The Helmholtzian Factorization, Pythagorean Quintuples, Fermion and Quark Architecture

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The Helmholtzian factorization shows that the heavy leptons and quarks are separated as:
$(0,0,0,m_0)$ heavy leptons satisfying the Dirac factoring, and $(m_1,m_2,m_3,0)$ quarks satisfying the Helmholtzian factoring, as pythagorean quadruples. Investigating pythagorean quadruples leads to one $(4,10,28,30)$ which leads to $\left(\frac{1}{3},\frac{5}{6},\frac{7}{3},\frac{5}{2}\right)$ and $\left(\frac{2}{3},\frac{5}{3},\frac{14}{3},5\right)$ further leading to the quark mass, color, and charge architecture.

In general, the Helmholtzian factorization yields the Helmholtz partial differential equation in four-vectors [4].

$$(\Box + |m|^2) = D_B D_A$$

In [1] I showed that the heavy leptons and quarks are separated as:

$$m \equiv (m_1,m_2,m_3,m_0) = (m_1,m_2,m_3,0) + (0,0,0,m_0) = m_q + m_\ell$$

$$m_q \equiv (m_1,m_2,m_3,0) \ , \ m_\ell \equiv (0,0,0,m_0)$$

such that:

$(\Box + |m_\ell|^2)$ satisfies the Dirac factoring, and:

$\left(\Box + |m_q|^2\right)$ satisfies the Helmholtzian factorization, as a pythagorean quadruple [5][6], yielding the

$(\Box + |m_\nu|^2)$ satisfies the Helmholtzian factorization, with color characteristics
along with Dirac characteristics; but more, as a pythagorean quintuple.

As noted in [1], the light leptons = neutrinos are not so easily separated, meaning that:

$$|m_\nu|^2 = m_1^2 + m_2^2 + m_3^2 + m_0^2$$

have chracteristics beyond simply adding the heavy lepton plus quark characteristics.

a small table of pythagorean quadruples [7]:

(1, 2, 2, 3)	(1, 4, 8, 9)	(1, 6, 18, 19)	(1, 12, 12, 17)	(1, 8, 32, 33)
(1, 18, 30, 35)	(2, 3, 6, 7)	(2, 4, 4, 6)	(2, 6, 9, 11)	(2, 10, 11, 15)
(2, 8, 16, 18)	(2, 5, 14, 15)	(3, 4, 12, 13)	(3, 6, 6, 9)	(3, 6, 22, 23)
(3, 8, 36, 37)	(3, 12, 24, 27)	(4, 5, 20, 21)	(4, 8, 8, 12)	(4, 7, 32, 33)
(4, 8, 19, 21)	(4, 10, 28, 30)	(4, 13, 16, 21)	(4, 17, 28, 33)	(5, 6, 30, 31)

(a parametrrzation: $a_j = 2u_j^2 \ , \ b_j = v_j^2 \ , \ c_j = 2u_jv_j \ , \ d_j = 2u_j^2 + 2v_j^2$)

a small table of pythagorean quintuples:

$(1,1,1,1,2)$	$(1,1,3,5,6)$	$(1,1,7,7,10)$	$(1,2,2,4,5)$	$(1,2,4,10,11)$	$(1,2,8,10,13)$
$(1,3,3,9,10)$	$(1,4,4,4,7)$	$(1,5,5,7,10)$	$(2,2,3,8,9)$	$(2,2,4,5,7)$	$(2,2,7,8,11)$
$(2,4,5,6,9)$	$(2,4,7,10,13)$	$(4,4,4,11,13)$	$(4,4,5,8,11)$	$(4,5,8,8,13)$	

As noted before [5]:

in general, from any pythagorean quadruple $(a_j,b_j,c_j,d_j) \Rightarrow a_j^2 + b_j^2 + c_j^2 = d_j^2 \propto |m_{q_j}|^2 = (\theta_j m_e)^2$:

$$\Rightarrow m_{q_j} = (a_j,b_j,c_j) \cdot \frac{\theta_j}{d_j} m_e$$

Similarly,

from any pythagorean quintuple $(a_j,b_j,c_j,d_j,e_j) \Rightarrow a_j^2 + b_j^2 + c_j^2 + d_j^2 = e_j^2 \propto |m_{q_j}|^2 = (\theta_j m_e)^2$:

$$\Rightarrow m_{q_j} = (a_j,b_j,c_j,d_j) \cdot \frac{\theta_j}{e_j} m_e$$

and likewise for neutrinos.

