Updating Maxwell's classical electromagnetic field theory with mutual energy theory of electromagnetic fields*

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The author comprehensively reviewed the mutual energy theory of electromagnetic field, including three axioms: (1) the law of conservation of electromagnetic energy, (2) the law that radiation does not overflow the universe, (3) the principle of mutual energy and self energy principle; the mutual energy theorem, mutual energy flow theorem, advanced wave existence theorem. According to the theory of mutual energy, electromagnetic radiation is an interaction between source and sink. The source is a primary coil of the transformer, transmitting antenna and the emitter charge. The sink is the secondary coil of the transformer, receiving antenna and absorbing body charge. The source emits retarded wave and the sink emits advanced wave. Electromagnetic radiation phenomena must include a transmitting antenna and a receiving antenna. At this time, the transmitting antenna is equivalent to the primary coil of the transformer, and the receiving antenna is equivalent to the secondary coil of the transformer. Essentially, an antenna system including a transmitting antenna and a receiving antenna is no different from a transformer system including a primary coil and a secondary coil. They all satisfy the same mutual energy theory. That is, retarded wave and advanced wave are synchronized and generate the mutual energy flow. Mutual energy flow is photons. The wave, including the retarded wave and the advanced wave, must collapse in reverse. The reverse collapse satisfies the Maxwell equation of time reversal. The retarded wave and time reversal wave can be replaced by a reactive power wave. The mutual energy flow and reactive power wave together describe the wave collapse phenomenon. The important conclusion of this theory is that the energy of an electromagnetic wave is transferred by mutual energy flow, not by self energy flow. A light source cannot produce radiation without the help of an environmental absorber. Photons are mutual energy flows. The author finds that Maxwell's equation is not automatically suitable for the theory of electromagnetic mutual energy, so it must be modified. This correction includes adding a time reversal wave, but considering that reactive power waves can also play the role of time reversal wave, the author thinks that reactive power waves can be used to replace time reversal waves in this correction. In this paper, the method of mutual energy theory is applied to two planar current transformer systems and two dipole antenna systems. These two examples verify the correctness of mutual energy theory.

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I. INTRODUCTION

A. Advanced wave

The solutions of Maxwell's equations include retarded potential and advanced potential, or are called retarded wave and advanced wave. The classical electromagnetic theory only recognizes the retarded wave, not the advanced wave. Because the advanced wave violates the causality. But a group of scientists also recognized the advanced wave. For example, Wheeler and Feynman put forward the absorber theory [1, 2], and the absorber theory is based on the principle of action-at-a-distance [8, 16, 18]. The absorber theory is further developed by Cramer and is called quantum mechanics transactional interpretation [5, 6]. Lawrence Stephenson also made important contributions to the advanced wave theory [17]. The theory of mutual energy proposed by the author involves the advanced wave. The author believes that the advanced wave must be a physical reality. The above authors all have great influence on the author, so the author firmly believes that there should be the position of the advanced wave in the electromagnetic field theory.

B. Cramer's transactional interpretation of quantum mechanics

In the following figure 1, the light source is represented by a red disc. The light sink in the figure is represented by a small blue disc. The horizontal axis represents distance and the vertical axis represents time. The source emits a retarded wave to the right, the source emits an advanced wave to the left, and the wave emitted by the source is represented by red. The sink sends an advanced wave to the left and a retarded wave to the right, which is represented by blue. Between the emitter and sink, the retarded wave and the advanced wave are synchronized, so they are superimposed. On the right side of

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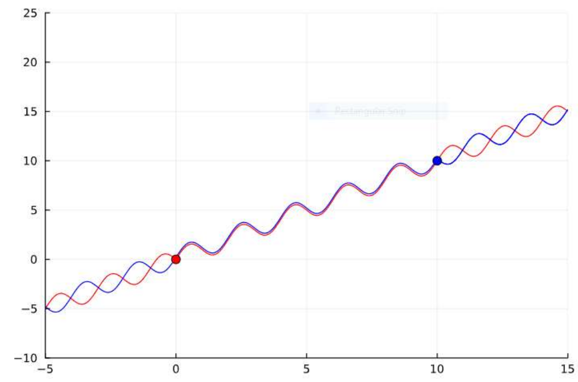

Figure 1. Cramer's photon model·

the sink, there is a 180 degree phase difference between the retarded wave from the sink and the retarded wave from the source, so they cancel. On the left side of the source, the advanced wave from the source and the advanced wave from the sink have a phase difference of 180 degrees, so they cancel each other out. Cramer's particle model is fantastic. However, it is not without problems. For example, the 180 degree phase difference is quite puzzling. What is the reason for the 180 degree phase difference? It is not mentioned in Cramer's model. In addition, this model is one-dimensional. In practice, we must face waves in three-dimensional space. The situation is much more complicated. In a word, Cramer's model is only a qualitative theory. This paper attempts to give a specific photon model based on the electromagnetic theory combined with the concept of mutual energy and mutual energy flow proposed by the author.

C. Reactive power and reactive power wave

The author proposed time reversal wave [11], but recently the author is not very satisfied with it. Recently, the author noted that it is difficult to confirm the time reversal wave. Especially if the time reversal wave exists, the time reversal wave can also generate the mutual energy flow, and the mutual energy flow of the time reversal wave may also offset the real mutual energy flow. It is difficult to make sense for time reversal wave to counteract self energy flow without creating the time-reversal mutual energy flow to counteract real mutual energy flow. The author notes that if the self energy flow is reactive power, the time reversal wave can be eliminated. Therefore, the author is keen to look for the modification of Maxwell's electromagnetic theory, so that the self energy flow can be converted into the wave of reactive power. Namely,

$$\Re(\oiint_{\Gamma} \boldsymbol{E}_1 \times \boldsymbol{H}_1^* \cdot \hat{n} d\Gamma) = 0 \tag{1}$$

$$\Re(\oiint_{\Gamma} \boldsymbol{E}_2 \times \boldsymbol{H}_2^* \cdot \hat{n} d\Gamma) = 0 \tag{2}$$

where $\Re$ is taking the real part of a complex number. This means that the electric and magnetic fields far away from the antenna should be 90 degrees in phase, not in phase. Note that the far field of the antenna is in phase according to Maxwell's equations. The plane wave solutions of Maxwell's equations are also in phase.

The author first notes that in the case of magnetic quasi-static field, the phase difference between the electric field and the magnetic field is 90 degrees. The question is why this phase becomes in phase when the radiation field satisfies Maxwell's equations? The author tries to add a phase to the electric field or magnetic field of Maxwell's solutions, and finds that it can just meet the requirements of mutual energy flow transferring energy, while self energy flow does not transfer energy [11–14].

D. Wave particle duality

The mutual energy flow itself is a point-to-point transfer of energy, and the mutual energy flow generated by the retarded wave and the advanced wave is particle. The particles are generated at the generation point of the retarded wave, that is, the source of light, and annihilated at the generation point of the advanced wave, that is, the sink. The particle has the property of wave between the source and sink, because the particle itself is a mutual energy flow composed of advanced wave and retarded wave. In this way, the author explains the problem of wave particle duality by using the mutual energy flow. The author thinks that the wave collapse can be explained by the mutual energy flow plus wave reverse collapse. The reverse collapse of wave and wave can be explained by reactive power wave. Therefore, the wave collapse can be explained by the combination of mutual energy flow and reactive power wave.

E. From reciprocity theorem to mutual energy theory

The earliest (around 1900) is the Lorentz reciprocity theorem[9]

[3, 4],

$$\iiint_{V_1} (\boldsymbol{J}_1(\omega) \cdot \boldsymbol{E}_2(\omega)) dV = \iiint_{V_2} (\boldsymbol{J}_2(\omega) \cdot \boldsymbol{E}_1(\omega)) dV \tag{3}$$

In 1960, there was Welch's reciprocity theorem [19],

$$-\int_{t=-\infty}^{\infty} dt \iiint_{V_1} (\boldsymbol{J}_1(t) \cdot \boldsymbol{E}_2(t)) dV$$

$$= \int_{t=-\infty}^{\infty} dt \iiint_{V_2} (\boldsymbol{J}_2(t) \cdot \boldsymbol{E}_1(t)) dV \quad (4)$$

In 1963, there was Rumsey's reciprocity theorem [15],

$$-\iiint_{V_1} (\boldsymbol{J}_1(\omega)\cdot\boldsymbol{E}_2^*(\omega))dV = \iiint_{V_2} (\boldsymbol{J}_2^*(\omega)\cdot\boldsymbol{E}_1(\omega))dV \quad (5)$$

In 1987, Zhao (the author) proposed the mutual energy theorem [10, 20, 21], which is the same as the above formula Eq.(5). The author didn't find Welch and Rumsey's paper at that time. The author felt that this theorem was different from Lorentz's reciprocity theorem, and should be an energy theorem. Not just a reciprocity theorems.

At the end of 1987, there was de Hoop's correlation reciprocity theorem [7].

$$-\int_{t=-\infty}^{\infty} dt \iiint_{V_1} (\boldsymbol{J}_1(t+\tau) \cdot \boldsymbol{E}_2(t)) dV$$

$$= \int_{t=-\infty}^{\infty} dt \iiint_{V_2} (\boldsymbol{J}_2(t) \cdot \boldsymbol{E}_1(t+\tau)) dV \quad (6)$$

The author found in (2017) that the formula (6) is the inverse Fourier transform of (5) [11–14]. Formula (4) is obviously a special case of formula (6) in $\tau = 0$, so they can be regarded as one law. These 4 theorems can be obtained from Lorentz reciprocity theorem through a conjugate transformation. The conjugate transformation in the frequency domain is,

$$\boldsymbol{E}(\omega), \boldsymbol{H}(\omega), \boldsymbol{J}(\omega) \rightarrow \boldsymbol{E}(\omega)^*, -\boldsymbol{H}(\omega)^*, -\boldsymbol{J}(\omega)^*$$

In the time domain is,

$$\boldsymbol{E}(t), \boldsymbol{H}(t), \boldsymbol{J}(t) \rightarrow \boldsymbol{E}(-t), -\boldsymbol{H}(-t), -\boldsymbol{J}(-t)$$

Therefore, these 4 theorems are closely related to Lorentz reciprocity theorem. These 4 theorems are not sub theorems of Lorentz's theorem because Maxwell's equation is used in conjugate transformation. The author thinks that these 4 theorems are energy theorems. The other three authors define these 4 theorems as reciprocity theorems, possibly because these 4 theorems involve advanced waves.

For example, when the author applies the mutual energy theorem to a pair of antennas, a transmitting antenna and a receiving antenna, it means that the electromagnetic wave generated by the transmitting antenna is a retarded wave and the receiving antenna is an advanced wave. The mutual energy theorem tells us that the energy obtained by the receiving antenna is exactly equal to the energy provided by the transmitting antenna to the receiving antenna. The advanced wave is not recognized in the classical theory, so most people thinks it is questionable that these 4 theorems are energy theorems. If they are not an energy theorems, they can only be used as a mathematical formula similar to Green's functions, so we call them reciprocity theorems. However, the author believes that the advanced wave is an objective physical existence. Hence, these 3 formulas are also energy theorems.

In 2017, the author popularized the mutual energy theorem (5) as the energy conservation theorem,

$$\sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \int_{t=-\infty}^{\infty} dt \iiint_{V} (\boldsymbol{J}_i \cdot \boldsymbol{E}_j) dV = 0 \quad (7)$$

At the same time, the author puts forward the principle of mutual energy,

$$-\oiint_{\Gamma} \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \boldsymbol{E}_i \times \boldsymbol{H}_j \cdot \hat{n} d\Gamma$$

$$= \iiint_{V} \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} (\boldsymbol{J}_i\cdot\boldsymbol{E}_j + \boldsymbol{E}_i\cdot\frac{\partial}{\partial t}\boldsymbol{D}_j + \boldsymbol{H}_i\cdot\frac{\partial}{\partial t}\boldsymbol{B}_j) dV \quad (8)$$

The theorem of mutual energy flow is also proved,

$$-\iiint_{V_1} (\boldsymbol{J}_1(\omega)\cdot\boldsymbol{E}_2^*(\omega))dV = (\xi_1, \xi_2) = \iiint_{V_2} (\boldsymbol{J}_2^*(\omega)\cdot\boldsymbol{E}_1(\omega))dV \quad (9)$$

Where (ξ_1, ξ_2) is defined as,

$$(\xi_1, \xi_2) \equiv \oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2 \times \boldsymbol{H}_1^*) \cdot \hat{n} d\Gamma \quad (10)$$

The author thinks that the mutual energy flow is the photon. Photons are normalized mutual energy flows [12]. In order to meet the principle that radiation does not overflow the universe, the author proposed time reversal wave [11]. In this way, the electromagnetic waves radiated from the universe can return. Finally, the author developed a complete set of electromagnetic theory, which is called mutual energy theory [11–14].

F. Work of this paper

This paper systematically reviews the whole theory of mutual energy. The author deduces the whole electromagnetic theory from the theory of magnetic quasi-static electromagnetic field according to the law of conservation of energy of mutual energy theory. Then, using two concrete examples, one is the transformer model of double plate currents, and the other is the model of double dipole antenna, the electromagnetic field solution satisfying the mutual energy flow equation is found. It is shown that this solution is different from the solution obtained from Maxwell's equations, and this solution is correct. Chapter 2 reviews the theory of electrostatic electromagnetic field and magnetic quasi-static electromagnetic field. Chapter 3 establishes the electromagnetic field mutual energy theory. Chapter 4 discusses the defects of classical electromagnetic field theory. Chapter 5 examples of solving electromagnetic field mutual energy theory of biplane current transformer. Chapter 6 is an example of solving the electromagnetic field mutual energy theory of double dipole antenna. From these two examples, readers should see that in addition to the theory of mutual energy flow and the principle of mutual energy, which are more than the classical electromagnetic field theory, the retarded wave and advanced wave involved in the theory of mutual energy are different from the retarded wave and advanced wave in the solution obtained by Maxwell's equations.

II. THEORY OF MAGNETIC QUASI-STATIC ELECTROMAGNETIC FIELD

Here we briefly review the theory of Magnetic quasi-static electromagnetic fields. Instead of starting from Maxwell's equations, the author derives all the electromagnetic field theories from Neumann's field induction theorem and the law of conservation of energy proposed by the author. All the magnetic quasi-static electromagnetic fields theories used are briefly reviewed.

