widetilde{\mathsf{C}}_{\mathsf{L}}\,] = +\mathsf{P} \quad . \end{aligned} \tag{14}$$

The parity P and chirality γ_5 are unitary, while left and right chiral operators are nilpotent

$$\mathsf{C}_{\mathsf{L}}^2 = \widetilde{\mathsf{C}}_{\mathsf{L}}{}^2 = 0 \quad , \qquad \mathsf{C}_{\mathsf{R}}^2 = \widetilde{\mathsf{C}}_{\mathsf{R}}{}^2 = 0 \quad , \tag{15}$$

antilinear and orthogonal

$$\mathsf{C}_{\mathsf{L}}\,\widetilde{\mathsf{C}}_{\mathsf{L}} = \widetilde{\mathsf{C}}_{\mathsf{L}}\,\mathsf{C}_{\mathsf{L}} = 0, \qquad \mathsf{C}_{\mathsf{R}}\,\widetilde{\mathsf{C}}_{\mathsf{R}} = \widetilde{\mathsf{C}}_{\mathsf{R}}\,\mathsf{C}_{\mathsf{R}} = 0 \quad . \tag{16}$$

The six elements set, namely two left chiral operators C_{L}, $\widetilde{\mathsf{C}}_{\mathsf{L}}$, two right chiral operators C_{R}, $\widetilde{\mathsf{C}}_{\mathsf{R}}$, P and γ_5 form a closed algebra under commutation relations 13 and 14. They are explicitly given by

$$\begin{aligned} \mathsf{C}_{\mathsf{L}} &= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \mathscr{C} \quad , & \mathsf{C}_{\mathsf{R}} &= \begin{pmatrix} 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & +i & 0 & 0 \end{pmatrix} \mathscr{C} \quad , \\ \widetilde{\mathsf{C}}_{\mathsf{L}} &= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +i \\ +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \mathscr{C} \quad , & \widetilde{\mathsf{C}}_{\mathsf{R}} &= \begin{pmatrix} 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{pmatrix} \mathscr{C} \quad , \\ \gamma_5 &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad , & \mathsf{P} &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad . \end{aligned} \tag{17}$$

3 Spinors under Spatial Reflection and Time Reversal

We explore the action of $\mathsf{P} = \mathscr{A}_2\,\mathscr{B}_2$ to the eigenfunctions of positive and negative energy

$$\begin{aligned} \mathsf{P}\Phi_1(p) &= +\Phi_1(\gamma_5\,p), & \mathsf{P}\Psi_1(p) &= -\Psi_1(\gamma_5\,p), \\ \mathsf{P}\Phi_2(p) &= -\Phi_2(\gamma_5\,p), & \mathsf{P}\Psi_2(p) &= +\Psi_2(\gamma_5\,p), \end{aligned} \tag{18}$$

and the action of $\mathsf{T} = i\,\mathscr{A}_1\,\mathscr{B}_2$ and $\widetilde{\mathsf{T}} = \mathscr{A}_1\,\mathscr{A}_2$ to the eigenfunctions of negative energy and positive energy, respectively

$$\begin{aligned} \mathsf{T}\Phi_1(p) &= -\Phi_2(\mathscr{A}_1\,\mathscr{B}_1\,p), & \widetilde{\mathsf{T}}\Psi_1(p) &= -\Psi_2(\mathscr{B}_2\,\mathscr{A}_2\,p), \\ \mathsf{T}\Phi_2(p) &= +\Phi_1(\mathscr{A}_1\,\mathscr{B}_1\,p), & \widetilde{\mathsf{T}}\Psi_2(p) &= +\Psi_1(\mathscr{B}_2\,\mathscr{A}_2\,p). \end{aligned} \tag{19}$$

While charge conjugation operators transform a spinor into another of opposite energy leaving its four-momentum unchanged, P transform a spinor with four-momentum $p = (\varepsilon, p^1, p^2, p^3)$ into the same spinor but with four-momentum $\gamma_5\,p = (\varepsilon, -p^1, -p^2, p^3)$. T acts on positive energy spinors by changing its four-momentum p into other spinor of positive energy and four-momentum $\mathscr{A}_1\mathscr{B}_1\,p = (-\varepsilon, p^1, -p^2, p^3)$. Similarly, $\widetilde{\mathsf{T}}$ acts on negative energy spinor by changing its four-momentum p into the other the spinor of negative energy and four-momentum $\mathscr{B}_2\mathscr{A}_2\,p = (-\varepsilon, -p^1, p^2, p^3)$. Operators T and $\widetilde{\mathsf{T}}$ are anti-Hermitian and satisfy $\mathsf{T}^2 = \widetilde{\mathsf{T}}^2 = -1$. They reverse the sign of energy $\boldsymbol{\varepsilon}$ and also of one spatial component of momentum, therefore we will denote time reversal operators as being T and $\widetilde{\mathsf{T}}$. Operator P is Hermitian, satisfy $\mathsf{P}^2 = 1$, reverse the sign of two components of momentum, therefore we will denote the parity operator as being P. The existence of two time reversal operators and the symmetry of left and right algebra 13 foresee the existence of two fundamental particles predicted by the Dirac equation. Parity and time reversal operators are anti-commuting and form a closed algebra under commutation

$$[\,\mathsf{T}\,,\,\widetilde{\mathsf{T}}\,] = 2i\,\mathsf{P} \quad , \qquad [\,\mathsf{P}\,,\,\mathsf{T}\,] = -2i\,\widetilde{\mathsf{T}} \quad , \qquad [\,\mathsf{P}\,,\,\widetilde{\mathsf{T}}\,] = 2i\,\mathsf{T} \quad . \tag{20}$$