For pythagorean triples: $(a_j,b_j,c_j) \Rightarrow a_j^2 + b_j^2 = c_j^2$
(parametrrzation: $a_j = u_j^2 - v_j^2 \ , \ b_j = 2u_jv_j \ , \ c_j = u_j^2 + v_j^2$)

Now, consider the fermions:

$m_e = m(3,1)$	$m_\mu = 5km_e$	$m_\tau = 1 \cdot \left[\left(\frac{2}{1450}\right)(5k)^2\right]^2 m_e$
$m_u = 5m_e$	$m_c = 5 \cdot 12km_e$	$m_b = 5 \cdot \left[\left(\frac{23}{25}\right)^{\frac{1}{2}} \cdot (k)^2\right] m_e$
$m_d = 10m_e$	$m_s = 10 \cdot \left(\frac{23}{50}\right)km_e$	$m_t = 10 \cdot \left[\left(\frac{3}{1004}\right)(6k)^2\right]^2 m_e$

$m_e \propto 1$	$m_\mu \propto 5k$	$m_\tau \propto \left[\left(\frac{2}{1450}\right)(5k)^2\right]^2$
$m_u \propto 5$	$m_c \propto 5 \cdot 12k$	$m_b \propto 5 \cdot \left(\frac{\sqrt{23}}{5}\right) \cdot k^2$
$m_d \propto 10$	$m_s \propto 10 \cdot \left(\frac{23}{50}\right) \cdot k$	$m_t \propto 10\left[\left(\frac{3}{1004}\right)(6k)^2\right]^2$

and quarks:

$m_u = (4,10,28)\frac{1}{6}m_e$	$m_c = (4,10,28)2km_e$	$m_b = (4,10,28)\frac{\sqrt{23}}{30} \cdot km_e$
$m_d = (4,10,28)\frac{1}{3}m_e$	$m_s = (4,10,28)\left(\frac{23}{30 \cdot 5}\right) \cdot km_e$	$m_t = (4,10,28)\frac{10}{30}\left[\left(\frac{3}{1004}\right)(6k)^2\right]^2 m_e$

$$\Downarrow$$

$$\left(-\frac{2}{3},-\frac{5}{3},-\frac{14}{3}\right) = (4,10,28)\frac{1}{6} \Rightarrow 30 \cdot \frac{1}{6} = 5 = \frac{15}{3}$$
$$(4,10,28)2k \Rightarrow 60k = 30 \cdot 2k = \frac{15}{3} \cdot 6 \cdot 2k = \frac{15}{3} \cdot 12k$$
$$\left(-\frac{2}{3},-\frac{5}{3},-\frac{14}{3}\right) = \left(-\frac{1}{3},-\frac{5}{6},-\frac{7}{3}\right)2 = (4,10,28)\frac{1}{6} \cdot 2 = (4,10,28)\frac{1}{3} = \left(\frac{4}{3},\frac{10}{3},\frac{28}{3}\right) \Rightarrow 30 \cdot \frac{1}{3} = 10 = \frac{30}{3}$$
$$\left(-\frac{2}{3},-\frac{5}{3},-\frac{14}{3}\right) \cdot 6 = \left(-\frac{1}{3},-\frac{5}{6},-\frac{7}{3}\right)2 \cdot 6 = (4,10,28)$$
$$\left(-\frac{1}{3},-\frac{5}{6},-\frac{7}{3}\right)2 \cdot 2 = \left(-\frac{2}{3},-\frac{5}{3},-\frac{14}{3}\right) \cdot 2 = (4,10,28)\frac{1}{3}$$