A. Electrostatic Field

The electrostatic field found by Coulomb's theorem can be expressed as,

$$\boldsymbol{E} = \iiint_V \rho \frac{\hat{r}}{r^2} dV$$

$$= \iiint_V \rho(-\nabla \frac{1}{r}) dV$$

$$= -\nabla \iiint_V \frac{\rho(\boldsymbol{x}')}{r} d^3x' \tag{11}$$

$$r = |\boldsymbol{x} - \boldsymbol{x}'| \tag{12}$$

$\rho(\boldsymbol{x}')$ is the charge density. $\boldsymbol{x}$ is the site location and $\boldsymbol{x}'$ is the source location. Define scalar potential,

$$\phi(\boldsymbol{x}) \equiv \iiint_V \frac{\rho(\boldsymbol{x}')}{r} d^3x' \tag{13}$$

Get,

$$\boldsymbol{E} = -\nabla \phi \tag{14}$$

B. Static magnetic field

The static magnetic field can be expressed as,

$$\boldsymbol{B} = \frac{\mu_0}{4\pi} \iiint_V \boldsymbol{J} \times \frac{\boldsymbol{r}}{r^3} dV$$

$$= \frac{\mu_0}{4\pi} \iiint_V \boldsymbol{J} \times (-\nabla \frac{1}{r}) dV$$

$$= \frac{\mu_0}{4\pi} \iiint_V \nabla \frac{1}{r} \times \boldsymbol{J}) dV$$

$$= \frac{\mu_0}{4\pi} \iiint_V \nabla \times \frac{\boldsymbol{J}}{r} dV$$

$$= \nabla \times \frac{\mu_0}{4\pi} \iiint_V \frac{\boldsymbol{J}}{r} dV \tag{15}$$

Define magnetic vector potential

$$\boldsymbol{A} \equiv \frac{\mu_0}{4\pi} \iiint_V \frac{\boldsymbol{J}}{r} dV \tag{16}$$

get

$$\boldsymbol{B} = \nabla \times \boldsymbol{A} \tag{17}$$

C. Magnetic quasi-static field

$$\mathcal{E} \equiv \oint_{C_2} \boldsymbol{E} \cdot d\boldsymbol{l} \tag{18}$$

$$\mathcal{E} = -\frac{d}{dt} MI \tag{19}$$

$$M = \oint_{C_2} \frac{\mu_0}{4\pi} \oint_{C_1} \frac{d\boldsymbol{l}_1}{r} \cdot d\boldsymbol{l}_2 \tag{20}$$

$$\oint_{C_2} \boldsymbol{E} \cdot d\boldsymbol{l} = -\frac{d}{dt} MI \tag{21}$$

$$M = \oint_{C_2} \frac{\mu_0}{4\pi} \oint_{C_1} \frac{d\boldsymbol{l}_1 \cdot d\boldsymbol{l}_2}{r} \tag{22}$$

$$\oint_{C_2} \boldsymbol{E} \cdot d\boldsymbol{l} = -\frac{d}{dt} \oint_{C_2} \frac{\mu_0}{4\pi} \oint_{C_1} I \frac{d\boldsymbol{l}_1}{r} \cdot d\boldsymbol{l}_2 \tag{23}$$

$$\oint_{C_2} \boldsymbol{E} \cdot d\boldsymbol{l} = -\oint_{C_2} \frac{\partial}{\partial t}\frac{\mu_0}{4\pi}\oint_{C_1} I\frac{d\boldsymbol{l}_1}{r} \cdot d\boldsymbol{l}_2 \quad (24)$$

Define

$$\boldsymbol{A} \equiv \frac{\mu_0}{4\pi}\oint_{C_1} I\frac{d\boldsymbol{l}_1}{r} \quad (25)$$

$$\oint_{C_2} \boldsymbol{E} \cdot d\boldsymbol{l} = -\oint_{C_2} \frac{\partial}{\partial t}\boldsymbol{A} \cdot d\boldsymbol{l}_2 \quad (26)$$

or

$$\oint_{C_2} (\boldsymbol{E} + \frac{\partial}{\partial t}\boldsymbol{A}) \cdot d\boldsymbol{l} = 0 \quad (27)$$

$$(\boldsymbol{E} + \frac{\partial}{\partial t}\boldsymbol{A}) = -\nabla\phi \quad (28)$$

$$\boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{A} - \nabla\phi \quad (29)$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}\nabla \times \boldsymbol{A} \quad (30)$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{B} \quad (31)$$

$$\boldsymbol{A} = \frac{\mu_0}{4\pi}\iiint_V \frac{\boldsymbol{J}}{r}dV \quad (32)$$

$$\nabla \cdot \boldsymbol{A} = \nabla \cdot \frac{\mu_0}{4\pi}\iiint_V \frac{\boldsymbol{J}}{r}dV$$

$$= \frac{\mu_0}{4\pi}\iiint_V \nabla(\frac{1}{r}) \cdot \boldsymbol{J}dV$$

$$= \frac{\mu_0}{4\pi}\iiint_V (-\nabla'(\frac{1}{r})) \cdot \boldsymbol{J}dV \quad (33)$$

$$\nabla' \cdot (\frac{\boldsymbol{J}}{r}) = \nabla'(\frac{1}{r}) \cdot \boldsymbol{J} + (\frac{1}{r})\nabla' \cdot \boldsymbol{J} \quad (34)$$

The volume integral of $\nabla' \cdot (\frac{\boldsymbol{J}}{r})$ can be converted to the outer surface, so it is zero,

$$-\nabla'(\frac{1}{r}) \cdot \boldsymbol{J} = (\frac{1}{r})\nabla' \cdot \boldsymbol{J}$$

$$\nabla \cdot \boldsymbol{A} = \iiint_V (\frac{1}{r})\nabla' \cdot \boldsymbol{J}dV$$

Considering,

$$\nabla' \cdot \boldsymbol{J} = -\frac{\partial}{\partial t}\rho = 0$$

The above consideration is steady-state, so there is $\frac{\partial}{\partial t}\rho = 0$, thus

$$\nabla \cdot \boldsymbol{A} = 0 \quad (35)$$

$$\nabla^2\boldsymbol{A} = \mu_0\iiint_V \nabla^2(\frac{1}{4\pi r})\boldsymbol{J}dV$$

$$= \mu_0\iiint_V (-\delta(\boldsymbol{x} - \boldsymbol{x'}))\boldsymbol{J}dV$$

$$= -\mu_0\boldsymbol{J} \quad (36)$$

$$\nabla \times \nabla \times \boldsymbol{A} = \nabla(\nabla \cdot \boldsymbol{A}) - \nabla^2\boldsymbol{A} \quad (37)$$

$$\nabla^2\boldsymbol{A} = -\nabla \times \nabla \times \boldsymbol{A} + \nabla(\nabla \cdot \boldsymbol{A}) \quad (38)$$

$$-\nabla \times \nabla \times \boldsymbol{A} + \nabla(\nabla \cdot \boldsymbol{A}) = -\mu_0\boldsymbol{J} \quad (39)$$

$$\nabla \times \nabla \times \boldsymbol{A} = \mu_0\boldsymbol{J} \quad (40)$$

$$\nabla \times \boldsymbol{B} = \mu_0\boldsymbol{J} \quad (41)$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} \quad (42)$$

So we get the magnetic quasi-static Maxwell equation,

$$\begin{cases} \nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{B} \\ \nabla \times \boldsymbol{H} = \boldsymbol{J} \end{cases} \quad (43)$$

This equation does not include Maxwell's displacement current. The corresponding Poynting theorem can be obtained from this equation,

$$\nabla \cdot (\boldsymbol{E} \times \boldsymbol{H}) = \nabla \times \boldsymbol{E} \cdot \boldsymbol{H} - \boldsymbol{E} \cdot \nabla \times \boldsymbol{H} \quad (44)$$

$$\nabla \cdot (\boldsymbol{E} \times \boldsymbol{H}) = -\frac{\partial}{\partial t}\boldsymbol{B} \cdot \boldsymbol{H} - \boldsymbol{E} \cdot \boldsymbol{J} \quad (45)$$

So we get Poynting's theorem,

$$-\oiint_\Gamma (\boldsymbol{E} \times \boldsymbol{H}) \cdot \hat{n}d\Gamma = \iiint (\frac{\partial}{\partial t}\boldsymbol{B} \cdot \boldsymbol{H} + \boldsymbol{E} \cdot \boldsymbol{J})dV$$

Since the magnetic quasi-static field is not a radiation field,

$$\boldsymbol{E}, \boldsymbol{H} \sim \frac{1}{r^2} \quad (46)$$

$$\lim_{r\to\infty} \oiint_\Gamma (\boldsymbol{E}\times\boldsymbol{H})\cdot\hat{n}d\Gamma = 0 \tag{47}$$

Similarly, if it is in frequency domain.

$$-\oiint_\Gamma (\boldsymbol{E}\times\boldsymbol{H}^*)\cdot\hat{n}d\Gamma = \iiint (j\omega\boldsymbol{B}\cdot\boldsymbol{H}^* + \boldsymbol{E}\cdot\boldsymbol{J}^*)dV \tag{48}$$

energy increase $j\omega\boldsymbol{B}\cdot\boldsymbol{H} = 0$,

$$-\oiint_\Gamma (\boldsymbol{E}\times\boldsymbol{H}^*)\cdot\hat{n}d\Gamma = \iiint (\boldsymbol{E}\cdot\boldsymbol{J}^*)dV \tag{49}$$

In the above formula, the surface can be taken arbitrarily. If it is infinite,

$$\lim_{r\to\infty} \oiint_\Gamma (\boldsymbol{E}\times\boldsymbol{H}^*)\cdot\hat{n}d\Gamma = 0 \tag{50}$$

This means that,

$$\oiint_\Gamma (\boldsymbol{E}\times\boldsymbol{H}^*)\cdot\hat{n}d\Gamma = 0 \tag{51}$$

The above formula Γ is any surface surrounding the current J.

$$\boldsymbol{E}\times\boldsymbol{H}^* = 0 \tag{52}$$

This shows that for the magnetic quasi-static field, the phase of the magnetic field and the electromagnetic field should be 90 degrees. This is different from the solution of the radiated electromagnetic field satisfying Maxwell's equation including displacement current. The solution electric field and magnetic field of the radiated electromagnetic field are in phase. The author thinks that it is not reliable that the electromagnetic radiation field and the magnetic field are in phase according to Maxwell's equations. The point that the magnetic quasi-static electromagnetic field develops to the 90 degree phase of the radiated electromagnetic field should be maintained. In addition, in the magnetic quasi-static field,

$$\iiint (\boldsymbol{E}\cdot\boldsymbol{J}^*)dV = 0 \tag{53}$$

III. ELECTROMAGNETIC MUTUAL ENERGY THEORY

In the circuit, we know that the power consumed on the resistance is

$$P_{load} = UI^* \tag{54}$$

The electromotive force of the battery can provide power of,

$$P_{output} = \mathcal{E}I^* = \int \boldsymbol{E}\cdot d\boldsymbol{l}I^* \to \iiint_V \boldsymbol{E}\cdot\boldsymbol{J}^*dV \tag{55}$$

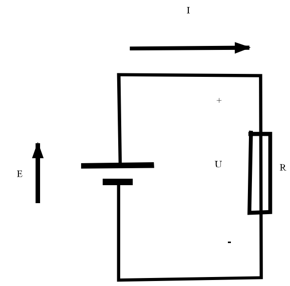

Figure 2. Circuit containing battery.

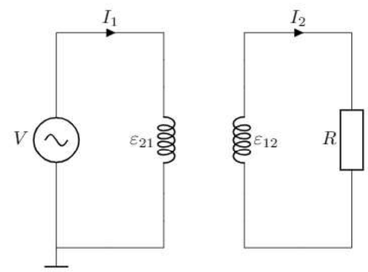

Figure 3. Transformer, assuming that the current of the primary coil and the current of the secondary coil are known.

$\boldsymbol{E}$ is the electric field of the induced electromotive force or the electric field of the battery. According to the conservation of energy,

$$P_{output} = P_{load} \tag{56}$$

For the above figure 3, there are

$$P_{2,1} = \int_{C_2} \boldsymbol{E}_1\cdot I_2^* d\boldsymbol{l} \tag{57}$$

$$P_{1,2} = \int_{C_1} \boldsymbol{E}_2^*\cdot I_1 d\boldsymbol{l} \tag{58}$$

$P_{2,1}$ is electrical power of the secondary coil or current I_2 provided by the electromagnetic field of the current I_1. Note that the conjugate sign of a complex number is marked either on a quantity containing the subscript sign 1 or on a quantity containing the subscript sign 2. This power is used for the consumption of the load circuit in the secondary circuit. If

$$\Re \int_{C_1} \boldsymbol{E}_2^*\cdot I_1 d\boldsymbol{l} < 0 \tag{59}$$

It indicates that the primary coil provides negative power to the primary circuit, so the primary coil actually obtains power from the primary circuit. This power will be supplied to the secondary coil of the transformer. If the secondary coil has,

$$\Re \int_{C_2} \boldsymbol{E}_1 \cdot I_2^* d\boldsymbol{l} > 0 \tag{60}$$

Description the secondary coil provides power to the secondary circuit. Therefore, power is drawn to the primary coil of the transformer. If

$$\Re \int_{C_2} \boldsymbol{E}_1 \cdot I_2^* d\boldsymbol{l} = 0 \tag{61}$$

It indicates that the power obtained from the transformer by the secondary coil of the transformer is reactive power. If current I_1 Transfer energy to I_2, energy $P_{2,1}$ of current I_2 increase, energy $P_{1,2}$ of the current I_1 decreases, but the total energy remains the same,

$$P_{2,1} + P_{1,2} = 0 \tag{62}$$

or

$$-P_{2,1} = P_{1,2} \tag{63}$$

or

$$-\int_{C_1} \boldsymbol{E}_2^* \cdot I_1 d\boldsymbol{l} = \int_{C_2} \boldsymbol{E}_1 \cdot I_2^* d\boldsymbol{l} \tag{64}$$

Note that the line current can replace the bulk current $Idl \to JdV$, and the above formula can be rewritten as,

$$-\iiint_V \boldsymbol{E}_2^* \cdot \boldsymbol{J}_1 dV = \iiint_V \boldsymbol{E}_1 \cdot \boldsymbol{J}_2^* dV \tag{65}$$

Negative sign on the left of the above formula, $-\iiint_V \boldsymbol{E}_2^* \cdot \boldsymbol{J}_1 dV$ refers to the power absorbed by the primary coil of the transformer from the primary circuit and supplied to the secondary coil. The right of the above formula $\iiint_V \boldsymbol{E}_1 \cdot \boldsymbol{J}_2^* dV$ refers to the power provided by the secondary coil to the secondary circuit and also the power obtained by the secondary circuit from the transformer. In the time domain, there is,

$$-\int_{t=-\infty}^{\infty} dt \iiint_V \boldsymbol{E}_2 \cdot \boldsymbol{J}_1 dV = \int_{t=-\infty}^{\infty} dt \iiint_V \boldsymbol{E}_1 \cdot \boldsymbol{J}_2 dV \tag{66}$$

A. The energy conservation law

The above formula can be rewritten as,

$$\sum_{i=1}^{2} \sum_{j=1, j\neq i}^{2} \int_{t=-\infty}^{\infty} dt \iiint_{V_i} (\boldsymbol{J}_i \cdot \boldsymbol{E}_j) dV = 0 \tag{67}$$

There are two current elements above. If there are N, there is the following law of conservation of energy,

$$\sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \int_{t=-\infty}^{\infty} dt \iiint_{V_i} (\boldsymbol{J}_i \cdot \boldsymbol{E}_j) dV = 0 \tag{68}$$

This law of conservation of energy is the starting point of the author's electromagnetic theory. This formula is proposed as an axiom. We use the knowledge of electrostatic and magnetic quasi-static fields, but do not assume Maxwell's equations. Maxwell's equations are derived from the above law of conservation of energy. Here, in the author's mutual energy theory, Maxwell's equations are only an auxiliary set of equations, not a physical law.

B. Law of conservation of energy in transformer

Figure 3 shows a transformer, which has a primary coil and a secondary coil. The primary coil gives a power to the secondary coil. The power of the secondary coil increases and the power of the primary coil decreases. However, the sum of the two coil powers remains unchanged. The author first verifies that the above energy conservation law is true for transformers.

