MINKOWSKI COMPLETE SPACE-TIME ROTATION HYPOTHESIS

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ABSTRACT. This model uses rotation of Minkowski space-time, but assumption is that there can be time flowing in any direction not just in normal time direction, from it comes S tensor field that represents possible states of light cones. From that tensor field I can figure out all elementary particles states that are combination of bosonic fields. Field interactions lead rise to all possible quantum fields.

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1. Minkowski rotated complete space-time

1.1. Light cones direction. Light cone direction is always a time direction, but what if there can be not one light cone directions but all four? It extends Minkowski space-time and is key to understanding elementary particles. Light cone can have four directions in four dimensional space-time as it's time axis. This will lead to eight possible parts of light cone, where I assume that light cone splits in future and past light cone. This can be represented by a matrix with eight scalar functions that represent some field, I will denote this matrix as $S_m^n(\mathbf{x})$ that is matrix in tensor notation, where I use plus sign for future light cone and minus sign for past light cone. I mark each direction by 1, 2, 3, 4 so i can write this mixed tensor field as:

$$S_{m}^{n}(\mathbf{x}) = \begin{pmatrix} S_{-}^{1}(\mathbf{x}) & S_{+}^{1}(\mathbf{x}) \\ S_{-}^{2}(\mathbf{x}) & S_{+}^{2}(\mathbf{x}) \\ S_{-}^{3}(\mathbf{x}) & S_{+}^{3}(\mathbf{x}) \\ S_{-}^{4}(\mathbf{x}) & S_{+}^{4}(\mathbf{x}) \end{pmatrix}$$
(1.1)

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Each state of this light cone field represents a physical state of some kind of field. Later on I will explore meaning of this field, but first i need to start with spin.

1.2. Spin. Spin is connected to mixed tensor field $S_m^n(\mathbf{x})$, I can calculate for field a spin as half sum of absolute value of sign function of that tensor field:

$$\sigma = \frac{1}{2} \left| \sum_{n} \sum_{m} \operatorname{sign} \left(S_{m}^{n} \left(\mathbf{x} \right) \right) \right|$$
(1.2)

Spin is number of light cones parts that a given field has, if there are many fields still if each field has one number of light cones those numbers do not sum for each particle field has to give same spin number for each elementary field. Each full light cone gives spin one, so half of light cone gives half spin. Where if that light cone function has positive value at given point it adds to spin, if it has negative value it will be with negative value so it subtracts. I will use rotation tensor that is rotation matrix tensor product with another rotation matrix, first I need to write it, then figure out angles and coordinates [1]:

$$R^{\mu\nu}_{\alpha\beta} = R^{\mu}_{\alpha} \otimes R^{\nu}_{\beta} \tag{1.3}$$

I will use coordinates of light cones that come from $S_m^n(\mathbf{x})$ tensor. I will denote those coordinates as that matrix elements. I will write angle dependence of those rotation matrix as, where subscript numbers means rotated axis that are six for four dimensional space-time (12, 13, 14, 42, 43, 23) i only write first axis I need to add probability for each rotation. If i denote probability of rotation as $\rho_{i_1,\ldots,i_{2\sigma}}(\mathbf{x}_m^n)$:

$$R^{\mu}_{\alpha}\left(\boldsymbol{\Theta}\left(\mathbf{x}_{m}^{n}\right)\right) = \sum_{i_{1},\dots,i_{2\sigma}} \rho_{i_{1},\dots,i_{2\sigma}}\left(\mathbf{x}_{m}^{n}\right) R^{\mu}_{\alpha}\left(\frac{1}{2}\left(\theta_{12i_{1}}\left(\mathbf{x}_{-}^{1}\right)\dots+\theta_{12i_{2\sigma}}\left(\mathbf{x}_{+}^{4}\right)\right),\dots\right)$$

$$(1.4)$$

For making this notation shorter i will just write rotation matrix as $R^{\mu}_{\alpha}\left(\Theta\left(\mathbf{x}^{n}_{m}\right)\right)$ where it's equal to equation 1.8. Summation represents each possible rotation angle that system can have. For each spin there can be twice times rotation angles for each spin part. It means that state of rotation matrix is not a single rotation angle but sum of all possible rotation angles. For each light cone there is rotation by half angle, so it sums to rotation by spin number angle as rotation that is full angle rotation. Coordinates are divided by what light cones part is used, for each rotation angle there is dependence only on one part of light cone that is one part of $S^{n}_{m}(\mathbf{x})$ tensor. It means that each part of rotation angle is matched with each part of $S^{n}_{m}(\mathbf{x})$ tensor field components. It means that this coordinates can be written as eight coordinates for four dimensional space-time:

$$(\mathbf{x}_m^n) = \left(\mathbf{x}_{-}^1, \mathbf{x}_{+}^1, .., \mathbf{x}_{-}^4, \mathbf{x}_{+}^4\right)$$
(1.5)