We can define two nilpotent operators, i.e. raising T_+, lowering T_- and T_3 operator as

$$\mathsf{T}_3 = \frac{1}{2}\mathsf{P} \quad , \qquad \mathsf{T}_+ = \frac{1}{2}(i\widetilde{\mathsf{T}} - \mathsf{T}) \quad , \qquad \mathsf{T}_- = \frac{1}{2}(i\widetilde{\mathsf{T}} + \mathsf{T}) \quad , \tag{21}$$

which commutes with γ_5, fulfill the algebra $\mathsf{SU}(2)$ of the angular momentum operators

$$[\,\mathsf{T}_3\,,\,\mathsf{T}_+\,] = +\mathsf{T}_+ \quad , \qquad [\,\mathsf{T}_3\,,\,\mathsf{T}_-\,] = -\mathsf{T}_- \quad , \qquad [\,\mathsf{T}_+\,,\,\mathsf{T}_-\,] = 2\mathsf{T}_3 \quad , \tag{22}$$

and form a closed algebra together with P and γ_5. Explicitly, time reversal operators defined by 19 are given by

$$\mathsf{T} = \begin{pmatrix} 0 & 0 & 0 & +1 \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \quad , \qquad \widetilde{\mathsf{T}} = \begin{pmatrix} 0 & 0 & 0 & i \\ 0 & 0 & -1 & 0 \\ 0 & +1 & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix} \quad . \tag{23}$$

The properties of parity and time reversal operators shed light on the spatial geometry of particles predicted. For both particles, the action of parity operator to reverse the sign of $\mathbf{x}$ and $\mathbf{y}$, but not all three spatial coordinates as we would have expected, is suggesting that at any moment these entities exist only in $\mathbf{x}-\mathbf{y}$ plane and this bear a resemblance with oscillation of electromagnetic radiation in a plane perpendicular to the direction of motion.The action of T is to reverse the sign of $\mathbf{t}$ and $\mathbf{y}$ while the action of $\widetilde{\mathsf{T}}$ is to reverse the sign of $\mathbf{t}$ and $\mathbf{x}$. Following the resemblance with the electromagnetic radiation where electric and magnetic field oscillate in planes perpendicular one to another, the action of time reversal operators as described above is suggesting that each of the two entities described by Dirac equation exist in planes one perpendicular to the other, like electric and magnetic fields.

4 Left Chiral and Right Chiral Particles

All combinations of simultaneous charge conjugation, parity and time reversal operators are of fundamental importance

$$\begin{aligned} \mathsf{C}_1\mathsf{P}\,\mathsf{T} &= \mathscr{A}_1\,\mathscr{C} \quad , & \mathsf{C}_2\,\mathsf{P}\,\mathsf{T} &= i\gamma_5\,\mathscr{B}_1\,\mathscr{C} \quad , \\ \widetilde{\mathsf{C}}_1\mathsf{P}\,\mathsf{T} &= \mathscr{B}_1\,\mathscr{C} \quad , & \widetilde{\mathsf{C}}_2\,\mathsf{P}\,\mathsf{T} &= -i\gamma_5\,\mathscr{A}_1\,\mathscr{C} \quad , \\ \mathsf{C}_1\mathsf{P}\widetilde{\mathsf{T}} &= i\gamma_5\,\mathscr{B}_1\,\mathscr{C} \quad , & \mathsf{C}_2\,\mathsf{P}\widetilde{\mathsf{T}} &= -\,\mathscr{A}_1\,\mathscr{C} \quad , \\ \widetilde{\mathsf{C}}_1\mathsf{P}\widetilde{\mathsf{T}} &= i\gamma_5\,\mathscr{A}_1\,\mathscr{C} \quad , & \widetilde{\mathsf{C}}_2\,\mathsf{P}\widetilde{\mathsf{T}} &= \mathscr{B}_1\,\mathscr{C} \quad , \end{aligned} \tag{24}$$

because none lead to Stückelberg-Feynman particle-antiparticle interpretation of the solutions of Dirac equations, i.e. a particle of mass m is equivalent to an anti-particle of mass m traveling backward in spacetime. Squaring the relations 24 we obtain

$$\begin{aligned} (\mathsf{C}_1\mathsf{P}\mathsf{T})^2 &= (\mathsf{C}_1\mathsf{P}\widetilde{\mathsf{T}})^2 = -\mathsf{P} \quad , & (\mathsf{C}_2\,\mathsf{P}\,\mathsf{T})^2 &= (\mathsf{C}_2\,\mathsf{P}\widetilde{\mathsf{T}})^2 = -\mathsf{P} \quad , \\ (\widetilde{\mathsf{C}}_1\mathsf{P}\,\mathsf{T})^2 &= (\widetilde{\mathsf{C}}_1\mathsf{P}\widetilde{\mathsf{T}})^2 = +\mathsf{P} \quad , & (\widetilde{\mathsf{C}}_2\,\mathsf{P}\,\mathsf{T})^2 &= (\widetilde{\mathsf{C}}_2\,\mathsf{P}\widetilde{\mathsf{T}})^2 = +\mathsf{P} \quad , \end{aligned} \tag{25}$$

which is different from result [1] of Wigner who obtained -1_4, and also different from that of Weinberg [2] who obtained $+1_4$. The commutation relation $[\,\gamma_5\,,\,\mathsf{P}\,] = 0$ imply that eigenvalues of chirality γ_5 and parity P can be used to classify their common eigenstates. The -1 and $+1$ eigenvalues of γ_5 will be denoted L and R i.e. -1 correspond to left chiral L and $+1$ to right chiral R, while the eigenvalues -1 and $+1$ of P will be denoted as $-$ and $+$ i.e. $-$ for negative parity eigenvalue -1 and $+$ for positive parity $+1$. If Ψ is a solution of Dirac equation then we define the four common eigenstates of chirality and parity as:

$$\begin{aligned} \Psi_{+\mathsf{L}} &= \frac{\widetilde{\mathsf{C}}_\mathsf{L} - \mathsf{C}_\mathsf{L}}{2}\Psi \quad , & \Psi_{+\mathsf{R}} &= \frac{\widetilde{\mathsf{C}}_\mathsf{R} + \mathsf{C}_\mathsf{R}}{2}\Psi \quad , \\ \Psi_{-\mathsf{L}} &= \frac{\widetilde{\mathsf{C}}_\mathsf{L} + \mathsf{C}_\mathsf{L}}{2}\Psi \quad , & \Psi_{-\mathsf{R}} &= \frac{\mathsf{C}_\mathsf{R} - \widetilde{\mathsf{C}}_\mathsf{R}}{2}\Psi \quad . \end{aligned} \tag{26}$$