The law of conservation of energy can be rewritten as,

$$-\int_{C_1} \boldsymbol{E}_2^*(\omega) \cdot I_1(\omega) d\boldsymbol{l} = \int_{C_2} \boldsymbol{E}_1(\omega) \cdot I_2^*(\omega) d\boldsymbol{l} \tag{69}$$

or

$$-\int_{C_1} \boldsymbol{E}_2^*(\omega) \cdot d\boldsymbol{l} I_1(\omega) = \int_{C_2} \boldsymbol{E}_1(\omega) \cdot d\boldsymbol{l} I_2^*(\omega) \tag{70}$$

define

$$\mathcal{E}_2 \equiv \int_{C_1} \boldsymbol{E}_2(\omega) \cdot d\boldsymbol{l}$$

$$\mathcal{E}_1 \equiv \int_{C_2} \boldsymbol{E}_1(\omega) \cdot d\boldsymbol{l}$$

there is

$$-\mathcal{E}_2^* I_1(\omega) = \mathcal{E}_1 I_2^*(\omega) \tag{71}$$

Considering (19,20) ,

$$\mathcal{E}_1 = -M_{2,1} \frac{dI_1}{dt} = -j\omega M_{2,1} I_1 \tag{72}$$

$$\mathcal{E}_2 = -M_{1,2} \frac{dI_2}{dt} = -j\omega M_{1,2} I_2 \tag{73}$$

$M_{2,1}$ is the mutual inductance generated by the current on coil 1 on coil 2. $M_{1,2}$ is the mutual inductance generated by current 2 on coil 1. By substituting these two formulas into (71), we get,

$$-(-j\omega M_{1,2} I_2)^* I_1(\omega) = (-j\omega M_{2,1} I_1) I_2^*(\omega) \tag{74}$$

Figure 4. Transformer, the secondary coil is far away from the primary coil, so the retarded effect must be considered.

or

$$(M_{1,2}I_2)^* I_1(\omega) = (M_{2,1}I_1) I_2^*(\omega) \tag{75}$$

or

$$M_{1,2}^* = M_{2,1} \tag{76}$$

The above is the energy conservation law for the transformer. Where,

$$M_{2,1} \equiv \oint_{C_2} \frac{\mu_0}{4\pi} \oint_{C_1} \frac{d\boldsymbol{l}_1 \cdot d\boldsymbol{l}_2}{r} \tag{77}$$

$$M_{1,2} \equiv \oint_{C_1} \frac{\mu_0}{4\pi} \oint_{C_2} \frac{d\boldsymbol{l}_2 \cdot d\boldsymbol{l}_1}{r} \tag{78}$$

The above formula (76) is the law of conservation of energy in the transformer, which is applicable to the magnetic quasi-static field,

$$M_{1,2} = M_{2,1} = M \tag{79}$$

The equation (76) obviously satisfies. This further verifies our law of conservation of energy ([68) in magnetic quasi-static situation.

C. Retarded potential

For a transformer, if the secondary coil is far away from the primary coil, the retared effect must be considered. We know,

$$M_{2,1}I_1 = \int_{C_2} \frac{\mu_0}{4\pi} \int_{C_1} \frac{I_1 d\boldsymbol{l}_1 \cdot d\boldsymbol{l}_2}{r} \tag{80}$$

If there is a certain distance between the secondary coil and the primary coil, as shown in the above figure 4. We have to consider the retarded potential, so there is,

$$M_{2,1}I_1 = \int_{C_2} \frac{\mu_0}{4\pi} \int_{C_1} \frac{\exp(-j\boldsymbol{k}\cdot\boldsymbol{r}) I_1 d\boldsymbol{l}_1 \cdot d\boldsymbol{l}_2}{r} \tag{81}$$

define retarded potential

$$\boldsymbol{A}_1 \equiv \frac{\mu_0}{4\pi} \int_{C_1} \frac{\exp(-j\boldsymbol{k}\cdot\boldsymbol{r}) I_1}{r} d\boldsymbol{l}_1 \tag{82}$$

D. Advanced potential exists

The following figure 4 is a transformer, but the secondary coil is far away from the primary coil. At this time, the primary coil becomes the transmitting antenna and the secondary coil becomes the receiving antenna. In order to satisfy the law of conservation of energy (76), there must be,

$$M_{1,2}I_2 = \int_{C_1} \frac{\mu_0}{4\pi} \int_{C_2} \frac{\exp(+j\boldsymbol{k}\cdot\boldsymbol{r}) I_2 d\boldsymbol{l}_2 \cdot d\boldsymbol{l}_1}{r} \tag{83}$$

Define

$$\boldsymbol{A}_2 \equiv \frac{\mu_0}{4\pi} \int_{C_2} \frac{\exp(+j\boldsymbol{k}\cdot\boldsymbol{r}) I_2}{r} d\boldsymbol{l}_2 \tag{84}$$

Only in this way can the energy conservation (76) be satisfied. This tells us that the dadvanced potential must exist, otherwise the law of conservation of energy cannot be satisfied. There are retarded potential and advanced potential. In the time domain, there is,

$$\boldsymbol{A}_1 = \frac{\mu_0}{4\pi} \iiint_{V_1} \frac{\boldsymbol{J}_1(x, t - r/c)}{r} dV \tag{85}$$

$$\boldsymbol{A}_2 = \frac{\mu_0}{4\pi} \iiint_{V_2} \frac{\boldsymbol{J}_2(x, t + r/c)}{r} dV \tag{86}$$

c is the speed of the retarded wave and advanced wave.

E. Electromagnetic field

$$\mathcal{E}_1 = -\frac{d}{dt} M_{2,1} I_1$$

$$= -\int_{C_2} \frac{\partial}{\partial t} \frac{\mu_0}{4\pi} \int_{C_1} \frac{I_1 d\boldsymbol{l}_1}{r} \cdot d\boldsymbol{l}_2$$

$$= -\int_{C_2} \frac{\partial}{\partial t} \boldsymbol{A}_1 \cdot d\boldsymbol{l}_2 \tag{87}$$

We known

$$\mathcal{E}_1 \equiv \int_{C_2} \boldsymbol{E}_1 \cdot d\boldsymbol{l}_2 \tag{88}$$

there is,

$$\int_{C_2} \boldsymbol{E}_1 \cdot d\boldsymbol{l}_2 = -\int_{C_2} \frac{\partial}{\partial t}\boldsymbol{A}_1 \cdot d\boldsymbol{l}_2 \tag{89}$$

or

$$\int_{C_2} (\boldsymbol{E}_1 + \frac{\partial}{\partial t}\boldsymbol{A}_1) \cdot d\boldsymbol{l}_2 = 0 \tag{90}$$

or

$$(\boldsymbol{E}_1 + \frac{\partial}{\partial t}\boldsymbol{A}_1) = -\nabla\phi_1 \tag{91}$$

or

$$\boldsymbol{E}_1 = -\frac{\partial}{\partial t}\boldsymbol{A}_1 - \nabla\phi_1 \tag{92}$$

ϕ_1 is any scalar function,

$$\nabla \times \boldsymbol{E}_1 = -\frac{\partial}{\partial t}\nabla \times \boldsymbol{A}_1 \tag{93}$$

define,

$$\boldsymbol{B}_1 \equiv \nabla \times \boldsymbol{A}_1 \tag{94}$$

$$\nabla \times \boldsymbol{E}_1 = -\frac{\partial}{\partial t}\boldsymbol{B}_1 \tag{95}$$

When the contribution of $\boldsymbol{A}_1$ can be ignored,

$$\boldsymbol{E}_1 = -\nabla\phi_1 \tag{96}$$

$$\nabla \cdot \boldsymbol{E}_1 = \rho/\epsilon_0 \tag{97}$$

$$\nabla \cdot (\nabla\phi_1) = -\rho/\epsilon_0 \tag{98}$$

$$\nabla^2\phi_1 = -\rho/\epsilon_0 \tag{99}$$

$$\phi_1 = \frac{1}{4\pi\epsilon_0} \iiint_V \frac{\rho}{r} dV \tag{100}$$

Considering that the scalar retarded potential is,

$$\phi_1 = \frac{1}{4\pi\epsilon_0} \iiint_V \frac{\rho(\boldsymbol{x}', t - r/c)}{r} dV \tag{101}$$

for the same reason

$$\phi_2 = \frac{1}{4\pi\epsilon_0} \iiint_V \frac{\rho}{r} dV \tag{102}$$

Consider advanced potential

$$\phi_2 = \frac{1}{4\pi\epsilon_0} \iiint_V \frac{\rho(\boldsymbol{x}', t + r/c)}{r} dV \tag{103}$$

The electric field,

$$\boldsymbol{E}_2 = -\frac{\partial}{\partial t}\boldsymbol{A}_2 - \nabla\phi_2 \tag{104}$$

$$\boldsymbol{B}_2 \equiv \nabla \times \boldsymbol{A}_2 \tag{105}$$

Because the author introduces the law of conservation of energy and applies it as axiom, we get that the advanced wave exists. Of course, if we admit the existence of advanced waves, we can derive the energy conservation law from Maxwell's equations by using the existence of advanced waves. However, because the advanced wave is controversial, the author chose to add an energy conservation law to Maxwell's electromagnetic theory as an axiom. Now the advanced wave can be derived from the axiom of energy conservation. As long as the reader accepts the law of conservation of energy, the advanced wave must exist now! So in fact, we have proved the existence of advanced wave by derivation.

F. Derivation of Maxwell's equations

Vector potential and scalar potential satisfy Poisson equation

$$\nabla^2\phi = -\rho/\epsilon_0 \tag{106}$$

$$\nabla^2\boldsymbol{A} = -\mu_0\boldsymbol{J} \tag{107}$$

The retarded potential and the advanced potential satisfy the wave equation

$$\nabla^2\phi - \mu_0\epsilon_0\frac{\partial}{\partial t}\phi = -\rho/\epsilon_0 \tag{108}$$

$$\nabla^2\boldsymbol{A} - \mu_0\epsilon_0\frac{\partial}{\partial t}\boldsymbol{A} = -\mu_0\boldsymbol{J} \tag{109}$$

For the wave equations, we omit the subscript $1, 2$ because the retarded wave and the advanced wave satisfy the same wave equations.

G. Lorenz gauge condition

Considering the current continuity equation,

$$\nabla \cdot \boldsymbol{J} = -\frac{\partial}{\partial t}\rho \tag{110}$$

The above formula (106, 107) can be verified to meet the Lorenz guage condition, and the wave equation (108,109]) also meets the Lorenz gauge condition,

$$\nabla \cdot \boldsymbol{A} + \mu_0\epsilon_0\frac{\partial}{\partial t}\phi = 0 \tag{111}$$

H. Derive Maxwell-Ampere circuital law from wave equations

Starting from the wave equation,

$$\nabla^2 \boldsymbol{A} - \mu_0\epsilon_0 \frac{\partial^2}{\partial t^2}\boldsymbol{A} = -\mu_0 \boldsymbol{J} \tag{112}$$

Considering the mathematical formula,

$$\nabla\times\nabla\times\boldsymbol{A} = \nabla\nabla\cdot\boldsymbol{A} - \nabla^2\boldsymbol{A} \tag{113}$$

or

$$\nabla^2\boldsymbol{A} = -\nabla\times\nabla\times\boldsymbol{A} + \nabla\nabla\cdot\boldsymbol{A} \tag{114}$$

Hence, there is

$$-\nabla\times\nabla\times\boldsymbol{A} + \nabla\nabla\cdot\boldsymbol{A} - \mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\boldsymbol{A} = -\mu_0\boldsymbol{J} \tag{115}$$

Considering $\boldsymbol{B} = \nabla\times\boldsymbol{A}$ and Lorenz gauge condition $\nabla\cdot\boldsymbol{A} = -\mu_0\epsilon_0\frac{\partial}{\partial t}\phi$,

$$-\nabla\times\boldsymbol{B} + \nabla(-\mu_0\epsilon_0\frac{\partial}{\partial t}\phi) - \mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\boldsymbol{A} = -\mu_0\boldsymbol{J} \tag{116}$$

$$-\nabla\times\boldsymbol{B} + \mu_0\epsilon_0\frac{\partial}{\partial t}(-\nabla\phi - \frac{\partial}{\partial t}\boldsymbol{A}) = -\mu_0\boldsymbol{J} \tag{117}$$

Consider $\boldsymbol{E} = -\nabla\phi - \frac{\partial}{\partial t}\boldsymbol{A}$

$$-\nabla\times\boldsymbol{B} + \mu_0\epsilon_0\frac{\partial}{\partial t}\boldsymbol{E} = -\mu_0\boldsymbol{J} \tag{118}$$

or

$$\mu_0\boldsymbol{J} + \mu_0\epsilon_0\frac{\partial}{\partial t}\boldsymbol{E} = \nabla\times\boldsymbol{B} \tag{119}$$

or

$$\nabla\times\boldsymbol{H} = \boldsymbol{J} + \frac{\partial}{\partial t}\epsilon_0\boldsymbol{E} \tag{120}$$

or

$$\nabla\times\boldsymbol{H} = \boldsymbol{J} + \frac{\partial}{\partial t}\boldsymbol{D} \tag{121}$$

considering (95), there is

$$\nabla\times\boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{B} \tag{122}$$

The above two formula are Ampere's circuital law and Faraday's law of Maxwell's equations.

I. Maxwell's equations

Mutual energy theory respects magnetic quasi-static electromagnetic field equations. They are skeptical of Maxwell's equations, especially those caused by displacement current. But we still start from Maxwell's equation, which is,

$$\begin{cases}\nabla\times\boldsymbol{H} = \boldsymbol{J} + \frac{\partial}{\partial t}\boldsymbol{D}\\ \nabla\times\boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{B}\end{cases} \tag{123}$$

or

$$\begin{cases}-\frac{\partial}{\partial t}\boldsymbol{D} + \nabla\times\boldsymbol{H} = \boldsymbol{J}\\ -\nabla\times\boldsymbol{E} - \frac{\partial}{\partial t}\boldsymbol{B} = 0\end{cases} \tag{124}$$

or

$$L\xi = \tau \tag{125}$$

where

$$L = \begin{bmatrix} -\frac{\partial}{\partial t}\epsilon_0, & \nabla\times \\ -\nabla\times & -\frac{\partial}{\partial t}\mu_0\end{bmatrix}, \quad \xi = \begin{bmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{bmatrix} \quad \tau = \begin{bmatrix}\boldsymbol{J}\\ 0\end{bmatrix} \tag{126}$$

Actually there are a group of Maxwell's equations,

$$\begin{cases}L\xi_1 = \tau_1\\ L\xi_2 = \tau_2\end{cases} \tag{127}$$

J. Green's function

Can prove a mathematical formula, similar to Green's function

$$-\oint\!\!\!\oint_\Gamma (\boldsymbol{E}_1\times\boldsymbol{H}_2 + \boldsymbol{E}_2\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma$$

$$\iiint_V (\xi_1\cdot L_2\xi_2 + \xi_2\cdot L_1\xi_1)dV$$

$$+\iiint_V (\epsilon_0\boldsymbol{E}_1\cdot\frac{\partial\boldsymbol{E}_2}{\partial t} + \epsilon_0\boldsymbol{E}_2\cdot\frac{\partial\boldsymbol{E}_1}{\partial t} + \epsilon_0\boldsymbol{H}_1\cdot\frac{\partial\boldsymbol{H}_2}{\partial t} + \epsilon_0\boldsymbol{H}_2\cdot\frac{\partial\boldsymbol{H}_1}{\partial t})dV \tag{128}$$

This formula is not a physical formula because Maxwell's equations has not been applied. It's just a mathematical formula.