1.3. Rotated Minkowski space-time. From spin I can move to rotating a Minkowski space-time [2], first I write rotation of that space-time:

$$ds^{2} = \eta_{\mu\nu}R^{\mu}_{\alpha}\left(\boldsymbol{\Theta}\left(\mathbf{x}_{m}^{n}\right)\right) \otimes R^{\nu}_{\beta}\left(\boldsymbol{\Theta}\left(\mathbf{x}_{m}^{n}\right)\right)dx^{\alpha}dx^{\beta}$$
(1.6)

Now I have half of equation, next half will change rotation mixed tensor into change of sum, of $S_m^n(\mathbf{x})$ tensor field components. I can write that new mixed tensor field as:

$$F^{\mu\nu}_{\alpha\beta}\left(\mathbf{x}^{n}_{m}\right) = \partial_{\alpha}\sum_{m}\delta^{\mu}_{n}S^{n}_{m}\left(\mathbf{x}^{n}_{m}\right) \otimes \partial_{\beta}\sum_{l}\delta^{\nu}_{k}S^{k}_{l}\left(\mathbf{x}^{n}_{m}\right)$$
(1.7)

For each field there is one of those tensor fields. So for each particle there are five fields of $S_m^n(\mathbf{x})$ tensor field. Now I can write full field equality:

$$ds^{2} = \eta_{\mu\nu}R^{\mu}_{\alpha}\left(\boldsymbol{\Theta}\left(\mathbf{x}_{m}^{n}\right)\right) \otimes R^{\nu}_{\beta}\left(\boldsymbol{\Theta}\left(\mathbf{x}_{m}^{n}\right)\right)dx^{\alpha}dx^{\beta}$$
$$= \eta_{\mu\nu}F^{\mu\nu}_{\alpha\beta}\left(\mathbf{x}_{m}^{n}\right)dx^{\alpha}dx^{\beta}$$
(1.8)

When i do measurement all probability numbers go to zero and one that was measured goes to one. I can denote field itself not metric part as:

$$\Psi^{\mu\nu}\left(\mathbf{x}_{m}^{n}\right) = \int R_{\alpha}^{\mu}\left(\mathbf{\Theta}\left(\mathbf{x}_{m}^{n}\right)\right) \otimes R_{\beta}^{\nu}\left(\mathbf{\Theta}\left(\mathbf{x}_{m}^{n}\right)\right) dx^{\alpha} dx^{\beta}$$
$$= \int F_{\alpha\beta}^{\mu\nu}\left(\mathbf{x}_{m}^{n}\right) dx^{\alpha} dx^{\beta}$$
(1.9)

Where sum of all probability functions squared has to give one:

$$\sum_{i_1,\dots,i_{2\sigma}} \rho_{i_1,\dots,i_{2\sigma}}^2 \left(\mathbf{x}_m^n \right) = 1$$
 (1.10)

Each of those function represents one of possible rotations.

1.4. Rotated light cone. For each observer it's casual structure is build upon it's light cones. Light cones rotate with angle equal to rotation of given field part. I can write it as equation of light cone:

$$ds^{2} = \eta_{\mu\nu}R^{\mu}_{\alpha}\left(\boldsymbol{\Theta}\left(\mathbf{x}_{m}^{n}\right)\right) \otimes R^{\nu}_{\beta}\left(\boldsymbol{\Theta}\left(\mathbf{x}_{m}^{n}\right)\right)dx^{\alpha}dx^{\beta} = 0$$
(1.11)

Where rotation is equal to solution to field equation. I can write full field equation being equal to zero as:

$$ds^{2} = \eta_{\mu\nu}R^{\mu}_{\alpha}\left(\boldsymbol{\Theta}\left(\mathbf{x}_{m}^{n}\right)\right) \otimes R^{\nu}_{\beta}\left(\boldsymbol{\Theta}\left(\mathbf{x}_{m}^{n}\right)\right)dx^{\alpha}dx^{\beta}$$
$$= \eta_{\mu\nu}F^{\mu\nu}_{\alpha\beta}\left(\mathbf{x}_{m}^{n}\right)dx^{\alpha}dx^{\beta} = 0$$
(1.12)