The action of parity and chirality operators on eigenstates are

$$
\begin{aligned}
\gamma_5\,\Psi_{+\mathsf{L}} &= -\Psi_{+\mathsf{L}} \quad , & \gamma_5\,\Psi_{+\mathsf{R}} &= +\Psi_{+\mathsf{R}} \quad , \\
\mathbf{P}\,\Psi_{+\mathsf{L}} &= +\Psi_{+\mathsf{L}} \quad , & \mathbf{P}\,\Psi_{+\mathsf{R}} &= +\Psi_{-\mathsf{R}} \quad , \\
\gamma_5\,\Psi_{-\mathsf{L}} &= -\Psi_{-\mathsf{L}} \quad , & \gamma_5\,\Psi_{-\mathsf{R}} &= +\Psi_{-\mathsf{R}} \quad , \\
\mathbf{P}\,\Psi_{-\mathsf{L}} &= -\Psi_{-\mathsf{L}} \quad , & \mathbf{P}\,\Psi_{-\mathsf{R}} &= -\Psi_{-\mathsf{R}} \quad .
\end{aligned} \tag{27}
$$

Starting with the complex conjugate of Dirac equation, we find that, for the eigenstates 26 the following coupled partial differential equations holds

$$
\begin{aligned}
&i(\gamma^0\frac{\partial}{\partial x^0}+\gamma^3\frac{\partial}{\partial x^3})\Psi_{+\mathsf{R}}+i(\gamma^1\frac{\partial}{\partial x^1}+\gamma^2\frac{\partial}{\partial x^2})\Psi_{-\mathsf{R}}-m\Psi_{+\mathsf{L}}=0\,,\\
&i(\gamma^0\frac{\partial}{\partial x^0}+\gamma^3\frac{\partial}{\partial x^3})\Psi_{-\mathsf{R}}+i(\gamma^1\frac{\partial}{\partial x^1}+\gamma^2\frac{\partial}{\partial x^2})\Psi_{+\mathsf{R}}-m\Psi_{-\mathsf{L}}=0\,,\\
&i(\gamma^0\frac{\partial}{\partial x^0}+\gamma^3\frac{\partial}{\partial x^3})\Psi_{+\mathsf{L}}+i(\gamma^1\frac{\partial}{\partial x^1}+\gamma^2\frac{\partial}{\partial x^2})\Psi_{-\mathsf{L}}-m\Psi_{+\mathsf{R}}=0\,,\\
&i(\gamma^0\frac{\partial}{\partial x^0}+\gamma^3\frac{\partial}{\partial x^3})\Psi_{-\mathsf{L}}+i(\gamma^1\frac{\partial}{\partial x^1}+\gamma^2\frac{\partial}{\partial x^2})\Psi_{+\mathsf{L}}-m\Psi_{-\mathsf{R}}=0\,.
\end{aligned} \tag{28}
$$

Introducing 16-dimmensional spinor $\Upsilon=(\Psi_{+\mathsf{L}},\Psi_{-\mathsf{L}},\Psi_{+\mathsf{R}},\Psi_{-\mathsf{R}})^{\mathsf{T}}$ and four 16×16 matrices

$$
\begin{aligned}
\Gamma^0 &= \begin{pmatrix} 0 & 0 & \gamma^0 & 0\\ 0 & 0 & 0 & \gamma^0\\ \gamma^0 & 0 & 0 & 0\\ 0 & \gamma^0 & 0 & 0\end{pmatrix}, &
\Gamma^1 &= \begin{pmatrix} 0 & 0 & 0 & \gamma^1\\ 0 & 0 & \gamma^1 & 0\\ 0 & \gamma^1 & 0 & 0\\ \gamma^1 & 0 & 0 & 0\end{pmatrix}, \\
\Gamma^2 &= \begin{pmatrix} 0 & 0 & 0 & \gamma^2\\ 0 & 0 & \gamma^2 & 0\\ 0 & \gamma^2 & 0 & 0\\ \gamma^2 & 0 & 0 & 0\end{pmatrix}, &
\Gamma^3 &= \begin{pmatrix} 0 & 0 & \gamma^3 & 0\\ 0 & 0 & 0 & \gamma^3\\ \gamma^3 & 0 & 0 & 0\\ 0 & \gamma^3 & 0 & 0\end{pmatrix},
\end{aligned} \tag{29}
$$

the coupled partial differential linear equations 28 can be written as a Dirac equations

$$
(i\Gamma^\alpha\partial_\alpha - m\,I_{16})\Upsilon = 0 \quad , \tag{30}
$$

where I_{16} is the 16×16 identity matrix and the Γ matrices satisfy the usual anticommutation relations $\Gamma^\alpha\Gamma^\beta+\Gamma^\beta\Gamma^\alpha = 2\,\mathsf{E}^{\alpha\beta} = 2\,diag(I_4,-I_4,-I_4,-I_4)$. Let Φ be the superposition of chirality and parity eigenstates

$$
\Phi = \Psi_{+\mathsf{L}}+\Psi_{-\mathsf{L}}+\Psi_{+\mathsf{R}}+\Psi_{-\mathsf{R}} \quad , \tag{31}
$$

wich is also a solution of Dirac equation. The solutions Φ and Ψ of Dirac equations are transformed one into another and vice-versa by the chirality and parity conjugation operator $\mathbf{C}$

$$
\begin{aligned}
\Phi &= \mathbf{C}\Psi \quad , \\
\Psi &= \mathbf{C}\Phi \quad ,
\end{aligned}
\qquad
\mathbf{C} = \begin{pmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & i\\ 1 & 0 & 0 & 0\\ 0 & i & 0 & 0\end{pmatrix}\mathscr{C} \quad . \tag{32}
$$

Chirality and parity conjugation operator is antilinear and anticommutes with the Hamiltonian, therefore if Ψ is a positive(negative) energy solution then $\Phi=\mathbf{C}\Psi$ is a negative (positive) solution.