K. Mutual energy principle

Consider the following figure 5 there are two current elements inside a closed surface Γ, one is the source and the other is the sink. The source emits retarded wave and the sink emits advanced wave.

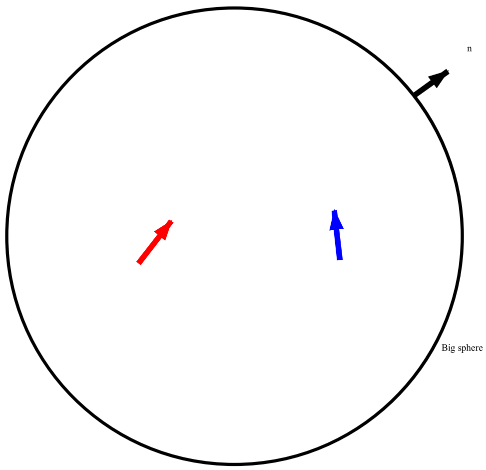

Figure 5. In the principle of mutual energy, there are two current elements, one is the source and the other is the sink.

By substituting Maxwell's equations (127) into Green's function,

$$-\oiint_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_2+\boldsymbol{E}_2\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma$$

$$=\iiint_V(\xi_1\cdot\tau_1+\xi_2\cdot\tau_2)dV$$

$$+\iiint_V(\epsilon_0\boldsymbol{E}_1\cdot\frac{\partial\boldsymbol{E}_2}{\partial t}+\epsilon_0\boldsymbol{E}_2\cdot\frac{\partial\boldsymbol{E}_1}{\partial t}+\mu_0\boldsymbol{H}_1\cdot\frac{\partial\boldsymbol{H}_2}{\partial t}+\mu_0\boldsymbol{H}_2\cdot\frac{\partial\boldsymbol{H}_1}{\partial t})dV \tag{129}$$

or

$$-\oiint_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_2+\boldsymbol{E}_2\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma$$

$$=\iiint_V(\boldsymbol{E}_1\cdot\boldsymbol{J}_1+\boldsymbol{E}_2\cdot\boldsymbol{J}_2)dV$$

$$+\iiint_V(\epsilon_0\boldsymbol{E}_1\cdot\frac{\partial\boldsymbol{E}_2}{\partial t}+\epsilon_0\boldsymbol{E}_2\cdot\frac{\partial\boldsymbol{E}_1}{\partial t}+\mu_0\boldsymbol{H}_1\cdot\frac{\partial\boldsymbol{H}_2}{\partial t}+\mu_0\boldsymbol{H}_2\cdot\frac{\partial\boldsymbol{H}_1}{\partial t})dV \tag{130}$$

or

$$-\sum_{i=1}^{2}\sum_{j=1,j\neq i}^{2}\oiint_{\Gamma}(\boldsymbol{E}_i\times\boldsymbol{H}_j)\cdot\hat{n}d\Gamma$$

$$=\sum_{i=1}^{2}\sum_{j=1,j\neq i}^{2}\iiint_V(\boldsymbol{E}_i\cdot\boldsymbol{J}_j+\epsilon_0\boldsymbol{E}_i\cdot\frac{\partial\boldsymbol{E}_j}{\partial t}+\mu_0\boldsymbol{H}_i\cdot\frac{\partial\boldsymbol{H}_j}{\partial t})dV \tag{131}$$

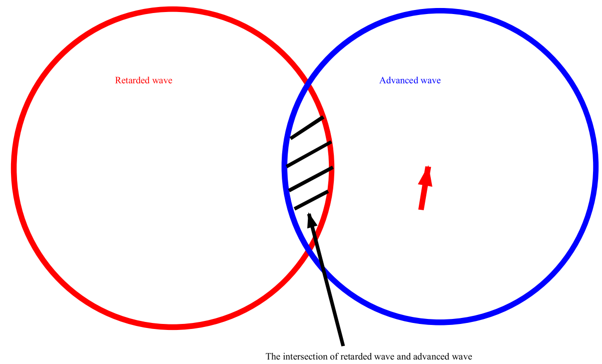

Figure 6. The union $R\cup A$ of retarded wave and advanced wave is the solutions of Maxwell's equations, and their intersection $R\cap A$ is the solutions of mutual energy principle.

From 2 generated to N,

$$-\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\oiint_{\Gamma}(\boldsymbol{E}_i\times\boldsymbol{H}_j)\cdot\hat{n}d\Gamma$$

$$=\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\iiint_V(\boldsymbol{E}_i\cdot\boldsymbol{J}_j+\epsilon_0\boldsymbol{E}_i\cdot\frac{\partial\boldsymbol{E}_j}{\partial t}+\mu_0\boldsymbol{H}_i\cdot\frac{\partial\boldsymbol{H}_j}{\partial t})dV \tag{132}$$

This leads to the principle of mutual energy. You can also derive Maxwell's equations from the principle of mutual energy (this step is omitted). However, the principle of mutual energy is different from Maxwell's equations, because for the principle of mutual energy

$$N\geq 2 \tag{133}$$

$$N\neq 1 \tag{134}$$

If $N=1$, the principle of mutual energy is not tenable, but Maxwell's equation is still tenable, and Maxwell's equations can still obtain the retarded wave and advanced wave. This kind of retarded wave and advanced wave are not the solution of the principle of mutual energy. The solution of the principle of mutual energy is the solution of a retarded wave and an advanced wave, which requires not only pairing but also synchronization. Here synchronization means that when the retarded wave from the light source reaches the light sink, the sink just sends out the advanced wave. Only in this way can the two waves be synchronized. Therefore, the solution of the principle of mutual energy is much less than that of Maxwell's equations. Therefore, the principle of mutual energy is not the equivalent of Maxwell's equations, but a limiting condition applied to Maxwell's equations. Therefore, the principle of mutual energy has its own physical meaning independent of Maxwell's equations. The figure below illustrates this view.

In the above figure 6, the red area represents the set R of retarded waves, the blue represents the set A of advanced waves, and the solutions of Maxwell's equations can be expressed as the union set $R \cup A$. The solutions of the mutual energy principle is expressed as the intersection $R \cap A$. because,

$$R \cap A \ll R \cup A \tag{135}$$

This shows that the solution of the principle of mutual energy is much less than that of Maxwell's equations. Therefore, the principle of mutual energy is not optional relative to Maxwell's equation.

L. Mutual energy flow does not overflow the universe

It is know that,

$$\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\iiint_V (\epsilon_0 \boldsymbol{E}_i \cdot \frac{\partial \boldsymbol{E}_j}{\partial t} + \mu_0 \boldsymbol{H}_i \cdot \frac{\partial \boldsymbol{H}_j}{\partial t})dV$$

$$= \frac{\partial}{\partial t}\sum_{i=1}^{N}\sum_{j=1}^{j<i}\iiint_V (\epsilon_0 \boldsymbol{E}_i \cdot \boldsymbol{E}_j + \mu_0 \boldsymbol{H}_i \cdot \partial \boldsymbol{H}_j)dV$$

$$= \frac{\partial}{\partial t}U \tag{136}$$

where

$$U = \sum_{i=1}^{N}\sum_{j=1}^{j<i}\iiint_V (\epsilon_0 \boldsymbol{E}_i \cdot \boldsymbol{E}_j + \mu_0 \boldsymbol{H}_i \cdot \partial \boldsymbol{H}_j)dV \tag{137}$$

Hence, there is

$$\int_{t=-\infty}^{\infty} dt \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\iiint_V (\epsilon_0 \boldsymbol{E}_i \cdot \frac{\partial \boldsymbol{E}_j}{\partial t} + \mu_0 \boldsymbol{H}_i \cdot \frac{\partial \boldsymbol{H}_j}{\partial t})dV$$

$$= \int_{t=-\infty}^{\infty} \frac{\partial}{\partial t} U dt = U(\infty) - U(-\infty) = 0 \tag{138}$$

For $U(-\infty)$, the process has not yet occurred, and for the $U(\infty)$ the process has been completed. Therefore, variety $U(\infty)$ =$U(-\infty)$. Therefore, we do time integration for (132) and subtract (138) to get,

$$-\int_{t=-\infty}^{\infty} dt \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} \oiint_\Gamma (\boldsymbol{E}_i \times \boldsymbol{H}_j)\cdot \hat{n} d\Gamma$$

$$= \int_{t=-\infty}^{\infty} dt \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\iiint_V (\boldsymbol{E}_i \cdot \boldsymbol{J}_j)dV \tag{139}$$

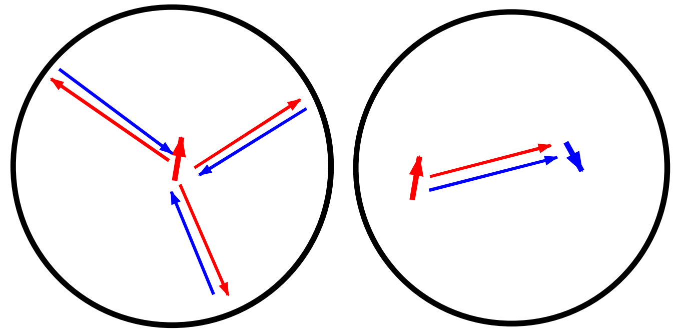

Figure 7. Neither self energy flow nor mutual energy flow can overflow the universe.

Subtract (68) from the above formula to obtain,

$$-\int_{t=-\infty}^{\infty} dt \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} \oiint_\Gamma (\boldsymbol{E}_i \times \boldsymbol{H}_j)\cdot \hat{n} d\Gamma = 0 \tag{140}$$

Or more strictly, requirements

$$\int_{t=-\infty}^{\infty} dt \oiint_\Gamma (\boldsymbol{E}_i \times \boldsymbol{H}_j)\cdot \hat{n} d\Gamma = 0 \tag{141}$$

Changed to the frequency domain,

$$\Re \oiint_\Gamma (\boldsymbol{E}_i(\omega) \times \boldsymbol{H}_j^*(\omega))\cdot \hat{n} d\Gamma = 0 \tag{142}$$

$\Re$ is to take the real part of the complex number. Γ can be a sphere with an infinite radius, or any closed surface surrounding all current elements. The above formula indicates that the mutual energy shall not overflow to the outside of the universe. In fact, not only the mutual energy flow can not overflow the universe, in fact, nothing can overflow the universe, otherwise the energy is not conserved, so the self energy flow can not overflow the universe.

The above figure 7 shows that the self energy flow should return to the source or sink from the sphere with infinite radius. Mutual energy flow is the point-to-point propagation of energy, which will not overflow the universe. In this way, all energy flows can be guaranteed not to overflow the universe. The Poynting theorem of classical electromagnetic field theory does not satisfy the requirement that the self energy flow does not overflow the universe, so the classical electromagnetic field theory is needed to be revised.

M. Mutual energy flow theorem

The principle of mutual energy when $N = 2$ (139) can be written as

$$-\int_{t=-\infty}^{\infty} dt \sum_{i=1}^{2}\sum_{j=1,j\neq i}^{2} \oiint_\Gamma (\boldsymbol{E}_i \times \boldsymbol{H}_j)\cdot \hat{n} d\Gamma$$

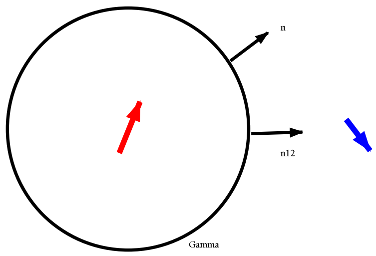

Figure 8. Current element $\boldsymbol{J}_2$ (the blue one) or the sink is not inside the closed surface Γ.

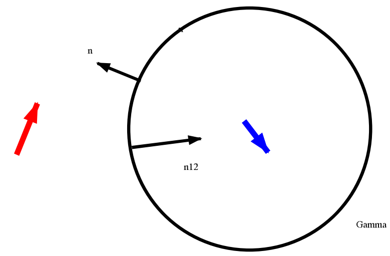

Figure 9. Current element 1 or source is not within the closed surface Γ.

$$=\int_{t=-\infty}^{\infty} dt \sum_{i=1}^{2} \sum_{j=1, j\neq i}^{2} \iiint_V (\boldsymbol{E}_i \cdot \boldsymbol{J}_j) dV \tag{143}$$

or

$$-\int_{t=-\infty}^{\infty} dt \oint\!\!\!\oint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma$$

$$=\int_{t=-\infty}^{\infty} dt \iiint_V (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2 + \boldsymbol{E}_2 \cdot \boldsymbol{J}_1) dV \tag{144}$$

Take the closed surface Γ_1 to surround only $\boldsymbol{J}_1$. $\boldsymbol{J}_2$ not inside the surface, as shown in figure 8, the above formula can be rewritten as

$$-\int_{t=-\infty}^{\infty} dt \oint\!\!\!\oint_{\Gamma_1} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma$$

$$=\int_{t=-\infty}^{\infty} dt \iiint_V (\boldsymbol{E}_2 \cdot \boldsymbol{J}_1) dV \tag{145}$$

or

$$-\int_{t=-\infty}^{\infty} dt \oint\!\!\!\oint_{\Gamma_1} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n}_{12} d\Gamma$$

$$=\int_{t=-\infty}^{\infty} dt \iiint_V (\boldsymbol{E}_2 \cdot \boldsymbol{J}_1) dV \tag{146}$$

$\hat{n}$ is the normal of the surface, $\hat{n}_{12}$ is also the normal of the surface, but this normal is from V_1 to V_2 and we know that there are,

$$\hat{n} = \hat{n}_{12}$$

Take closed surface Γ_2 to surround $\boldsymbol{J}_2$. $\boldsymbol{J}_1$ is not inside the surface Γ, see figure 9,

Mutual energy principle can be written as,

$$-\int_{t=-\infty}^{\infty} dt \oint\!\!\!\oint_{\Gamma_2} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma$$

$$=\int_{t=-\infty}^{\infty} dt \iiint_V (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2) dV \tag{147}$$

or

$$\int_{t=-\infty}^{\infty} dt \oint\!\!\!\oint_{\Gamma_2} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n}_{1,2} d\Gamma$$

$$=\int_{t=-\infty}^{\infty} dt \iiint_V (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2) dV \tag{148}$$

Where we have consider

$$\hat{n} = -\hat{n}_{12}$$

We already know the law of conservation of energy,

$$-\int_{t=-\infty}^{\infty} dt \iiint_{V_1} (\boldsymbol{J}_1 \cdot \boldsymbol{E}_2) dV = \int_{t=-\infty}^{\infty} dt \iiint_{V_2} (\boldsymbol{J}_2 \cdot \boldsymbol{E}_1) dV \tag{149}$$

Considering (147, 148), we get

$$-\int_{t=-\infty}^{\infty} dt \iiint_{V_1} (\boldsymbol{J}_1 \cdot \boldsymbol{E}_2) dV$$

$$=\int_{t=-\infty}^{\infty} dt \oint\!\!\!\oint_{\Gamma_1} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n}_{12} d\Gamma$$

$$=\int_{t=-\infty}^{\infty} dt \oint\!\!\!\oint_{\Gamma_2} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n}_{1,2} d\Gamma$$

$$=\int_{t=-\infty}^{\infty} dt \iiint_{V_2} (\boldsymbol{J}_2 \cdot \boldsymbol{E}_1) dV \tag{150}$$

Γ_1 and Γ_2 can be merged so that,

$$-\int_{t=-\infty}^{\infty} dt \iiint_{V_1} (\boldsymbol{J}_1 \cdot \boldsymbol{E}_2) dV$$

$$= (\xi_1, \xi_2)$$

$$=\int_{t=-\infty}^{\infty} dt \iiint_{V_2} (\boldsymbol{J}_2 \cdot \boldsymbol{E}_1) dV \tag{151}$$

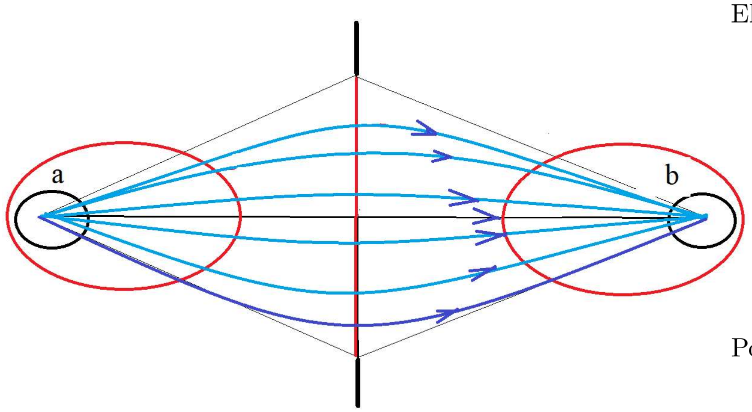

Figure 10. The mutual energy flow from the source to the sink.

where (ξ_1, ξ_2) is defined as,

$$(\xi_1, \xi_2) \equiv \int_{t=-\infty}^{\infty} dt \oiint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n}_{1,2} d\Gamma \tag{152}$$

Γ is any surface which divided V_1 and V_2, Γ can be a closed surface the surrounding V_1 or V_2 It can also be an infinite plane that divides two volumes V_1 and V_2. Refer to figure 10 for the shape of mutual energy flow.