But all observers need to see speed of light as constant what about speed of light for rotated observer and non rotated one? There is need for transformations that conserve speed of light, those transformation need to obey [3]:

$$ds^{2'} = ds^{2} = \eta_{\mu\nu}R^{\mu}_{\alpha}\left(\boldsymbol{\Theta}\left(\mathbf{x}_{m}^{n}\right)\right) \otimes R^{\nu}_{\beta}\left(\boldsymbol{\Theta}\left(\mathbf{x}_{m}^{n}\right)\right) dx^{\alpha}dx^{\beta}$$
$$= \eta_{\mu\nu}R^{\mu\prime}_{\alpha}\left(\boldsymbol{\Theta}\left(\mathbf{x}_{m}^{n}\right)\right) \otimes R^{\nu\prime}_{\beta}\left(\boldsymbol{\Theta}\left(\mathbf{x}_{m}^{n}\right)\right) dx^{\alpha\prime}dx^{\beta\prime}$$
(1.13)

So they conserve speed of light being same for all rotated frame of reference. So for any observer I can write equality between field equation:

$$ds^{2'} = ds^{2} = \eta_{\mu\nu} R^{\mu\prime}_{\alpha} \left(\boldsymbol{\Theta} \left(\mathbf{x}_{m}^{n} \right) \right) \otimes R^{\nu\prime}_{\beta} \left(\boldsymbol{\Theta} \left(\mathbf{x}_{m}^{n} \right) \right) dx^{\alpha\prime} dx^{\beta\prime} = \eta_{\mu\nu} F^{\mu\nu\prime}_{\alpha\beta} \left(\mathbf{x}_{m}^{n} \right) dx^{\alpha\prime} dx^{\beta\prime} = = \eta_{\mu\nu} R^{\mu}_{\alpha} \left(\boldsymbol{\Theta} \left(\mathbf{x}_{m}^{n} \right) \right) \otimes R^{\nu}_{\beta} \left(\boldsymbol{\Theta} \left(\mathbf{x}_{m}^{n} \right) \right) dx^{\alpha} dx^{\beta} = \eta_{\mu\nu} F^{\mu\nu}_{\alpha\beta} \left(\mathbf{x}_{m}^{n} \right) dx^{\alpha} dx^{\beta}$$
(1.14)

So for any observer space-time interval has to be same. Additional condition is that rotation matrix need to conserve space-time interval [4]. I can express it as rotated and non rotated space-time interval are equal:

$$ds^{2\prime} = ds^{2} = \eta_{\mu\nu} R^{\mu\prime}_{\alpha} \left(\boldsymbol{\Theta} \left(\mathbf{x}_{m}^{n} \right) \right) \otimes R^{\nu\prime}_{\beta} \left(\boldsymbol{\Theta} \left(\mathbf{x}_{m}^{n} \right) \right) dx^{\alpha\prime} dx^{\beta\prime} = \eta_{\mu\nu} dx^{\mu} dx^{\nu}$$
(1.15)

2. Elementary fields

2.1. Virtual bosonic fields. There are five elementary bosonic fields [5], I will write them in matrix form as spin components without sum so each matrix will be equal to $\hat{\sigma} = \frac{1}{2} \left| \text{sign} \left(S_m^n(\mathbf{x}) \right) \right|$: First I write Higgs virtual field as and photonic field as one for each axis where I write only for first axis states:

$$\hat{\sigma}_{\tilde{H}^0} = \begin{bmatrix} -\frac{1}{2} & +\frac{1}{2} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad \hat{\sigma}_{\tilde{\gamma}} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$
(2.1)

Now graviton virtual field- that has more states, three:

$$\hat{\sigma}_{\tilde{G}} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 0 & 0 \\ \frac{1}{2} & \frac{1}{2} \\ 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$
(2.2)

Now i move to strong virtual field and six for each axis states of gluons, where i write first three and rest is all combination of them that gives total twenty four:

$$\hat{\sigma}_{\tilde{g}} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 0 & 0 \\ \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \qquad \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \\ 0 & 0 \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$
(2.3)

And as last i get W and Z bosons so weak virtual force, where again I write only three states rest are combination of them so both give total twelve states for Z boson and twenty-four for W boson:

$$\hat{\sigma}_{\tilde{Z}} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ +\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 0 & 0 \\ +\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 0 & 0 \\ +\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

$$\hat{\sigma}_{\tilde{W}} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \qquad \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & +\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \qquad \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & +\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \qquad (2.4)$$

$$\hat{\sigma}_{\tilde{W}} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \qquad \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & +\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \qquad (2.5)$$

From three base fields, Higgs field , photonic field and gravity field I can build rest fields as combination of them, where im allowed to use anti-fields of those base fields.