$$\Downarrow$$

$m_u = \left(+\frac{2}{3},+\frac{5}{3},+\frac{14}{3}\right)m_e$	$m_c = \left(+\frac{2}{3},+\frac{5}{3},+\frac{14}{3}\right)12km_e$	$m_b = \left(+\frac{2}{3},+\frac{5}{3},+\frac{14}{3}\right)\frac{\sqrt{23}}{5} \cdot km_e$
$m_d = \left(-\frac{1}{3},-\frac{5}{6},-\frac{7}{3}\right)4m_e$	$m_s = \left(-\frac{1}{3},-\frac{5}{6},-\frac{7}{3}\right)\left(\frac{23}{30 \cdot 5}\right) \cdot 4km_e$	$m_t = \left(-\frac{1}{3},-\frac{5}{6},-\frac{7}{3}\right)\left[\left(\frac{3}{1004}\right)(6k)^2\right]^2 \frac{4}{3}m_e$

$$\|$$

$m_u = \left(+\frac{1}{3},+\frac{5}{6},+\frac{7}{3}\right)2m_e$	$m_c = \left(+\frac{1}{3},+\frac{5}{6},+\frac{7}{3}\right)24km_e$	$m_b = \left(+\frac{1}{3},+\frac{5}{6},+\frac{7}{3}\right)\frac{2}{5}\sqrt{23} \cdot km_e$
$m_d = \left(-\frac{1}{3},-\frac{5}{6},-\frac{7}{3}\right)4m_e$	$m_s = \left(-\frac{1}{3},-\frac{5}{6},-\frac{7}{3}\right)\left(\frac{23}{30 \cdot 5}\right) \cdot 4km_e$	$m_t = \left(-\frac{1}{3},-\frac{5}{6},-\frac{7}{3}\right)\left[\left(\frac{3}{1004}\right)(6k)^2\right]^2 \frac{4}{3}m_e$

Note:

$$u\bar{u} \sim \left(+\frac{2}{3},+\frac{5}{3},+\frac{14}{3}\right) + \left(-\frac{2}{3},-\frac{5}{3},-\frac{14}{3}\right) = (0,0,0) = \left(\frac{1}{3},\frac{5}{6},\frac{7}{3}\right)0 \equiv 0(\text{mod}\,3)$$
$$u\bar{d} \sim \left(+\frac{2}{3},+\frac{5}{3},+\frac{14}{3}\right) + \left(\frac{1}{3},\frac{5}{6},\frac{7}{3}\right) = \left(+1,+\frac{5}{2},+7\right) = (2,5,14)\frac{1}{2} = (4,10,28)\frac{1}{4} = \left(\frac{1}{3},\frac{5}{6},\frac{7}{3}\right)3 \equiv 0(\text{mod}\,3)$$
$$u\bar{c} \sim \left(+\frac{2}{3},+\frac{5}{3},+\frac{14}{3}\right) + \left(\frac{1}{3},\frac{5}{6},\frac{7}{3}\right) = \left(+1,+\frac{5}{2},+7\right) = (2,5,14)\frac{1}{2} = (4,10,28)\frac{1}{4} = \left(\frac{1}{3},\frac{5}{6},\frac{7}{3}\right)3 \equiv 0(\text{mod}\,3)$$
$$d\bar{c} \sim \left(-\frac{1}{3},-\frac{5}{6},-\frac{7}{3}\right) + \left(-\frac{2}{3},-\frac{5}{3},-\frac{14}{3}\right) + \left(-1,-\frac{5}{2},-7\right) \equiv \left(-\frac{1}{3},-\frac{5}{6},-\frac{7}{3}\right)3 \equiv 0(\text{mod}\,3)$$
$$d\bar{s} \sim \left(-\frac{1}{3},-\frac{5}{6},-\frac{7}{3}\right) + \left(-\frac{1}{3},-\frac{5}{6},-\frac{7}{3}\right) = (0,0,0) = \left(\frac{1}{3},\frac{5}{6},\frac{7}{3}\right)0 \equiv 0(\text{mod}\,3)$$