Note that although the mutual energy flow theorem can be derived from Maxwell's equations through the mutual energy principle, the solutions satisfying the mutual energy flow theorem may not satisfy the mutual energy principle and Maxwell's equations. The mutual energy flow theorem still leaves a degree of freedom for the solution of electromagnetic field. Later, we can see that we need to limit this degree of freedom by the condition that self energy flow does not radiate. In fact, the solution of electromagnetic field obtained according to the mutual energy flow theorem is different from that obtained according to Maxwell's equations. This will be shown in the example in the following sections.

N. Poynting theorem

In addition, we know that Maxwell's equations can derive Poynting's theorem,

$$-\oiint_\Gamma \boldsymbol{E} \times \boldsymbol{H} \cdot \hat{n} d\Gamma = \iiint_V (\boldsymbol{J} \cdot \boldsymbol{E} + \boldsymbol{E} \cdot \frac{\partial}{\partial t} \boldsymbol{D} + \boldsymbol{H} \cdot \frac{\partial}{\partial t} \boldsymbol{B}) dV \tag{153}$$

If N current elements are considered,

$$\boldsymbol{J} = \sum_{i=1}^{N} \boldsymbol{J}_i \tag{154}$$

Electromagnetic field shall meet

$$\boldsymbol{E} = \sum_{i=1}^{N} \boldsymbol{E}_i \tag{155}$$

$$\boldsymbol{H} = \sum_{i=1}^{N} \boldsymbol{H}_i \tag{156}$$

Poynting's theorem for N current elements is,

$$-\oiint_\Gamma \sum_{i=1}^{N} \sum_{j=1}^{N} \boldsymbol{E}_i \times \boldsymbol{H}_j \cdot \hat{n} d\Gamma$$

$$= \iiint_V \sum_{i=1}^{N} \sum_{j=1}^{N} (\boldsymbol{J}_i \cdot \boldsymbol{E}_j + \boldsymbol{E}_i \cdot \frac{\partial}{\partial t} \boldsymbol{D}_j + \boldsymbol{H}_i \cdot \frac{\partial}{\partial t} \boldsymbol{B}_j) dV \tag{157}$$

O. Contradiction between the principle of mutual energy and Poynting's theorem

Comparing formulas (157) and (132), both are energy conservation theorems of N current elements. If both are true, there must be a difference between the two,

$$-\oiint_\Gamma \sum_{i=1}^{N} \boldsymbol{E}_i \times \boldsymbol{H}_i \cdot \hat{n} d\Gamma$$

$$= \iiint_V \sum_{i=1}^{N} (\boldsymbol{J}_i \cdot \boldsymbol{E}_i + \boldsymbol{E}_i \cdot \frac{\partial}{\partial t} \boldsymbol{D}_i + \boldsymbol{H}_i \cdot \frac{\partial}{\partial t} \boldsymbol{B}_i) dV \tag{158}$$

does not transmit energy, or,

$$-\oiint_\Gamma \boldsymbol{E}_i \times \boldsymbol{H}_i \cdot \hat{n} d\Gamma =$$

$$\iiint_V (\boldsymbol{J}_i \cdot \boldsymbol{E}_i + \boldsymbol{E}_i \cdot \frac{\partial}{\partial t} \boldsymbol{D}_i + \boldsymbol{H}_i \cdot \frac{\partial}{\partial t} \boldsymbol{B}_i) dV \quad i = 1, 2 \cdots N \tag{159}$$

Does not transmit energy. But the above formula is the Poynting theorem of the current element $\boldsymbol{J}_i$, the classical electromagnetic theory tells us $\oiint_\Gamma \boldsymbol{E}_i \times \boldsymbol{H}_i \cdot \hat{n} d\Gamma$ transfers the energy. In the frequency domain, that means,

$$\Re \oiint_\Gamma \boldsymbol{E}_i \times \boldsymbol{H}_i^* \cdot \hat{n} d\Gamma \neq 0 \tag{160}$$

This leads a contradiction.

P. Time reversal transformation

The first attempt to correct the contradiction of the classical electromagnetic theory is to add a time reversal wave to the classical electromagnetic theory[11]. First, the time reversal transformation is defined as,

$$\boldsymbol{E}(t),\boldsymbol{H}(t),\frac{\partial}{\partial t}\boldsymbol{E}(t),\frac{\partial}{\partial t}\boldsymbol{H}(t),\boldsymbol{J}(t)$$

$$\rightarrow\boldsymbol{E}^r(-t),\boldsymbol{H}^r(-t),-\frac{\partial}{\partial t}\boldsymbol{E}^r(-t),-\frac{\partial}{\partial t}\boldsymbol{H}^r(-t),-\boldsymbol{J}^r(-t) \quad (161)$$

or

$$\boldsymbol{E},\boldsymbol{H},\frac{\partial}{\partial t},\frac{\partial}{\partial t}\boldsymbol{H},\boldsymbol{J}\rightarrow\boldsymbol{E}^r,\boldsymbol{H}^r,-\frac{\partial}{\partial t}\boldsymbol{E}^r,-\frac{\partial}{\partial t}\boldsymbol{H}^r,-\boldsymbol{J}^r \quad (162)$$

Maxwell's equations,

$$\begin{cases}\nabla\times\boldsymbol{H}=\boldsymbol{J}+\frac{\partial}{\partial t}\boldsymbol{D}\\ \nabla\times\boldsymbol{E}=-\frac{\partial}{\partial t}\boldsymbol{B}\end{cases} \quad (163)$$

After time reversal transformation, it is called time reversal Maxwell equation,

$$\begin{cases}\nabla\times\boldsymbol{H}^r=-\boldsymbol{J}-\frac{\partial}{\partial t}\boldsymbol{D^r}\\ \nabla\times\boldsymbol{E}^r=\frac{\partial}{\partial t}\boldsymbol{B}^r\end{cases} \quad (164)$$

Poynting's theorem of time reversal,

$$-\oiint_\Gamma \boldsymbol{E}_i^r\times\boldsymbol{H}_i^r\cdot\hat{n}d\Gamma$$

$$=\iiint_V(-\boldsymbol{J}_i^r\cdot\boldsymbol{E}_i^r-\boldsymbol{E}_i^r\cdot\frac{\partial}{\partial t}\boldsymbol{D}_i^r-\boldsymbol{H}_i^r\cdot\frac{\partial}{\partial t}\boldsymbol{B}_i^r)dV \quad i=1,2\cdots N \quad (165)$$

Q. Self energy principle

Self energy flow should not transfer energy, but according to Maxwell's equations, self energy flow transfers energy. We can add a time reversal wave to Maxwell's equation so that the self energy flow does not transfer energy. The self energy flow is canceled by the time reversal wave, which is called the self energy principle, that is,

$$-\int_{t=-\infty}^{\infty}dt(\oiint_\Gamma\boldsymbol{E}_i\times\boldsymbol{H}_i\cdot\hat{n}d\Gamma+\oiint_\Gamma\boldsymbol{E}_i^r\times\boldsymbol{H}_i^r\cdot\hat{n}d\Gamma)=0 \quad (166)$$

$$\int_{t=-\infty}^{\infty}dt\iiint_V(\boldsymbol{J}_i\cdot\boldsymbol{E}_i-\boldsymbol{J}_i^r\cdot\boldsymbol{E}_i^r)=0 \quad (167)$$

$$\int_{t=-\infty}^{\infty}dt\iiint_V(\boldsymbol{E}_i\cdot\frac{\partial}{\partial t}\boldsymbol{D}_i+\boldsymbol{H}_i\cdot\frac{\partial}{\partial t}\boldsymbol{B}_i-\boldsymbol{E}_i^r\cdot\frac{\partial}{\partial t}\boldsymbol{D}_i^r-\boldsymbol{H}_i^r\cdot\frac{\partial}{\partial t}\boldsymbol{B}_i^r)dV$$

$$=0 \quad (168)$$

When we add the time reversal wave, the self energy flow radiation becomes zero. The above formula is called the self energy principle.

R. Reactive power wave

The electromagnetic and magnetic fields of this wave have a phase of 90 degrees. If it's a plane wave, for example

$$\boldsymbol{H}=H_0\exp(j\omega-jkx)\hat{y} \quad (169)$$

$$\boldsymbol{E}=j\eta_0H_0\exp(j\omega-jkx)(-\hat{z}) \quad (170)$$

$$\boldsymbol{E}\times\boldsymbol{H}^*=j\eta_0H_0\exp(j\omega-jkx)(-\hat{z})\times(H_0\exp(j\omega-jkx)\hat{y})^* \quad (171)$$

$$=j\eta_0H_0{H_0}^*\hat{x} \quad (172)$$

Hence,

$$\Re(\boldsymbol{E}\times\boldsymbol{H}^*)=0 \quad (173)$$

$\Re$ is taking the real part. This wave is a reactive power wave. The real part of the Poynting vector of the reactive power wave is zero. The average radiated power of the wave of reactive power is 0.

S. Equivalence of reactive power wave and time reversal wave

We know that the Maxwell's equations without considering the source is,

$$\nabla\times\boldsymbol{E}=-\frac{\partial}{\partial t}\boldsymbol{B}$$

$$\nabla\times\boldsymbol{H}=\frac{\partial}{\partial t}\boldsymbol{D}$$

If $\boldsymbol{H}$ and $\boldsymbol{E}$ have a phase difference of 90 degrees, half the time $\boldsymbol{H}$ and $\boldsymbol{E}$ are in the same phase, and half the time $\boldsymbol{H}$ and $\boldsymbol{E}$ are in the opposite phase (180 degree phase difference). When the phase is opposite, we can adjust the phase of magnetic field and electromagnetic field in Maxwell's equations as,

$$\nabla\times\boldsymbol{E}=\frac{\partial}{\partial t}\boldsymbol{B}$$

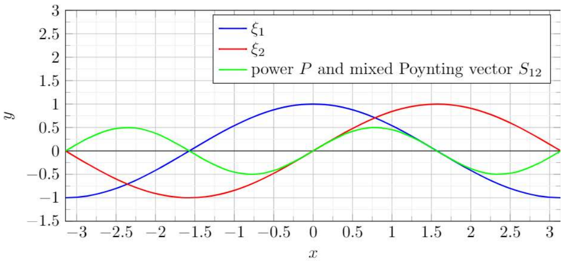

Figure 11. Energy flow curve of reactive power wave.

$$\nabla \times \boldsymbol{H} = -\frac{\partial}{\partial t}\boldsymbol{D}$$

This is actually a time reversal wave. The Poynting vector of the time reversal wave is in the opposite direction to that of the normal wave. Therefore, in half of a cycle, the energy flow emitted by the normal wave shoots into the whole space. In the other half of a cycle, the time reversal wave can be used to return the energy flow from the whole space to the light source or the light sink.

We know that there is a certain time difference between the time reversal wave and the original wave. Even if there is a wavelength error between the time reversal wave and the original wave, the energy can still be returned. So all the energy still returns to the light source.

We find that the period of reactive power wave can be divided into four parts. Two of them are positive, and the other two are negative. Therefore, this wave is a wave with positive energy propagation for half of a period, and a wave with negative energy propagation for the other half. This wave can transmit energy to space, but the energy is not lost, because the energy returns in a time reversal process. This is called reverse collapse. Reactive power wave can replace the combination of normal wave and time reversal wave.

In the figure below, red is the curve of electric field and blue is the curve of magnetic field. There is a 90 degree phase between them. Green is the magnitude of Poynting vector of electric and magnetic fields. We will find that the green power changes sign twice in a cycle. Twice is positive and twice is negative. Therefore, half the time is normal wave and half time is time reversal wave. The power flow of time reversal wave is negative. It indicates that although a wave is emitted, it returns or collapses inversely in time.

In the above figure 11, the electric field $\boldsymbol{E}$ is represented by red lines, the magnetic field $\boldsymbol{H}$ is represented by blue lines, and the power of Poynting vector is represented by green. In one cycle, two $\frac{1}{4}$ cycles of power are positive and two $\frac{1}{4}$ cycles of power are negative. So the average power is 0. This indicates that Poynting vector $\boldsymbol{E} \times \boldsymbol{H}$ is reactive power.

T. Self energy flow theorem

Since we can add a time reversal wave, it means that we can properly adjust the phase of the electric field and the magnetic field to meet,

$$-\Re \oiint_{\Gamma} \boldsymbol{E}_i(\omega) \times \boldsymbol{H}_i^*(\omega) \cdot \hat{n} d\Gamma = 0 \quad i = 1, 2 \cdots N \quad (174)$$

$\Re$ is the real part of the complex number. This indicates that the self energy flow is a pure imaginary number, i.e. reactive power.