2.2. **Real quark fields.** From virtual bosonic fields I can create real particle states. All quarks have spin half so I will take only states of those fields that gives spin one half. For any axis *a* I can write general quarks states:

$$\hat{\sigma}_{i}^{a_{1/2}} = \hat{\sigma}_{\tilde{H}_{a}^{0}}^{a_{1/2}} + \hat{\sigma}_{\tilde{\gamma}_{a}}^{a_{1/2}} + \hat{\sigma}_{\tilde{G}_{i}}^{a_{1/2}} + \hat{\sigma}_{\tilde{X}_{i}^{0}}^{a_{1/2}} + \hat{\sigma}_{\tilde{Z}_{i}^{0}}^{a_{1/2}} + \frac{1}{\sqrt{2}} \left(\hat{\sigma}_{\tilde{g}_{i}}^{a_{1/2}} + \hat{\sigma}_{\tilde{g}_{i}}^{a_{1/2}} \right)$$

$$\hat{\sigma}_{j}^{a_{1/2}} = \hat{\sigma}_{\tilde{H}_{a}^{0}}^{a_{1/2}} + \hat{\sigma}_{\tilde{\gamma}_{a}}^{a_{1/2}} + \hat{\sigma}_{\tilde{G}_{j}}^{a_{1/2}} + \hat{\sigma}_{\tilde{W}_{j}^{-}}^{a_{1/2}} + \hat{\sigma}_{\tilde{Z}_{j}^{0}}^{a_{1/2}} + \frac{1}{\sqrt{2}} \left(\hat{\sigma}_{\tilde{g}_{j}}^{a_{1/2}} + \hat{\sigma}_{\tilde{g}_{j}}^{a_{1/2}} \right)$$

$$(2.6)$$

$$(2.6)$$

$$(2.7)$$

So quarks interact by all fields, I used subscript with number that represents I take only component of $S_m^n(\mathbf{x})$ tensor field that give value one half in this matrix. Strong interaction consists of gluon-antigluon pair interaction where I represent antigluon by bar notation. In general antifield is just a field with all signs reversed [6]:

$$S_m^n\left(\mathbf{x}\right) = -\overline{S}_m^n\left(\mathbf{x}\right) \tag{2.8}$$

It means that all light cones directions are reversed. That field represents antimatter. I did use two indexes i and j that represents both particles and possible states of virtual fields. First one gives all quarks that have electric charge two thirds, and second one- one third. Both indexes run from one to three but i distinct them for masses of quarks are not the same.

2.3. Rest of lepton family fields. Now I can move to rest of lepton family of particles. I start with electron, moun and tau, then with neutrinos. All of those fermions follow simple rule, quarks interact with all fields. Electron, moun and tau with all fields without strong one, neutrinos all without strong and photonic and finally last dark matter particles only interact by Higgs field and gravity. But still there is one part lacking in electron, moun and tau family. And one lacking in neutrinos family. If this thinking is correct there could be three more neutrinos and three more charged particles with positive charge. First I write know leptons:

$$\hat{\sigma}_{i}^{a_{1/2}} = \hat{\sigma}_{\tilde{H}_{a}^{0}}^{a_{1/2}} + \hat{\sigma}_{\tilde{\gamma}_{a}}^{a_{1/2}} + \hat{\sigma}_{\tilde{G}_{i}}^{a_{1/2}} + \hat{\sigma}_{\tilde{W}_{i}^{-}}^{a_{1/2}} + \hat{\sigma}_{\tilde{Z}_{i}^{0}}^{a_{1/2}}$$
(2.9)

$$\hat{\sigma}_{j}^{a_{1/2}} = \hat{\tilde{\sigma}}_{\tilde{H}_{a}^{0}}^{a_{1/2}} + \hat{\tilde{\sigma}}_{\tilde{G}_{j}}^{a_{1/2}} + \hat{\sigma}_{\tilde{W}_{j}^{+}}^{a_{1/2}} + \hat{\sigma}_{\tilde{Z}_{j}^{0}}^{a_{1/2}}$$
(2.10)