$$u_R u_G u_B \sim \left(+\frac{2}{3}+\frac{5}{3}+\frac{14}{3}\right) + \left(-\frac{2}{3},-\frac{5}{3},-\frac{14}{3}\right) + \left(-\frac{2}{3},-\frac{5}{3},-\frac{14}{3}\right) = \left(-\frac{2}{3},-\frac{5}{3},-\frac{14}{3}\right)3 \equiv 0(\text{mod}\,3)$$
$$u_R u_G d_B \sim \left(+\frac{2}{3}+\frac{5}{3}+\frac{14}{3}\right) + \left(+\frac{2}{3},+\frac{5}{3},+\frac{14}{3}\right) + \left(-\frac{1}{3},-\frac{5}{6},-\frac{7}{3}\right)$$
$$= \left(+\frac{1}{3},+\frac{5}{6},+\frac{7}{3}\right)2 + \left(+\frac{1}{3},+\frac{5}{6},+\frac{7}{3}\right)2 + \left(-\frac{1}{3},-\frac{5}{6},-\frac{7}{3}\right) = \left(+\frac{3}{3},+\frac{15}{6},+\frac{21}{3}\right) = \left(+\frac{1}{3},+\frac{5}{6},+\frac{7}{3}\right)3 \equiv 0(\text{mod}$$
$$u_R d_G d_B \sim \left(+\frac{2}{3}+\frac{5}{3}+\frac{14}{3}\right) + \left(-\frac{1}{3},-\frac{5}{6},-\frac{7}{3}\right) + \left(-\frac{1}{3},-\frac{5}{6},-\frac{7}{3}\right) = (0,0,0) = \left(\frac{1}{3},\frac{5}{6},\frac{7}{3}\right)0 \equiv 0(\text{mod}\,3)$$
$$d_R d_G d_B \sim \left(-\frac{1}{3},-\frac{5}{6},-\frac{7}{3}\right) + \left(-\frac{1}{3},-\frac{5}{6},-\frac{7}{3}\right) + \left(-\frac{1}{3},-\frac{5}{6},-\frac{7}{3}\right) = \left(-\frac{3}{3},-\frac{15}{6},-\frac{21}{3}\right) = \left(-\frac{1}{3},-\frac{5}{6},-\frac{7}{3}\right)3 \equiv 0(\text{mod}\,3$$

Note:

although the mass constituents may be in any order the addition is equivalent to aligning/rearranging the constituents from least to largest and adding and adding the resultant tuple , constituent-wise

so:

the primitive triples of the quark mass/color constituent pythagorean quadruples add up to $0\ (\text{mod}\,3)$, as **color** , and

the first/least of the primitive triples of the quark mass/color constituent pythagorean quadruples add up to $0\ (\text{mod}\,2)$ as **charge** , and

the total sum of the squares of the primitive triples is the square of the quark **mass magnitude** (the last/largest constituent pythagorean quadruple/quintuple)

Thus, the quark mass/color/charge constituent pythagorean quadruple/quintuple is fully determinant of of the quark mass, color, and charge.

However, because each quark has a different multiple, the fermion decay/interaction combination of mass magnitudes is more complex.

These multiple corroborations attest to the quark primitive pythagorean triples/quadruples/quintuples

$$\left(\frac{1}{3},\frac{5}{6},\frac{7}{3},0,\frac{5}{2}\right) \ \& \ \left(\frac{2}{3},\frac{5}{3},\frac{14}{3},0,5\right)$$

and to the Helmholtzian factorization yielding the fermion and quark architectures.

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