$$
\mathbf{C}^2 = 1 \quad , \quad \mathbf{HC} = -\mathbf{CH} \quad . \tag{33}
$$

Closed algebra under commutation can be constructed by extending LA and RA with chirality and parity conjugation operator $\mathbf{C}$ and the following commutation relations holds

$$
\begin{aligned}
[\,\mathbf{C}_\mathsf{L}\,,\,\mathbf{C}\,] &= -\mathbf{P} \quad , & [\,\mathbf{C}_\mathsf{R}\,,\,\mathbf{C}\,] &= +\mathbf{P} \quad , \\
[\,\widetilde{\mathbf{C}}_\mathsf{L}\,,\,\mathbf{C}\,] &= -\gamma_5 \quad , & [\,\widetilde{\mathbf{C}}_\mathsf{R}\,,\,\mathbf{C}\,] &= +\gamma_5 \quad , \\
[\,\mathbf{P}\,,\,\mathbf{C}\,] &= 2(\widetilde{\mathbf{C}}_\mathsf{R}-\mathbf{C}_\mathsf{L}\,) \quad , & [\,\gamma_5\,,\,\mathbf{C}\,] &= 2(\mathbf{C}_R-\widetilde{\mathbf{C}}_\mathsf{L}) \quad .
\end{aligned} \tag{34}
$$

In the same manner we defined the common eigenstates of chirality for Ψ, we define them for Φ which also has four common eigenstates of chirality and parity

$$
\begin{aligned}
\Phi_{+\mathsf{L}} &= \frac{\widetilde{\mathbf{C}}_{\mathsf{L}} - \mathbf{C}_{\mathsf{L}}}{2}\Phi \quad , &\qquad \Phi_{+\mathsf{R}} &= \frac{\widetilde{\mathbf{C}}_{\mathsf{R}} + \mathbf{C}_{\mathsf{R}}}{2}\Phi \quad , \\
\Phi_{-\mathsf{L}} &= \frac{\widetilde{\mathbf{C}}_{\mathsf{L}} + \mathbf{C}_{\mathsf{L}}}{2}\Phi \quad , &\qquad \Phi_{-\mathsf{R}} &= \frac{\mathbf{C}_{\mathsf{R}} - \widetilde{\mathbf{C}}_{\mathsf{R}}}{2}\Phi \quad ,
\end{aligned}
\tag{35}
$$

The action of parity and chirality operators on Φ eigenstates are

$$
\begin{aligned}
\mathbf{P}\,\Phi_{-\mathsf{L}} &= -\Phi_{-\mathsf{L}} \quad , &\qquad \mathbf{P}\,\Phi_{-\mathsf{R}} &= -\Phi_{-\mathsf{R}} \quad , \\
\mathbf{P}\,\Phi_{+\mathsf{L}} &= +\Phi_{-\mathsf{L}} \quad , &\qquad \mathbf{P}\,\Phi_{+\mathsf{R}} &= +\Phi_{-\mathsf{R}} \quad , \\
\gamma_5\,\Phi_{-\mathsf{L}} &= -\Phi_{-\mathsf{L}} \quad , &\qquad \gamma_5\,\Phi_{-\mathsf{R}} &= +\Phi_{-\mathsf{R}} \quad , \\
\gamma_5\,\Phi_{+\mathsf{L}} &= -\Phi_{+\mathsf{L}} \quad , &\qquad \gamma_5\,\Phi_{+\mathsf{R}} &= +\Phi_{+\mathsf{R}} \quad ,
\end{aligned}
\tag{36}
$$

the sum of Φ eigenvalues of parity and chirality is equal to Ψ

$$
\Psi = \Phi_{+\mathsf{L}} + \Phi_{-\mathsf{L}} + \Phi_{+\mathsf{R}} + \Phi_{-\mathsf{R}} \quad . \tag{37}
$$

Chirality and parity conjugation operator $\mathbf{C}$ transform Φ eigenstates of chirality and parity into Ψ eigenstates with inverse eigenvalues of chirality and parity, and vice-versa

$$
\begin{aligned}
\Psi_{+\mathsf{L}} &= \mathbf{C}\Phi_{-\mathsf{R}} \quad , &\qquad \Phi_{-\mathsf{R}} &= \mathbf{C}\Psi_{+\mathsf{L}} \quad , \\
\Psi_{-\mathsf{L}} &= \mathbf{C}\Phi_{+\mathsf{R}} \quad , &\qquad \Phi_{+\mathsf{R}} &= \mathbf{C}\Psi_{-\mathsf{L}} \quad , \\
\Psi_{+\mathsf{R}} &= \mathbf{C}\Phi_{-\mathsf{L}} \quad , &\qquad \Phi_{-\mathsf{L}} &= \mathbf{C}\Psi_{+\mathsf{R}} \quad , \\
\Psi_{-\mathsf{R}} &= \mathbf{C}\Phi_{+\mathsf{L}} \quad , &\qquad \Phi_{+\mathsf{L}} &= \mathbf{C}\Psi_{-\mathsf{R}} \quad .
\end{aligned}
\tag{38}
$$

The Φ eigenstates satisfies coupled partial differential equations similar with equations 28 in which $\Psi_{-\mathsf{L}}$, $\Psi_{+\mathsf{L}}$, $\Psi_{+\mathsf{R}}$ and $\Psi_{-\mathsf{R}}$ are replaced by $\Phi_{+\mathsf{R}}$, $\Phi_{-\mathsf{R}}$, $\Phi_{-\mathsf{L}}$ and respectively $\Phi_{+\mathsf{L}}$. All 8 left and right eigenstates can be expressed in terms of parity and chirality operators as

$$
\begin{aligned}
\Psi_{\pm\mathsf{L}} &= \frac{1 \pm \mathbf{P}}{2}\frac{1-\gamma_5}{2}\Phi \quad , &\qquad \Psi_{\pm\mathsf{R}} &= \frac{1 \pm \mathbf{P}}{2}\frac{1+\gamma_5}{2}\Phi \quad , \\
\Phi_{\pm\mathsf{L}} &= \frac{1 \pm \mathbf{P}}{2}\frac{1-\gamma_5}{2}\Psi \quad , &\qquad \Phi_{\pm\mathsf{R}} &= \frac{1 \pm \mathbf{P}}{2}\frac{1+\gamma_5}{2}\Psi \quad ,
\end{aligned}
\tag{39}
$$

and similarly their adjoints

$$
\begin{aligned}
\overline{\Psi}_{\pm\mathsf{L}} &= \overline{\Phi}\;\frac{1 \pm \mathbf{P}}{2}\frac{1+\gamma_5}{2} \quad , &\qquad \overline{\Psi}_{\pm\mathsf{R}} &= \overline{\Phi}\;\frac{1 \pm \mathbf{P}}{2}\frac{1-\gamma_5}{2} \quad , \\
\overline{\Phi}_{\pm\mathsf{L}} &= \overline{\Psi}\;\frac{1 \pm \mathbf{P}}{2}\frac{1+\gamma_5}{2} \quad , &\qquad \overline{\Phi}_{\pm\mathsf{R}} &= \overline{\Psi}\;\frac{1 \pm \mathbf{P}}{2}\frac{1-\gamma_5}{2} \quad .
\end{aligned}
\tag{40}
$$