U. A guess

We know that Maxwell's equation is,

$$\nabla \times \boldsymbol{E} = -\mu_0 \frac{\partial}{\partial t}\boldsymbol{H} \quad (175)$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \epsilon_0 \frac{\partial}{\partial t}\boldsymbol{E} \quad (176)$$

For a transformer system this actually means

$$\nabla \times \boldsymbol{E}_1 = -\mu_0 \frac{\partial}{\partial t}(\boldsymbol{H}_1 + \boldsymbol{H}_2) \quad (177)$$

That is to say, the induced electric field of the primary coil is not only related to the magnetic field change of the primary coil, but also related to the magnetic field of the secondary coil. The magnetic field of the secondary coil also contributes to the induced potential of the primary coil. Therefore,

$$\nabla \times \boldsymbol{E}_2 = -\mu_0 \frac{\partial}{\partial t}(\boldsymbol{H}_1 + \boldsymbol{H}_2) \quad (178)$$

Similarly, there should to have,

$$\nabla \times \boldsymbol{H}_1 = \boldsymbol{J}_1 + \epsilon_0 \frac{\partial}{\partial t}(\boldsymbol{E}_1 + \boldsymbol{E}_2) \quad (179)$$

$$\nabla \times \boldsymbol{H}_2 = \boldsymbol{J}_2 + \epsilon_0 \frac{\partial}{\partial t}(\boldsymbol{E}_1 + \boldsymbol{E}_2) \quad (180)$$

This is a coupled equations, and we may need to know the impedance conditions at the secondary coil. At present, it is not known how to find the solution of this coupled equations. Therefore, it is considered to simplify the equations and ignore the magnetic field coupling item from above two equations,

$$\nabla \times \boldsymbol{H}_1 = \boldsymbol{J}_1 + \epsilon_0 \frac{\partial}{\partial t}\boldsymbol{E}_1 \quad (181)$$

$$\nabla \times \boldsymbol{H}_2 = \boldsymbol{J}_2 + \epsilon_0 \frac{\partial}{\partial t}\boldsymbol{E}_2 \quad (182)$$

Let's split (177,178) into two terms,

$$\nabla \times \boldsymbol{E}_{1,1} = -\mu_0 \frac{\partial}{\partial t} \boldsymbol{H}_1 \tag{183}$$

$$\nabla \times \boldsymbol{E}_{1,2} = -\mu_0 \frac{\partial}{\partial t} \boldsymbol{H}_2 \tag{184}$$

$$\nabla \times \boldsymbol{E}_{2,1} = -\mu_0 \frac{\partial}{\partial t} \boldsymbol{H}_1 \tag{185}$$

$$\nabla \times \boldsymbol{E}_{2,2} = -\mu_0 \frac{\partial}{\partial t} \boldsymbol{H}_2 \tag{186}$$

$\boldsymbol{E}_{1,1}$ is the magnetic field $\boldsymbol{H}_1$ The magnetic field generated on the primary coil actually has,

$$\boldsymbol{E}_{1,1} \rightarrow \boldsymbol{E}_1 \tag{187}$$

$$\nabla \times \boldsymbol{E}_1 = -\mu_0 \frac{\partial}{\partial t} \boldsymbol{H}_1 \tag{188}$$

$$\nabla \times \boldsymbol{H}_1 = \boldsymbol{J}_1 + \epsilon_0 \frac{\partial}{\partial t} \boldsymbol{E}_1 \tag{189}$$

Similarly, there is,

$$\boldsymbol{E}_{2,2} \rightarrow \boldsymbol{E}_2 \tag{190}$$

$$\nabla \times \boldsymbol{E}_2 = -\mu_0 \frac{\partial}{\partial t} \boldsymbol{H}_2 \tag{191}$$

$$\nabla \times \boldsymbol{H}_2 = \boldsymbol{J}_2 + \epsilon_0 \frac{\partial}{\partial t} \boldsymbol{E}_2 \tag{192}$$

So we get two Maxwell equations, solve them separately, and finally use

$$\nabla \times \boldsymbol{E}_{1,2} = -\mu_0 \frac{\partial}{\partial t} \boldsymbol{H}_2$$

$$\nabla \times \boldsymbol{E}_{2,1} = -\mu_0 \frac{\partial}{\partial t} \boldsymbol{H}_1$$

Get $\boldsymbol{E}_{1,2}$ and $\boldsymbol{E}_{2,1}$, they are the contribution of the secondary coil magnetic field to the primary coil electric field and the primary coil magnetic field to the secondary coil electric field. In this way, the coupling problem is solved separately. The above two equations are equivalent to

$$\mathcal{E}_{1,2} = -M_{1,2} \frac{dI_2}{dt} \tag{193}$$

$$\mathcal{E}_{2,1} = -M_{2,1} \frac{dI_1}{dt} \tag{194}$$

The problem is simplified. Now let's look back at how we achieved this simplification. We ignored the contribution that $\frac{\partial}{\partial t}\boldsymbol{E}_2$ gives to the magnetic field $\boldsymbol{H}_1$. We also ignored the contribution that $\frac{\partial}{\partial t}\boldsymbol{E}_1$ gives to the magnetic field $\boldsymbol{H}_2$.

This makes the magnetic field $\boldsymbol{H}_1$, $\boldsymbol{H}_2$ to have some deviation in the calculation. This provides a basis for us to adjust the phase of the magnetic field. Because we have not solved the coupled equations, we have neglected some important things. Therefore, Maxwell's equation must be adjusted.

V. Another guess

Here we give another explanation of Maxwell's equation, in the equation

$$\nabla \times \boldsymbol{E}_1 = -\mu_0 \frac{\partial}{\partial t} (\boldsymbol{H}_1 + \boldsymbol{H}_2) \tag{195}$$

The electric field is determined by the magnetic field, $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$, where $\boldsymbol{H}_1$ Is its own magnetic field, which we assume can be ignored,

$$\nabla \times \boldsymbol{E}_1 = -\mu_0 \frac{\partial}{\partial t} (\boldsymbol{H}_2) \tag{196}$$

Do the same for the formula (177-180), and get,

$$\begin{cases} \nabla \times \boldsymbol{E}_1 = -\mu_0 \frac{\partial}{\partial t}(\boldsymbol{H}_2) \\ \nabla \times \boldsymbol{E}_2 = -\mu_0 \frac{\partial}{\partial t}(\boldsymbol{H}_1) \\ \nabla \times \boldsymbol{H}_1 = \boldsymbol{J}_1 + \epsilon_0 \frac{\partial}{\partial t}(\boldsymbol{E}_2) \\ \nabla \times \boldsymbol{H}_2 = \boldsymbol{J}_2 + \epsilon_0 \frac{\partial}{\partial t}(\boldsymbol{E}_1) \end{cases} \tag{197}$$

In this case, the above Maxwell's equations can be divided into two groups,

$$\begin{cases} \nabla \times \boldsymbol{E}_2 = -\mu_0 \frac{\partial}{\partial t}(\boldsymbol{H}_1) \\ \nabla \times \boldsymbol{H}_1 = \boldsymbol{J}_1 + \epsilon_0 \frac{\partial}{\partial t}(\boldsymbol{E}_2) \end{cases} \tag{198}$$

$$\begin{cases} \nabla \times \boldsymbol{E}_1 = -\mu_0 \frac{\partial}{\partial t}(\boldsymbol{H}_2) \\ \nabla \times \boldsymbol{H}_2 = \boldsymbol{J}_2 + \epsilon_0 \frac{\partial}{\partial t}(\boldsymbol{E}_1) \end{cases} \tag{199}$$

Each of the above groups is Maxwell's equations, which can be solved. After these two sets of equations are solved, $\boldsymbol{E}_2$ and $\boldsymbol{H}_1$ have the same phase, $\boldsymbol{E}_1$ and $\boldsymbol{H}_2$have the same phase. This means that the solution of Maxwell's equation can guarantee that the Poynting vector is active power. In this case, it is actually guaranteed,

$$\Re \boldsymbol{E}_1 \times \boldsymbol{H}_2^* > 0 \tag{200}$$

$$\Re \boldsymbol{E}_2 \times \boldsymbol{H}_1^* > 0 \tag{201}$$

This result is correct. In other words, Maxwell's equations can be interpreted by other methods.

IV. SOLVE THE ELECTROMAGNETIC FIELD ACCORDING TO THE AUTHOR'S MUTUAL ENERGY FLOW THEOREM AND SELF ENERGY FLOW THEOREM

Finally, we give a method to solve the electromagnetic field problems according to the mutual energy theory. We find the electromagnetic field solution satisfying the following equation,

(1) Mutual energy flow theorem,

$$-\iiint_{V_1} \boldsymbol{e_2^*}\cdot\boldsymbol{J}_1 dV$$

$$=\oiint_{\Gamma}(\boldsymbol{e}_1\times\boldsymbol{h}_2^*+\boldsymbol{e}_2^*\times\boldsymbol{h}_1)\cdot\hat{n}d\Gamma=\iiint_{V_2}\boldsymbol{e_1}\cdot\boldsymbol{J}_2^* dV \quad (202)$$

Where Γ is a closed surface or infinite plane which is the segmentation of V_1 and V_2. $\boldsymbol{e}_1$,$\boldsymbol{h}_1$,$\boldsymbol{e}_2$,$\boldsymbol{h}_2$ are still calculated according to the retarded wave and advanced wave of Maxwell's equations, but when the results are obtained, the phase between the magnetic field and the electromagnetic field can be adjusted to meet the following conditions. Note that the reason why we can adjust this way is that the author has supplemented Maxwell's classical electromagnetic theory with the time reversal waves, which provides additional degrees of freedom and allows an appropriate phase difference between the magnetic field and the electric field.

(2) Self energy flow does not radiate,

$$\Re\oiint_{\Gamma}(\boldsymbol{e}_1\times\boldsymbol{h}_1^*)\cdot\hat{n}d\Gamma=0 \quad (203)$$

$$\Re\oiint_{\Gamma}(\boldsymbol{e}_2\times\boldsymbol{h}_2^*)\cdot\hat{n}d\Gamma=0 \quad (204)$$

Above Γ is a infinite sphere surrounding V_1 or V_2. It is worth mentioning that our above assumptions are different from the solutions of Maxwell's equations in classical electromagnetic theory, such as the electromagnetic field of dipole antenna. For the solution of Maxwell's equations, the formula (203, 204) is not satisfied. A dipole antenna radiates energy into space. However, the author thinks that the classical electromagnetic theory is flawed. Therefore, we need to have a time reversal wave, and increasing the time reversal wave is equivalent to allowing a phase adjustment between the magnetic field and the electric field. We still keep $\boldsymbol{E}$,$\boldsymbol{H}$for the electromagnetic fields satisfy the Maxwell's equation. $\boldsymbol{E}$,$\boldsymbol{H}$ is still useful in many situations. We use $\boldsymbol{e}$ and $\boldsymbol{h}$ expresses the electromagnetic field of the author suggested.

This is the most important correction of Maxwell's theory by the mutual energy theory. The fundamental reason for this correction is that under the condition of magnetic quasi-static electromagnetic field, there are

$$\mathcal{E}=-j\omega LI \quad (205)$$

$$\Re\iiint\boldsymbol{E}\cdot\boldsymbol{J}^*dV=\Re\int_C\boldsymbol{E}\cdot dlI^*=0 \quad (206)$$

$$\Re(\mathcal{E}I^*)=\Re(-j\omega LII^*)=0 \quad (207)$$

The above formula requires the inductance L to be a positive real number. Mutual energy theory requires that this clause can still be guaranteed when the transition from magnetic quasi-static field to radiated electromagnetic field occurs. Thus, according to Poynting's theorem, there are formulas (203, 204). These equations also makes the retarded wave and advanced wave obtained from the mutual energy theory different from the solution of Maxwell's equations. According to the theory of mutual energy, the retarded wave is the reactive power wave with 90 degree phase difference between the electromagnetic field and the magnetic field, rather than the active power wave of the solution of Maxwell's equations. The task of energy transfer in the theory of mutual energy has been completely entrusted to the mutual energy flow. Self energy flow must be reactive power. If in the time domain, the above equation is,

$$-\int_{t=-\infty}^{\infty}dt\iiint_{V_1}\boldsymbol{e_2}\cdot\boldsymbol{J}_1dV$$

$$=\int_{t=-\infty}^{\infty}dt\oiint_{\Gamma}(\boldsymbol{e}_1\times\boldsymbol{h}_2+\boldsymbol{e}_2\times\boldsymbol{h}_1)\cdot\hat{n}d\Gamma$$

$$=\int_{t=-\infty}^{\infty}dt\iiint_{V_2}\boldsymbol{e_1}\cdot\boldsymbol{J}_2dV \quad (208)$$

and

$$\int_{t=-\infty}^{\infty}dt\oiint_{\Gamma}(\boldsymbol{e}_1\times\boldsymbol{h}_1)\cdot\hat{n}d\Gamma=0 \quad (209)$$

$$\int_{t=-\infty}^{\infty}dt\oiint_{\Gamma}(\boldsymbol{e}_2\times\boldsymbol{h}_2)\cdot\hat{n}d\Gamma=0 \quad (210)$$

The above three formulas are the basic equations used by the author to solve electromagnetic field problems, which are replace to Maxwell's equations in classical electromagnetic field theory. The author finds that the formula (208) has exactly one degree of freedom, which is defined by the conditions (209, 210). Therefore, the problem of electromagnetic field can just be solved, and there is no situation of over restriction. It is worth mentioning that the solution of this set of equations is exactly a set of retarded waves from the source and advanced waves from the sink. This retarded wave and advanced wave are obviously different from the retarded wave solved by Maxwell's equations. Because the retarded wave and advanced wave of the author are reactive power waves, the solution of Maxwell's equations is active power wave. For reactive power wave, although the wave still carries the energy of electromagnetic field to any place in space, these energy are returned to the source or sink by time reversal. The average Poynting energy flow of reactive power wave is zero, so it carries energy but does not transfer energy. This is totally different from the active power wave satisfying by the solution of Maxwell's equations.

However, it does not mean that Maxwell's equation is useless. At present, the author has not found a method to directly solve the above equations. Therefore, the electric field and magnetic field are still obtained from Maxwell's equations, and then the phases of the electromagnetic and magnetic fields are adjusted to meet the above three formulas (208, 209, 210). Therefore, Maxwell's equation still plays an auxiliary role.

It should noticed that the definition of electric field

$$\boldsymbol{e}=-\frac{\partial}{\partial t}\boldsymbol{A}+\nabla\phi \tag{211}$$

is still OK generally. We still can use above formula to find the electric field. But

$$\boldsymbol{h}=\frac{1}{\mu_0}\nabla\times\boldsymbol{A} \tag{212}$$

is not effective. This means all formula related to magnetic field is not effective any more, for example this two formula,

$$\nabla\times\boldsymbol{e}=\frac{\partial}{\boldsymbol{\partial t}}\mu_0\boldsymbol{h} \tag{213}$$

$$\nabla\times\boldsymbol{h}=\boldsymbol{J}+\frac{\partial}{\boldsymbol{\partial t}}\epsilon_0\boldsymbol{e} \tag{214}$$

are not effective. This is also the reason the author use Lowercase letters $\boldsymbol{e},\boldsymbol{h}$ to express the new defined electric and magnetic fields. Capital letters $\boldsymbol{E},\boldsymbol{H}$are left to express the electric field and magnetic field satisfy the Maxwell's equations.

V. EXAMPLES OF CONFLICTS BETWEEN POYNTING'S THEOREM AND THE PRINCIPLE OF MUTUAL ENERGY

We use the wave of infinite plane current to study the conflict between Poynting's theorem and the principle of mutual energy, which is also the conflict between Poynting's theorem and the theorem of mutual energy flow. Suppose there are two infinite plane current sheets close to each other. Plate current 1 is on the left and plate current 2 is on the right. It is assumed that plane current 1 radiates retarded waves and plane current 2 radiates advanced waves, so these two waves are running to the right.