Now I can write those additional ones that could come out of this reasoning:

$$\hat{\sigma}_{k}^{a_{1/2}} = \hat{\sigma}_{\tilde{H}_{a}^{0}}^{a_{1/2}} + \hat{\sigma}_{\tilde{\gamma}_{a}}^{a_{1/2}} + \hat{\sigma}_{\tilde{G}_{k}}^{a_{1/2}} + \hat{\sigma}_{\tilde{W}_{k}^{+}}^{a_{1/2}} + \hat{\sigma}_{\tilde{Z}_{k}^{0}}^{a_{1/2}}$$
(2.11)

$$\hat{\sigma}_{l}^{a_{1/2}} = \hat{\tilde{\sigma}}_{\tilde{H}_{a}^{0}}^{a_{1/2}} + \hat{\tilde{\sigma}}_{\tilde{G}_{l}}^{a_{1/2}} + \hat{\sigma}_{\tilde{W}_{l}^{-}}^{a_{1/2}} + \hat{\sigma}_{\tilde{Z}_{l}^{0}}^{a_{1/2}}$$
(2.12)

I use indexes k and l to distinct them from know leptons. They would vary by mass, and for electric charged particles by charge. Now last left particles are candidate for dark matter in this model, last particles of this ladder, that interact only by gravity and Higgs field. There would be only three of them it comes from fact that there is only three states of gravity virtual field. They would be spin one half, so I can write them as:

$$\hat{\sigma}_{i}^{a_{1/2}} = \hat{\sigma}_{\tilde{H}_{a}^{0}}^{a_{1/2}} + \hat{\sigma}_{\tilde{G}_{i}}^{a_{1/2}} \tag{2.13}$$

2.4. Real bosonic fields. I presented virtual bosonic fields, but there are real ones. Higgs boson will interact not only with gravity virtual field but with antigravity field to sum up give zero spin. Photon will only interact with gravity field. Same as graviton but their spin states possible will be not the same. Gluons will interact with gluon field and gravity field. More complex are weak force real particles, W bosons interact with Higgs, gravity, photonic and themself. Z boson with Higgs, gravity and itself. I can write all this as before :

$$\hat{\sigma}_{H_i^0}^{a_0} = \hat{\sigma}_{\tilde{H}_a^0}^{a_0} + \frac{1}{\sqrt{2}} \left(\hat{\sigma}_{\tilde{G}_a}^{a_+} + \hat{\sigma}_{\tilde{G}_a}^{a_-} \right)$$
(2.14)

$$\hat{\sigma}_{\gamma_i}^{a_1} = \hat{\sigma}_{\tilde{G}_i}^{a_1} \tag{2.15}$$

$$\hat{\sigma}_{G_i}^{a_2} = \hat{\sigma}_{\tilde{G}_i}^{a_2} \tag{2.16}$$

$$\hat{\sigma}_{g_i}^{a_1} = \hat{\sigma}_{\tilde{G}_i}^{a_1} + \hat{\sigma}_{\tilde{g}_i}^{a_1} \tag{2.17}$$

$$\hat{\sigma}_{W_i^+}^{a_1} = \hat{\sigma}_{\tilde{H}_a^0}^{a_1} + \hat{\sigma}_{\tilde{G}_i}^{a_1} + \hat{\sigma}_{\tilde{\gamma}_a}^{a_1} + \hat{\sigma}_{\tilde{W}_i^+}^{a_1} \tag{2.18}$$

$$\hat{\sigma}_{W_i^-}^{a_1} = \hat{\sigma}_{\tilde{H}_a^0}^{a_1} + \hat{\sigma}_{\tilde{G}_i}^{a_1} + \hat{\sigma}_{\tilde{\gamma}_a}^{a_1} + \hat{\sigma}_{\tilde{W}_i^-}^{a_1}$$
(2.19)

$$\hat{\sigma}_{Z_i^0}^{a_1} = \hat{\sigma}_{\tilde{H}_a^0}^{a_1} + \hat{\sigma}_{\tilde{G}_i}^{a_1} + \hat{\sigma}_{\tilde{Z}_i}^{a_1} \tag{2.20}$$

Now I have all possible real and virtual fields.

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References

- [1] https://mathworld.wolfram.com/RotationMatrix.html
- [2] https://mathworld.wolfram.com/MinkowskiMetric.html
- [3] https://en.wikipedia.org/wiki/Lorentz_transformation
- [4] https://www.sciencedirect.com/science/article/pii/ S221137971832076X
- [5] https://www3.nd.edu/~cjessop/research/overview/particle_chart.pdf
- [6] http://scipp.ucsc.edu/outreach/23FeynmanDiagrams.pdf