Next, we define left X_{L} and right X_{R} eigenstates of chiral operator by superposition of all left chiral and respectively right eigenstates

$$
X_{\mathsf{L}} = \Phi_{-\mathsf{L}} + \Phi_{+\mathsf{L}} + \Psi_{-\mathsf{L}} + \Psi_{+\mathsf{L}} \quad , \qquad X_{\mathsf{R}} = \Phi_{-\mathsf{R}} + \Phi_{+\mathsf{R}} + \Psi_{-\mathsf{R}} + \Psi_{+\mathsf{R}} \quad , \tag{41}
$$

which form an orthogonal set

$$
\begin{aligned}
\overline{X}_{\mathsf{L}}X_{\mathsf{L}} &= 0 \quad , &\qquad \overline{X}_{\mathsf{R}}X_{\mathsf{R}} &= 0 \quad , \\
\overline{X}_{\mathsf{L}}X_{\mathsf{R}} &= 0 \quad , &\qquad \overline{X}_{\mathsf{R}}X_{\mathsf{L}} &= 0 \quad ,
\end{aligned}
\tag{42}
$$

mix positive (negative) energy solutions with negative (positive) energy solutions of Dirac equation

$$
\begin{aligned}
X_{\mathsf{L}} &= \frac{1-\gamma_5}{2}(\Psi + \Phi) \quad , &\qquad X_{\mathsf{R}} &= \frac{1+\gamma_5}{2}(\Psi + \Phi) \quad , \\
\overline{X}_{\mathsf{L}} &= (\overline{\Psi} + \overline{\Phi})\frac{1+\gamma_5}{2} \quad , &\qquad \overline{X}_{\mathsf{R}} &= (\overline{\Psi} + \overline{\Phi})\frac{1-\gamma_5}{2} \quad ,
\end{aligned}
\tag{43}
$$

and they satisfy Klein-Gordon equation

$$(\Box + m^2) X_{\mathsf{L}} = 0 \quad , \qquad\qquad (\Box + m^2) X_{\mathsf{R}} = 0 \quad , \tag{44}$$

and the following partial differential coupled equations

$$i\,\partial_\alpha \gamma^\alpha X_{\mathsf{L}} - m X_{\mathsf{R}} = 0 \quad , \qquad\qquad i\,\partial_\alpha \gamma^\alpha X_{\mathsf{R}} - m X_{\mathsf{L}} = 0 \quad . \tag{45}$$

Out of 64 currents between all 8 common eigenstates of chirality and parity defined by 39 and 40, 32 of them, which are between L and R or R and L are zero. The 32 non-zero currents are between eigenstates of the same chirality, but not being conserved we introduce left and right conserved four-currents (probability densities) for eigenstates of the same chirality defined by 41

$$j^\alpha(X_{\mathsf{L}}, X_{\mathsf{L}}) = \overline{X}_{\mathsf{L}} \gamma^\alpha X_{\mathsf{L}} \quad , \qquad\qquad j^\alpha(X_{\mathsf{R}}, X_{\mathsf{R}}) = \overline{X}_{\mathsf{R}} \gamma^\alpha X_{\mathsf{R}} \quad , \tag{46}$$

which are equal and positively defined

$$\mathscr{C} j^\alpha(X_{\mathsf{L}}, X_{\mathsf{L}}) = j^\alpha(X_{\mathsf{L}}, X_{\mathsf{L}}) \quad , \qquad\qquad \mathscr{C} j^\alpha(X_{\mathsf{R}}, X_{\mathsf{R}}) = j^\alpha(X_{\mathsf{R}}, X_{\mathsf{R}}) \quad . \tag{47}$$

We interpret the conservation of the left and right four-currrent

$$\partial_\alpha\, j^\alpha(X_{\mathsf{L}}, X_{\mathsf{L}}) = 0 \quad , \qquad\qquad \partial_\alpha\, j^\alpha(X_{\mathsf{R}}, X_{\mathsf{R}}) = 0 \quad , \tag{48}$$

and the zero current between eigenstates of left an right chirality

$$j^\alpha(X_{\mathsf{L}}, X_{\mathsf{R}}) = j^\alpha(X_{\mathsf{R}}, X_{\mathsf{L}}) = 0 \quad , \tag{49}$$

as the existence of chiral particles L and R. The two chiral particles are transformed one into another by chirality and parity operator $\mathbf{C}$

$$X_{\mathsf{L}} = \mathbf{C} X_{\mathsf{R}} \quad , \qquad\qquad X_{\mathsf{R}} = \mathbf{C} X_{\mathsf{L}} \quad , \tag{50}$$

and are two different states of the particle described by $\Psi + \Phi = X_{\mathsf{L}} + X_{\mathsf{R}}$, which has no definite chirality nor parity and is an eigenstate of chirality and conjugation operator, i.e. is transformed into itself by chirality and parity operator,

$$\mathbf{C}(\Psi + \Phi) = \Psi + \Phi \quad , \tag{51}$$

therefore is a Majorana particle. All three particles mix positive (negative) energy solutions of Dirac equation with negative (positive) energy solutions, and their currents are explicitly given by

$$j^\alpha(X_{\mathsf{L}}, X_{\mathsf{L}}) = (\overline{\Psi} + \overline{\Phi}) \gamma^\alpha \frac{1 - \gamma_5}{2} (\Psi + \Phi) \quad , \qquad j^\alpha(X_{\mathsf{R}}, X_{\mathsf{R}}) = (\overline{\Psi} + \overline{\Phi}) \gamma^\alpha \frac{1 + \gamma_5}{2} (\Psi + \Phi) \quad , \tag{52}$$

and

$$j^\alpha(\Psi + \Phi, \Psi + \Phi) = (\overline{\Psi} + \overline{\Phi}) \gamma^\alpha (\Psi + \Phi) \quad . \tag{53}$$