A. Electromagnetic field of current $\boldsymbol{J}_1$

$$\boldsymbol{J}_1(x=0)=\exp(j\omega t)\hat{z}=J_{10}\exp(j\omega t)\hat{z} \tag{215}$$

$$\boldsymbol{J}_2(x=L)=\exp(j\omega t)(-\hat{z})=J_{20}(-\hat{z}) \tag{216}$$

$$L\to 0 \tag{217}$$

According to Maxwell equations, there are,

$$\boldsymbol{H}_1=\frac{J_{10}}{2}\exp(j\omega t-jkx)\hat{y} \tag{218}$$

$$\boldsymbol{E}_1=\eta_0\frac{J_{10}}{2}\exp(j\omega t-jkx)(-\hat{z}) \tag{219}$$

At this time, the electric field and the magnetic field are in phase.

$$\boldsymbol{S}_{11right}=\boldsymbol{E}_1\times\boldsymbol{H}_1^*|_{right}$$

$$=(\eta_0\frac{J_{10}}{2}\exp(j\omega t-jkx)(-\hat{z}))\times(\frac{J_{10}}{2}\exp(j\omega t-jkx)\hat{y})^*$$

$$=\frac{\eta_0 J_{10}J_{10}^*}{4}\hat{x} \tag{220}$$

$$\boldsymbol{S}_{11left}=\boldsymbol{E}_1\times\boldsymbol{H}_1^*|_{left}=\frac{\eta_0 J_{10}J_{10}^*}{4}(-\hat{x}) \tag{221}$$

and

$$-\boldsymbol{E}_1(x=0)\cdot\boldsymbol{J}_1^*=-(\eta_0\frac{J_{10}}{2}\exp(j\omega t-jk0)(-\hat{z}))\cdot(J_{10}\exp(j\omega t)\hat{z})^*$$

$$=-(\eta_0\frac{J_{10}}{2}(-\hat{z}))\cdot(J_{10}\hat{z})^*$$

$$=-(\eta_0\frac{J_{10}J_{10}^*}{2}) \tag{222}$$

Hence, there is,

$$\boldsymbol{E}_1\times\boldsymbol{H}_1^*|_{right}\cdot\hat{x}+\boldsymbol{E}_1\times\boldsymbol{H}_1^*|_{left}\cdot(-\hat{x})$$

$$=\frac{\eta_0 J_{10}J_{10}^*}{4}\hat{x}\cdot\hat{x}+\frac{\eta_0 J_{10}J_{10}^*}{4}(-\hat{x})\cdot(-\hat{x})$$

$$=\frac{\eta_0 J_{10}J_{10}^*}{2} \tag{223}$$

So we verified that,

$$\boldsymbol{E}_1\times\boldsymbol{H}_1^*|_{left}\cdot(-\hat{x})+\boldsymbol{E}_1\times\boldsymbol{H}_1^*|_{right}\cdot\hat{x}=-\boldsymbol{E}_1(x=0)\cdot\boldsymbol{J}_1^* \tag{224}$$

This means that Poynting's theorem has been verified,

$$\iint_\sigma\boldsymbol{E}_1\times\boldsymbol{H}_1^*\cdot\hat{n}d\sigma=-\iint_\sigma\boldsymbol{E}_1(x=0)\cdot\boldsymbol{J}_1^*d\sigma \tag{225}$$

It can be seen that Poynting's theorem holds.

B. The electromagnetic field of the current $\boldsymbol{J}_2$

Assume,

$$x < L \tag{226}$$

$$|x-L| = -(x-L) \tag{227}$$

The field of subscript 2 is a leading wave, so it has,

$$\boldsymbol{H}_2 = \frac{J_{20}}{2}\exp(j\omega t + jk|x-L|)\hat{y}$$

$$= \frac{J_{20}}{2}\exp(j\omega t - jk(x-L))\hat{y} \tag{228}$$

$$\boldsymbol{E}_2 = \eta_0\frac{J_{20}}{2}\exp(j\omega t - jk(x-L))(-\hat{z}) \tag{229}$$

$$J_2 = J_{20}\exp(j\omega t)(-\hat{z}) = \boldsymbol{E}_1(x=L)/Z_2$$

$$= \eta_0\frac{J_{10}}{2Z_2}\exp(j\omega t - jkL)(-\hat{z}) \tag{230}$$

$$J_{20} = \eta_0\frac{J_{10}}{2Z_2}\exp(-jkL) \tag{231}$$

$$\boldsymbol{H}_2 = \frac{J_{20}}{2}\exp(j\omega t - jk(x-L))\hat{y} \tag{232}$$

$$= \frac{1}{2}(\eta_0\frac{J_{10}}{2Z_2}\exp(-jkL))\exp(j\omega t - jk(x-L))\hat{y}$$

$$= \frac{1}{4}\frac{\eta_0 J_{10}}{Z_2}\exp(j\omega t - jkx)\hat{y} \tag{233}$$

$$\boldsymbol{E}_2 = \eta_0\frac{1}{2}(\eta_0\frac{J_{10}}{2Z_2}\exp(-jkL))\exp(j\omega t - jk(x-L))(-\hat{z})$$

$$= \frac{1}{4}(\frac{\eta_0^2 J_{10}}{Z_2})\exp(j\omega t - jkx)(-\hat{z}) \tag{234}$$

In the same way, we can verify

$$\iint_\sigma \boldsymbol{E}_2 \times \boldsymbol{H}_2^* \cdot \hat{n} d\sigma = -\iint_\sigma \boldsymbol{E}_2(x=0)\cdot \boldsymbol{J}_2^* d\sigma \tag{235}$$

C. Calculation of the mutual energy flow

$$S_m = \boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1$$

$$= (\eta_0\frac{J_{10}}{2}\exp(j\omega t - jkx)(-\hat{z}))\times(\frac{1}{4}\frac{\eta_0 J_{10}}{Z_2}\exp(j\omega t - jkx)\hat{y})^*$$

$$+(\frac{1}{4}(\frac{\eta_0^2 J_{10}}{Z_2})\exp(j\omega t - jkx)(-\hat{z}))^*\times(\frac{J_{10}}{2}\exp(j\omega t - jkx)\hat{y})$$

$$= (\eta_0\frac{J_{10}}{2}(-\hat{z}))\times(\frac{1}{4}\frac{\eta_0 J_{10}}{Z_2}\hat{y})^* + (\frac{1}{4}(\frac{\eta_0^2 J_{10}}{Z_2})(-\hat{z}))^*\times(\frac{J_{10}}{2}\hat{y})$$

$$= ((\eta_0\frac{J_{10}}{2})(\frac{1}{4}\frac{\eta_0 J_{10}}{Z_2})^* + (\frac{1}{4}(\frac{\eta_0^2 J_{10}}{Z_2}))^*(\frac{J_{10}}{2}))\hat{x}$$

$$= \frac{\eta_0^2}{4}\frac{J_{10}J_{10}^*}{Z_2^*}\hat{x} \tag{236}$$

$$-\boldsymbol{E}_2^*(x=0)\cdot \boldsymbol{J}_1$$

$$= -(\frac{1}{4}(\frac{\eta_0^2 J_{10}}{Z_2})\exp(j\omega t - jk0)(-\hat{z}))^*(J_{10}\exp(j\omega t)\hat{z})$$

$$= -(\frac{1}{4}(\frac{\eta_0^2 J_{10}}{Z_2})(-\hat{z}))^*(J_{10}\hat{z})$$

$$= \frac{1}{4}\frac{\eta_0^2 J_{10}J_{10}^*}{Z_2^*}) \tag{237}$$

$$\boldsymbol{E}_1(x=L)\cdot \boldsymbol{J}_2^*$$

$$= (\eta_0\frac{J_{10}}{2}\exp(j\omega t - jkL)(-\hat{z}))\cdot(\eta_0\frac{J_{10}}{2Z_2}\exp(j\omega t - jkL)(-\hat{z}))^*$$

$$= (\eta_0\frac{J_{10}}{2}(-\hat{z}))\cdot(\eta_0\frac{J_{10}}{2Z_2}(-\hat{z}))^*$$

$$= (\eta_0\frac{J_{10}}{2})(\eta_0\frac{J_{10}}{2Z_2})^*$$

$$= \frac{\eta_0^2}{4}\frac{J_{10}J_{10}^*}{Z^*} \tag{238}$$

So we verified that,

$$-\boldsymbol{E}_2^*(x=0)\cdot \boldsymbol{J}_1$$

$$= \boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1$$

$$= \boldsymbol{E}_1(x = L) \cdot \boldsymbol{J}_2^* \tag{239}$$

This means that,

$$-\iint_\sigma \boldsymbol{E}_2^*(x = 0) \cdot \boldsymbol{J}_1 d\sigma =$$

$$\iint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma$$

$$= \iint_\sigma \boldsymbol{E}_1(x = L) \cdot \boldsymbol{J}_2^* d\sigma \tag{240}$$

This verifies the mutual energy flow theorem.

D. Comments on mutual energy flow and self energy flow

That is to say, we have verified that the mutual energy flow theorem is valid. In this way, according to Maxwell's theory, Poynting's theorem and the mutual energy flow theorem are verified. Because the principle of mutual energy can appear as a sub theorem of Poynting's theorem. If Poynting's theorem holds, the principle of mutual energy must hold. From the principle of mutual energy, we can deduce the theorem of mutual energy flow, so the theorem of mutual energy flow holds. In this way, Poynting theorem and mutual energy flow theorem are all established. The problem is that the energy transferred by Poynting vector plus the energy transferred by mutual energy flow is more than the energy generated by current, that is

$$\iint_\sigma \boldsymbol{E}_1 \times \boldsymbol{H}_1^* \cdot \hat{n} d\sigma + \iint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma$$

$$> -\iint_\sigma \boldsymbol{E}_1(x = 0) \cdot \boldsymbol{J}_1^* d\sigma$$

$$\iint_\sigma \boldsymbol{E}_1 \times \boldsymbol{H}_1^* \cdot \hat{n} d\sigma + \iint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma$$

$$> -\iint_\sigma \boldsymbol{E}_2^*(x = 0) \cdot \boldsymbol{J}_1 d\sigma$$

So the conservation of energy is broken. Therefore, the mutual energy theorem and Poynting's theorem cannot be both true! The classical electromagnetic theory chooses Poynting theorem. But the choice is self contradictory, because the mutual energy flow theorem can be derived from Poynting's theorem. So if Poynting's theorem holds, the mutual energy flow theorem holds. In this way, both Poynting's theorem and the mutual energy flow theorem are valid. And this means that the law of conservation of energy does not hold for Maxwell's theory!

The author chooses the latter, that is, the mutual energy flow theorem is established. Time reversal wave is added to Maxwell's equations. When the time reversal wave is added, the self energy flow does not transfer energy. The time reversal wave actually provides us with a degree of freedom to adjust the electromagnetic field. We add a degree of freedom between the electric and magnetic fields. Then it is proved that the time reversal wave can be formed by properly adjusting the phase of the electric field and the magnetic field. After adjusting the phase, the electromagnetic field becomes a reactive power wave. So the Poynting vector doesn't transfer energy. This is why the author advocates that the self energy flow is reactive power wave.

E. Phase of electric field and magnetic field

As we said earlier, the phase of the electromagnetic and magnetic fields can be properly selected to make the self energy flow into reactive power. Here, there are two options: keep the electric field unchanged and change the magnetic field. 2 select the constant magnetic field and change the electric field.

The author chose to change the magnetic field without changing the electric field. This is because for a dipole antenna, the current element is,

$$\boldsymbol{J} = J_0 \Delta z \hat{z} \delta(y) \delta(x) \tag{241}$$

The far field obtained by solving Maxwell's equation is,

$$\boldsymbol{E}\tilde{\ } j \frac{\exp(j\omega t - jkr)}{r} \sin(\theta) \hat{\theta} \tag{242}$$

$$\boldsymbol{H}\tilde{\ } j \frac{\exp(j\omega t - jkr)}{r} \sin(\theta) \hat{\phi} \tag{243}$$

$\sim$ stands for proportional and only cares about the direction of phase and vector When $r = 0$, $\theta = \frac{\pi}{2}$

$$\boldsymbol{E}\tilde{\ } j \frac{\exp(j\omega t)}{r} \hat{\theta}\tilde{\ } j \frac{\exp(j\omega t)}{r} (-\hat{z}) \tag{244}$$

$$\boldsymbol{H}\tilde{\ } j \frac{\exp(j\omega t)}{r} \hat{\phi}\tilde{\ } j \frac{\exp(j\omega t)}{r} \hat{y} \tag{245}$$

We know that if the magnetic quasi-static field

$$\boldsymbol{E}(r = 0) = -\frac{\partial}{\partial t} \boldsymbol{A} = -\frac{\partial}{\partial t} \iint_\sigma \frac{\boldsymbol{J}}{r} d\sigma = -j\omega \iint_\sigma \frac{\boldsymbol{J}}{r} d\sigma \sim j(-\hat{z}) \tag{246}$$

$$\boldsymbol{H}(r = 0) \sim \hat{y} \tag{247}$$

After comparison, we found that for the far field from the magnetic quasi-static field to the radiated electromagnetic field, the initial value of the electric field remained the same when $r = 0$, while the magnetic field changed. Therefore, the electric field was kept unchanged and the phase of the magnetic field was changed during adjustment. Thus, the far-field electromagnetic field of the dipole antenna is,

$$\boldsymbol{e} \sim j\frac{\exp(j\omega t - kr)}{r}\sin(\theta)\hat{\theta} \tag{248}$$

$$\boldsymbol{h} \sim \frac{\exp(j\omega t - kr)}{r}\sin(\theta)\hat{\phi} \tag{249}$$

$\sim$ means proportional to, only cares about the phase and the direction of the vector. $\boldsymbol{e}$,$\boldsymbol{h}$ are author suggested electromagnetic field according the mutual energy theory.

F. Possible plane wave solutions of mutual energy flow

1. *Find the plane wave solution according to Maxwell's equations*

For Maxwell's equation, if the source $\boldsymbol{J}$ is not considered, there is,

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{B} \tag{250}$$

$$\nabla \times \boldsymbol{H} = \frac{\partial}{\partial t}\boldsymbol{D} \tag{251}$$

If we already know that the electromagnetic field is a plane wave

$$\boldsymbol{H} = H_0 \exp(j\omega t - jkx)\hat{y}$$

for the electric field,

$$-jk\hat{x} \times \boldsymbol{H} = j\omega\epsilon_0 \boldsymbol{E}$$

$$-k\hat{x} \times \boldsymbol{H} = \omega\epsilon_0 \boldsymbol{E} \tag{252}$$

$$\boldsymbol{E} = -\hat{x} \times \eta_0 \boldsymbol{H}$$

$$= \eta_0 H_0 \exp(j\omega t - jkx)(-\hat{z}) \tag{253}$$

$$\boldsymbol{E}\times\boldsymbol{H}^* = (\eta_0 H_0 \exp(j\omega t - jkx)(-\hat{z}))\times(H_0 \exp(j\omega t - jkx)\hat{y})^*$$

$$= \eta_0 H_0 {H_0}^* \hat{x} \tag{254}$$

2. *Plane wave solution based on mutual energy theory*

According to the mutual energy theory, energy is transmitted by mutual energy flow, which is also composed of retarded and advance waves. Assuming that both retarded and advance waves are reactive power waves and plane waves, we can see if they can form a mutual energy flow to transfer energy. Because the retarded and advance waves must be synchronized, we assume that they are both plane waves and propagate in the x direction.