Next, we calculate in the ultrarelativistic regime $\boldsymbol{\varepsilon} \gg m$ the helicity of L and R particles moving along the z axis. The helicity operator is given by

$$\mathbf{h} = \frac{1}{p}(p_1 S^1 + p_2 S^2 + p_3 S^3) \quad , \tag{54}$$

where $p = (p_1^2 + p_2^2 + p_3^2)^{1/2}$ is momentum and $S^1 = \Sigma^{23}$, $S^2 = \Sigma^{31}$, $S^3 = \Sigma^{12}$ are spatial components of spin, related to the generators of Lorentz transformations of gamma matrices by $\Sigma^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu]$. Helicity commutes with parity and chirality operator and for a particle moving along z axis we get $\mathbf{h} = -\frac{1}{2}\mathbf{P}$. For each of the four solution of Dirac equations we calculate the action of helicity operator on ultrarelativistic left and right particles moving along z axis and get correct predictions

$$\mathbf{h} X_{\mathsf{L}} = -\frac{1}{2} X_{\mathsf{L}} \quad , \qquad\qquad \mathbf{h} X_{\mathsf{R}} = +\frac{1}{2} X_{\mathsf{R}} \quad , \tag{55}$$

when $\varepsilon \gg m$. We can now identify the neutrino as the left chiral particle X_{L} having helicity $-\frac{1}{2}$ and the antineutrino as the right chiral particle X_{R} having helicity $-\frac{1}{2}$. Conserved currents are formed only by left X_{L} and right X_{R} eigenstates of chirality, not by the parity eigenstates. The superposition of all negative parity eigenstates

$$\mathbf{P}(\Psi_{-\mathsf{L}} + \Psi_{-\mathsf{R}} + \Phi_{-\mathsf{L}} + \Phi_{-\mathsf{R}}) = -(\Psi_{-\mathsf{L}} + \Psi_{-\mathsf{R}} + \Phi_{-\mathsf{L}} + \Phi_{-\mathsf{R}}) \quad , \tag{56}$$

and of all positive parity eigenstates

$$\mathbf{P}(\Psi_{+\mathsf{L}} + \Psi_{+\mathsf{R}} + \Phi_{+\mathsf{L}} + \Phi_{+\mathsf{R}}) = +(\Psi_{+\mathsf{L}} + \Psi_{+\mathsf{R}} + \Phi_{+\mathsf{L}} + \Phi_{+\mathsf{R}}) \quad , \tag{57}$$

gives currents which are are not conserved, i.e. we can not built conserved currents using linear combinations of parity eigenstates. Therefore, it is the chirality which is the fundamental symmetry of nature, because the left chiral and right chiral currents are conserved and identified with L and right R particle, i.e. neutrino and respectively antineutrino and the question is if chirality, not parity, is violated, i.e. if there is a field (particle) which couple with different strengths to L and R currents. The property of parity operator, in Weyl basis, to transform the left-handed Weyl spinor into the right-handed Weyl spinor and vice-versa requires that right-handed neutrino and left-handed antineutrino should also exist. The absence of right-handed neutrino and left-handed antineutrino had led to the conclusion that parity is not conserved. The mistake was to associate left-handed Weyl spinor, which gives a four current that is not conserved, to neutrino, and similarly, a right-handed Weyl spinor, which gives a four current that is not conserved, to antineutrino. Our definition of currents associated to left and right eigenstates of chirality are conserved 48 and transformed one into another by the chirality and parity conjugation operator 38, not by the parity operator as in Weyl representation. Therefore, the puzzling absence of positive helicity neutrino and respectively negative helicity antineutrino, i.e. parity violation, is a consequence wrong association of spinors to particles. The $-1/2$ helicity of the neutrino and $+1/2$ helicity of antineutrino are a result of their ultrarelatistic regime in which eigenstates X_{L} and X_{R} of chirality operator γ_5 are also eigenstates of parity operator $\mathbf{P}$. We can also construct conserved currents with first order derivatives of left eigenstates and conserved currents with first order derivatives of right eigenstates

$$\begin{aligned} j^{\alpha}_{(1)}(X_{\mathsf{L}}, X_{\mathsf{L}}) &= \partial_{\mu}\overline{X}_{\mathsf{L}}\gamma^{\alpha}\,\partial^{\mu}X_{\mathsf{L}} \quad , & j^{\alpha}_{(1)}(X_{\mathsf{R}}, X_{\mathsf{R}}) &= \partial_{\mu}\overline{X}_{\mathsf{R}}\gamma^{\alpha}\,\partial^{\mu}X_{\mathsf{R}} \quad , \\ \partial_{\alpha}\, j^{\alpha}_{(1)}(X_{\mathsf{L}}, X_{\mathsf{L}}) &= 0 \quad , & \partial_{\alpha}\, j^{\alpha}_{(1)}(X_{\mathsf{R}}, X_{\mathsf{R}}) &= 0 \quad , \end{aligned} \tag{58}$$

as well as conserved currents with second order derivatives of left and respectively right eigenstates

$$\begin{aligned} j^{\alpha}_{(2)}(X_{\mathsf{L}}, X_{\mathsf{L}}) &= \partial_{\mu}\partial_{\nu}\overline{X}_{\mathsf{L}}\gamma^{\alpha}\,\partial^{\mu}\partial^{\nu}X_{\mathsf{L}} \quad , & j^{\alpha}_{(2)}(X_{\mathsf{R}}, X_{\mathsf{R}}) &= \partial_{\mu}\partial_{\nu}\overline{X}_{\mathsf{R}}\gamma^{\alpha}\,\partial^{\mu}\partial^{\nu}X_{\mathsf{R}} \quad , \\ \partial_{\alpha}\, j^{\alpha}_{(2)}(X_{\mathsf{L}}, X_{\mathsf{L}}) &= 0 \quad , & \partial_{\alpha}\, j^{\alpha}_{(2)}(X_{\mathsf{R}}, X_{\mathsf{R}}) &= 0 \quad , \end{aligned} \tag{59}$$

and also conserved currents with derivatives of higher order than 2. For both left and right particles, only j^{α} and $j^{\alpha}_{(1)}$ are independent, all other currents built with derivatives of higher order than one can be expressed in term of j^{α} or $j^{\alpha}_{(1)}$ as follows

$$\begin{aligned} j^{\alpha}_{(2)}(X_{\mathsf{L}}, X_{\mathsf{L}}) &= m^4 j^{\alpha}(X_{\mathsf{L}}, X_{\mathsf{L}}) \quad , & j^{\alpha}_{(2)}(X_{\mathsf{R}}, X_{\mathsf{R}}) &= m^4 j^{\alpha}(X_{\mathsf{R}}, X_{\mathsf{R}}) \quad , \\ j^{\alpha}_{(3)}(X_{\mathsf{L}}, X_{\mathsf{L}}) &= m^4 j^{\alpha}_{(1)}(X_{\mathsf{L}}, X_{\mathsf{L}}) \quad , & j^{\alpha}_{(3)}(X_{\mathsf{R}}, X_{\mathsf{R}}) &= m^4 j^{\alpha}_{(1)}(X_{\mathsf{R}}, X_{\mathsf{R}}) \quad , \end{aligned} \tag{60}$$

and so on. The currents $j^{\alpha}(X_{\mathsf{L}}, X_{\mathsf{L}})$ and $j^{\alpha}_{(1)}(X_{\mathsf{L}}, X_{\mathsf{L}})$ satisfy coupled second order differential equations

$$\begin{aligned} \Box\, j^{\alpha}(X_{\mathsf{L}}, X_{\mathsf{L}}) &= -2m^2 j^{\alpha}(X_{\mathsf{L}}, X_{\mathsf{L}}) + 2 j^{\alpha}_{(1)}(X_{\mathsf{L}}, X_{\mathsf{L}}) \quad , \\ \Box\, j^{\alpha}_{(1)}(X_{\mathsf{L}}, X_{\mathsf{L}}) &= -2m^2 j^{\alpha}_{(1)}(X_{\mathsf{L}}, X_{\mathsf{L}}) + 2m^4 j^{\alpha}(X_{\mathsf{L}}, X_{\mathsf{L}}) \quad , \end{aligned} \tag{61}$$

and similar equations holds for $j^{\alpha}(X_{\mathsf{R}}, X_{\mathsf{R}})$ and $j^{\alpha}_{(1)}(X_{\mathsf{R}}, X_{\mathsf{R}})$. Next we define four vector fields A^{α}_{L}, A^{α}_{R}, B^{α}_{L}, and B^{α}_{R}

$$\begin{aligned} A^{\alpha}_{\mathsf{L}} &= m^2 j^{\alpha}(X_{\mathsf{L}}, X_{\mathsf{L}}) - j^{\alpha}_{(1)}(X_{\mathsf{L}}, X_{\mathsf{L}}) \quad , & A^{\alpha}_{\mathsf{R}} &= m^2 j^{\alpha}(X_{\mathsf{R}}, X_{\mathsf{R}}) - j^{\alpha}_{(1)}(X_{\mathsf{R}}, X_{\mathsf{R}}) \quad , \\ B^{\alpha}_{\mathsf{L}} &= m^2 j^{\alpha}(X_{\mathsf{L}}, X_{\mathsf{L}}) + j^{\alpha}_{(1)}(X_{\mathsf{L}}, X_{\mathsf{L}}) \quad , & B^{\alpha}_{\mathsf{R}} &= m^2 j^{\alpha}(X_{\mathsf{R}}, X_{\mathsf{R}}) + j^{\alpha}_{(1)}(X_{\mathsf{R}}, X_{\mathsf{R}}) \quad , \end{aligned} \tag{62}$$

and find that A_{L}^{α} and A_{R}^{α} fields satisfy Proca equations for a particle with mass $2m$

$$(\Box + 4m^2)A_{\mathsf{L}}^{\alpha} = 0 \quad , \qquad (\Box + 4m^2)A_{\mathsf{R}}^{\alpha} = 0 \quad ,$$
$$\partial_\alpha A_{\mathsf{L}}^{\alpha} = 0 \quad , \qquad \partial_\alpha A_{\mathsf{R}}^{\alpha} = 0 \quad , \tag{63}$$

while B_L^α and B_R^α satisfy Mawxell equations

$$\Box B_{\mathsf{L}}^{\alpha} = 0 \quad , \qquad \Box B_{\mathsf{R}}^{\alpha} = 0 \quad ,$$
$$\partial_\alpha B_{\mathsf{L}}^{\alpha} = 0 \quad , \qquad \partial_\alpha B_{\mathsf{R}}^{\alpha} = 0 \quad . \tag{64}$$

Using the solutions of Dirac equation for free field 5 and 7, by direct calculations we find that left and right B fields are equal and constant in space-time

$$B_{\mathsf{L}}^{\alpha} = B_{\mathsf{R}}^{\alpha} = \frac{2m^2}{\varepsilon} p^\alpha. \tag{65}$$

This result should be compared with Higgs Kibble cumbersome mechanism [3] that postulates a scalar field ϕ which couples to a massless boson to obtain a factor in the wave equation for W_- boson which play the same role as a mass term, and it is assumed that the scalar field is constant in space, while in our representation we have proved, not postulated, the existence of two fields derived from Dirac equation that are constant in space, and the symmetry $\mathsf{SU}(2) \times \mathsf{LA} \times \mathsf{RA}$ has the mass built-in, while Higgs Kibble mechanism requires spontaneous symmetry breaking. The α index in 65 is obvious not a covariant one, due to ε at the denominator. We multiply B fields as well as A fields with $\varepsilon/(2m^2)$ to get covariant fields without changing all previous results. Explicitly, the left and right A-fields are

$$A_{\mathsf{L}}^{\alpha} = \frac{\varepsilon}{2}\left[(\overline{\Psi}+\overline{\Phi})\gamma^\alpha \frac{1-\gamma_5}{2}(\Psi+\Phi) - \frac{1}{m^2}\partial_\lambda(\overline{\Psi}+\overline{\Phi})\gamma^\alpha\frac{1-\gamma_5}{2}\partial^\lambda(\Psi+\Phi)\right] \quad ,$$
$$A_{\mathsf{R}}^{\alpha} = \frac{\varepsilon}{2}\left[(\overline{\Psi}+\overline{\Phi})\gamma^\alpha \frac{1+\gamma_5}{2}(\Psi+\Phi) - \frac{1}{m^2}\partial_\lambda(\overline{\Psi}+\overline{\Phi})\gamma^\alpha\frac{1+\gamma_5}{2}\partial^\lambda(\Psi+\Phi)\right] \quad . \tag{66}$$

Since L particle current equals R particle current, and left and right currents built with first derivatives of eigenstates are equal, it follows that A-fields are also equal $A_{\mathsf{L}}^{\alpha} = A_{\mathsf{R}}^{\alpha}$. In the present framework, we postulate that for weak interaction processes (i) all particles are represented by the left fields A_{L}, all antiparticles by the right A_{R} fields and (ii) the Hamiltonian of interaction is

$$\mathcal{H}_{int} = g\int d^3x\, \left(A_{\mathsf{L}}^{\lambda}(x)\,,\, A_{\mathsf{R}}(x)^{\lambda}\right)_{out} \mathcal{G} \begin{pmatrix} A_{\lambda}^{\mathsf{L}}(x) \\ A_{\lambda}^{\mathsf{R}}(x)\end{pmatrix}_{inc} \quad , \tag{67}$$

where left and right fields for incoming particles are superposed separately, as well as left and right fields for outgoing particles, $\mathcal{G} = diag(1, -1)$ is an 2×2 matrix whose elements were determined by requiring that the Hamiltonian of B-fields for a free particle is zero, and g is a constant which determines the strength of interaction. The postulated Hamiltonian of interaction has resemblance with that of F.J. Hasert [4] for scattering of a muon neutrino by an electron, constructed with neutral currents, rather than those of Fermi and V-A theory, constructed with charge transition currents. As an application of postulated interaction Hamiltonian 67 to weak interactions processes, for muon decay $\mu^- \longrightarrow \nu_\mu + \bar{\nu}_e + e^-$ the A-fields for incoming and outgoing particles are given by

$$A_{\mathsf{L}}^{\lambda}(x)_{inc} = A_{\mathsf{L}\,\mu^-}^{\lambda}(p,x) \quad , \qquad A_{\mathsf{R}}^{\lambda}(x)_{inc} = 0 \quad ,$$
$$A_{\mathsf{L}}^{\lambda}(x)_{out} = A_{\mathsf{L}\,e^-}^{\lambda}(p',x) + A_{\mathsf{L}\,\nu_\mu}^{\lambda}(k',x) \quad , \qquad A_{\mathsf{R}}^{\lambda}(x)_{out} = A_{\mathsf{L}\,\bar{\nu}_\mu}^{\lambda}(k,x) \quad , \tag{68}$$

and the B-fields by

$$B_{\mathsf{L}}^{\lambda}(x)_{inc} = p^\lambda \quad , \qquad B_{\mathsf{R}}^{\lambda}(x)_{inc} = 0 \quad ,$$
$$B_{\mathsf{L}}^{\lambda}(x)_{out} = p'^\lambda + k'^\lambda \quad , \qquad B_{\mathsf{R}}^{\lambda}(x)_{out} = k^\lambda \quad , \tag{69}$$

where p, p', k' and k are the momenta of muon, electron, muon neutrino and respectively electron antineutrino. Detailed calculations and comparison with the V-A theory and the standard model of electroweak interactions will be given in a future paper. Using redefined fields we construct the

Lorentz invariant for left and right B-fields as $B^{\lambda}_{\mathsf{L}} B^{\mathsf{L}}_{\lambda} = m^2$ and $B^{\lambda}_{\mathsf{R}} B^{\mathsf{R}}_{\lambda} = m^2$. The particle mass m can not be arbitrary small because the product of left and right invariant is proportional to the trace of energy-momentum tensor, which accordingly to Einstein field equations is a positive constant equal to $\Lambda/(2\pi G)$ for flat Minkowski space-time, where G is the gravitational constant and Λ the cosmological constant. Therefore, the cosmological constant is related to the lightest particle mass predicted by the following relation

$$\frac{\Lambda}{2\pi G} = m^4 \quad , \tag{70}$$

in natural units, with $\hbar = 1$ and $c = 1$. Considering neutrino as the lightest particle of the theory we get $\Lambda = 2.38 \times 10^{-30} s^{-2}$ which is 5 orders bigger than current value of cosmological constant, and vice-versa, the mass of the lightest particle predicted by the theory $m = 3.80 \times 10^{-3} eV/c^2$ is calculated using cosmological constant $\Lambda = 2.036 \times 10^{-35} s^{-2}$ obtained by the High-Z Supernova Team and the Supernova Cosmology Project [5].

5 Conclusions

This result, that Dirac equation is equivalent with two $A^{\alpha}_{\mathsf{L}}(x)$ and $A^{\alpha}_{\mathsf{R}}(x)$ Proca fields with mass $2m$ and two constant massless fields B^{α}_{L} and B^{α}_{R}, suggest that this representation unifies fermion and bossons fields, its symmetry given by left and right algebra could be used to unify the description of weak and strong interactions. While the gauge symmetry $\mathsf{U}(1) \times \mathsf{SU}(2) \times \mathsf{SU}(3)$ of the Standard Model is exact only when the particles are massless, the $\mathsf{SU}(2) \times \mathsf{LA} \times \mathsf{RA}$ is built with 4 antiunitary and 4 nilpotent operators, and have the mass built in the symmetry, i.e. it require neither cumbersome mechanism with arbitrary chosen fields and parameters nor spontaneous breaking symmetry to generate mass. The theory predicts 5 orders of magnitude discrepancy for cosmological constant by using the neutrino as the lightest particle, otherwise for the current cosmological constant it predicts the value of the lightest particle as being $m = 3.80 \times 10^{-3} eV/c^2$.

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