If the source $\boldsymbol{J}$ is not considered,

$$\boldsymbol{h}_1 = H_{10} \exp(j\omega t - jkx)\hat{y} \tag{255}$$

We assume that there is a phase difference between the electric and magnetic fields $\exp(-j\phi_1)$

$$\boldsymbol{e}_1 = \eta_0 H_{10} \exp(j\omega t - jkx - j\phi_1)(-\hat{z}) \tag{256}$$

We assume that there is a phase difference $\exp(-j\psi)$ between the two magnetic fields, which is arbitrarily variable,

$$\boldsymbol{h}_2 = H_{20} \exp(j\omega t - jkx - j\psi)\hat{y} \tag{257}$$

Where H_{10}, H_{20} are a positive constants. Suppose that the electric field also has a starting phase $\eta_0 H_{20} \exp(-j\phi_2)$

$$\boldsymbol{e}_2 = \eta_0 H_{20} \exp(j\omega t - jkx - j\psi - j\phi_2)(-\hat{z}) \tag{258}$$

For the mutual energy theory, we know

$$\Re \boldsymbol{e}_1 \times \boldsymbol{h}_1^* \cdot \hat{x} = 0 \tag{259}$$

$$\Re \boldsymbol{e}_2 \times \boldsymbol{h}_2^* \cdot \hat{x} = 0 \tag{260}$$

$$\Re(\boldsymbol{e}_1 \times \boldsymbol{h}_2^* + \boldsymbol{e}_2^* \times \boldsymbol{h}_1) \cdot \hat{x} > 0 \tag{261}$$

First calculate

$$\boldsymbol{e}_1 \times \boldsymbol{h}_2^* + \boldsymbol{e}_2^* \times \boldsymbol{h}_1$$

$$= (\eta_0 H_{10} \exp(j\omega t - jkx - j\phi_1)(-\hat{z}))\times(H_{20} \exp(j\omega t - jkx - j\psi)\hat{y})^*$$

$$+(\eta_0 H_{20} \exp(j\omega t - jkx - j\psi - j\phi_2)(-\hat{z}))^*\times(H_{10} \exp(j\omega t - jkx)\hat{y})$$

$$= (\eta_0 \exp(-j\phi_1)\exp(-j\psi)^* + \exp(-j\psi - j\phi_2)^*)\eta_0 H_{10} H_{20}^* \hat{x}$$

$$= (\exp(-j(\phi_1 - \psi) + \exp(-j(\psi + \phi_2))^*)\eta_0 H_{10} H_{20}^* \hat{x} \tag{262}$$

To make the upper form positive, you get the following,

$$\phi_1 - \psi = 0 \rightarrow \psi = \phi_1 \tag{263}$$

$$\psi + \phi_2 = 0 \rightarrow \phi_2 = -\psi = -\phi_1 \tag{264}$$

Substitute the values above to,

$$\boldsymbol{e}_1 \times \boldsymbol{h}_1^* =$$

$$(\eta_0 H_{10} \exp(j\omega t - jkx - j\phi_1)(-\hat{z})) \times (H_{10} \exp(j\omega t - jkx)\hat{y})^*$$

$$= (\eta_0 H_{10} H_{10}{}^* \exp(-j\phi_1)\hat{x} \tag{265}$$

$$\Re \boldsymbol{e}_1 \times \boldsymbol{h}_1^* \cdot \hat{x} = 0 \rightarrow \exp(-j\phi_1) = \pm j \tag{266}$$

Select,

$$\exp(-j\phi_1) = j \tag{267}$$

$$\boldsymbol{h}_1 = H_{10} \exp(j\omega t - jkx)\hat{y} \tag{268}$$

$$\boldsymbol{e}_1 = j\eta_0 H_{10} \exp(j\omega t - jkx)(-\hat{z}) \tag{269}$$

$$\boldsymbol{h}_2 = H_{20} \exp(j\omega t - jkx - j\psi)\hat{y}$$

$$= H_{20} \exp(j\omega t - jkx - j\phi_1)\hat{y}$$

$$= jH_{20} \exp(j\omega t - jkx)\hat{y} \tag{270}$$

$$\boldsymbol{e}_2 = \eta_0 H_{20} \exp(j\omega t - jkx - j\psi - j\phi_2)(-\hat{z})$$

$$= \eta_0 H_{20} \exp(j\omega t - jkx - j\phi_1 + j\phi_1)(-\hat{z})$$

$$= \eta_0 H_{20} \exp(j\omega t - jkx)(-\hat{z}) \tag{271}$$

Hence,

$$\boldsymbol{h}_1 = H_{10} \exp(j\omega t - jkx)\hat{y} \tag{272}$$

$$\boldsymbol{e}_1 = j\eta_0 H_{10} \exp(j\omega t - jkx)(-\hat{z}) \tag{273}$$

$$\boldsymbol{h}_2 = jH_{20} \exp(j\omega t - jkx)\hat{y} \tag{274}$$

$$\boldsymbol{e}_2 = \eta_0 H_{20} \exp(j\omega t - jkx)(-\hat{z}) \tag{275}$$

Hence,

$$\boldsymbol{e}_1 \times \boldsymbol{h}_1^*$$

$$= (j\eta_0 H_{10} \exp(j\omega t - jkx)(-\hat{z})) \times (H_{10} \exp(j\omega t - jkx)\hat{y})^*$$

$$= j\eta_0 H_{10} H_{10}{}^* \hat{x} \tag{276}$$

$$\boldsymbol{e}_2 \times \boldsymbol{h}_2^*$$

$$= \eta_0 H_{20} \exp(j\omega t - jkx)(-\hat{z}) \times (jH_{20} \exp(j\omega t - jkx)\hat{y})^*$$

$$= -j\eta_0 H_{20} H_{20} \hat{x} \tag{277}$$

$$S_m = \boldsymbol{e}_1 \times \boldsymbol{h}_2^* + \boldsymbol{e}_2^* \times \boldsymbol{h}_1$$

$$= (j\eta_0 H_{10} \exp(j\omega t - jkx)(-\hat{z})) \times (jH_{20} \exp(j\omega t - jkx)\hat{y})^*$$

$$+ (\eta_0 H_{20} \exp(j\omega t - jkx)(-\hat{z}))^* \times (H_{10} \exp(j\omega t - jkx)\hat{y})$$

$$= (\eta_0 H_{10} H_{20}{}^* + \eta_0 H_{20}{}^* H_{10})\hat{x}$$

$$= (2\eta_0 H_{10} H_{20}{}^*)\hat{x} \tag{278}$$

In this way, the self-energy flow is indeed reactive power, and the mutual energy flow is indeed active power. Above we get a plane wave solution based on the theory of mutual energy.

If we choose,

$$\exp(-j\phi_1) = -j \tag{279}$$

The result is equivalent to aligning the positions of $[\boldsymbol{e}_1, \boldsymbol{h}_1]$ and $[\boldsymbol{e}_2, \boldsymbol{h}_2]$, so there is no difference.

Thus, according to the theory of mutual energy, there is indeed a plane wave solution. In this case the self-energy flows are reactive power waves, but the mutual energy flow are with active power.

VI. PHOTON MODEL OF MUTUAL ENERGY FLOW FOR PLANAR WAVE TRANSFORMER WITH ELECTROMAGNETIC RADIATION

Firstly, we assume that the electromagnetic field in photons is a plane wave, so we first establish the photon model of plane wave.

A. Plane wave generated by light source

It is assumed that a wave propagating to the right is generated on both sides of the plane current, the retarded wave is on the right side of the current, and the advanced wave is on the left side of the current,

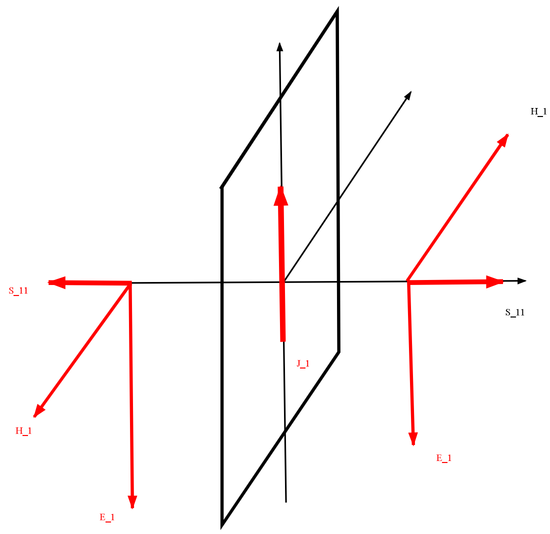

Figure 12. Plane wave generated by plane current sheet.

1. *Right side of current*

Plane wave generated by plane current sheet can be see as following figure 12.

Set the plate current density as

$$\boldsymbol{J}_1 = J_{10} \exp(j\omega t)\hat{z} \tag{280}$$

The magnetic field on the right side of the false current is,

$$\boldsymbol{H}_1 = H_{10} \exp(j\omega t)\hat{y} \tag{281}$$

The magnetic field can be obtained from the Ampere circuital theorem,

$$H_{10} = \frac{1}{2} J_{10} \tag{282}$$

The transmitted electromagnetic field is,

$$\boldsymbol{H}_1 = H_{10} \exp(j\omega t - jkx)\hat{y} \tag{283}$$

If, according to Maxwell's equation, the electric field has

$$\nabla \times \boldsymbol{H}_1 = j\omega\epsilon_0 \boldsymbol{E}_1$$

$$-jk\hat{x} \times \boldsymbol{H}_1 = j\omega\epsilon_0 \boldsymbol{E}_1 \tag{284}$$

$$-\frac{k}{\omega\epsilon_0}\hat{x} \times \boldsymbol{H}_1 = \boldsymbol{E}_1 \tag{285}$$

$$\boldsymbol{E}_1 = -\eta_0 \hat{x} \times \boldsymbol{h}_1 \tag{286}$$

If the result of Maxwell's equation is correct, it means that

$$\boldsymbol{E}_1 = -\eta_0 \hat{x} \times \boldsymbol{H}_1 \tag{287}$$

Therefore,

$$\boldsymbol{E}_1 = -\eta_0 \hat{x} \times (H_{10} \exp(j\omega t - jkx)\hat{y})$$

$$= \eta_0 H_{10} \exp(j\omega t - jkx)(-\hat{z}) \tag{288}$$

$$\boldsymbol{E}_1(x = 0) = \eta_0 H_{10} \exp(j\omega t)(-\hat{z}) \tag{289}$$

However, the author considers that according to the magnetic quasi-static situation,

$$\boldsymbol{E}_1 = -j\omega \boldsymbol{A}_1 \tag{290}$$

$$\boldsymbol{A}_1 = \iint_\sigma J_{10} \frac{d\sigma}{r} \hat{z} \tag{291}$$

$$\boldsymbol{E}_1 \sim -j\hat{z} = j(-\hat{z}) \tag{292}$$

$\sim$ stands for proportional. This formula only relates to the phase and direction, not the magnitude. In this paper, it is considered that Maxwell's equation is wrong, and the initial phase value of the electric field should be considered according to the magnetic quasi-static field of the above formula. Therefore,

$$\boldsymbol{e}_1(x = 0) = j\eta_0 H_{10} \exp(j\omega t)(-\hat{z}) \tag{293}$$

In this way,

$$\boldsymbol{e}_1 = j\eta_0 H_{10} \exp(j\omega t - jkx)(-\hat{z}) \tag{294}$$

$$\boldsymbol{e}_1 \times \boldsymbol{h}_1^*$$

$$= j\eta_0 H_{10} \exp(j\omega t - jkx)(-\hat{z}) \times (H_{10} \exp(j\omega t - jkx)\hat{y})^*$$

$$= j\eta_0 H_{10} H_{10}{}^* \hat{x} \tag{295}$$

$$\boldsymbol{e}_1 \times \boldsymbol{h}_1^* \cdot \hat{x} \sim j \tag{296}$$

the above formula is satisfied,

$$\Re(\boldsymbol{e}_1 \times \boldsymbol{h}_1^*) = 0 \tag{297}$$

$\Re$ is the real part.

2. Left side of current

$$H_{10} = \frac{1}{2} J_{10} \tag{298}$$

The transmitted electromagnetic field is,

$$\boldsymbol{h}_{1l} = H_{10} \exp(j\omega t - jkx)(-\hat{y}) \tag{299}$$

$$\boldsymbol{h}_{1l} = j\eta_0 H_{10} \exp(j\omega t - jkx)(-\hat{z}) \tag{300}$$

The above formula is correct. The electric field on the left is the same as that on the right. In addition,

$$\boldsymbol{e}_{1l} \times \boldsymbol{h}_{1l}^*$$

$$= j\eta_0 H_{10} \exp(j\omega t - jkx)(-\hat{z}) \times (H_{10} \exp(j\omega t - jkx)(-\hat{y}))^*$$

$$\sim -j\hat{x} \tag{301}$$

$$\boldsymbol{e}_{1l} \times \boldsymbol{h}_{1l}^* \cdot (-\hat{x}) \sim j \tag{302}$$

The above formula is correct. The Poynting vector on the right side of the current should be the same as that on the left.

B. Plane wave generated by light sink

Figure 13shows the light sink which is plane current sheet. On its left the advanced wave is produced. On its right regarded wave is produced.

Two factors should be considered for the wave generated by the plane current of the sink. The direction of the current is specified as $-\hat{z}$, current and electric field $\boldsymbol{E}_1$ In direct proportion. The position of the current is at $x = L$.

$$\boldsymbol{e}_1(x = L) = \boldsymbol{e}_1 = j\eta_0 H_{10} \exp(j\omega t - jkL)(-\hat{z}) \tag{303}$$

hence,

$$J_2 = \frac{1}{Z_2} j\eta_0 H_{10} \exp(j\omega t - jkL) \tag{304}$$

where J_2 is the current intensity of the second plane current sheet. Z_2 is the impedance of the secondary coil.

$$J_{20} = \frac{1}{Z_2} j\eta_0 H_{10} \exp(-jkL) = \frac{1}{Z_2} j\eta_0 \frac{I_{10}}{2} \exp(-jkL) \tag{305}$$

$$J_2 = J_{20} \exp(j\omega t) \tag{306}$$

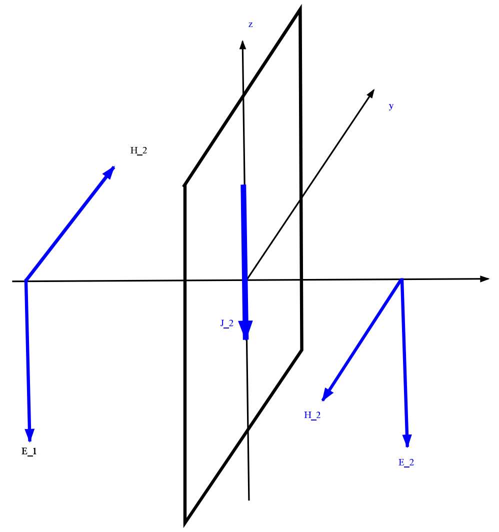

Figure 13. The left side of the plate current produces the plane wave of the leading wave.

1. Right side of current

Considering that the right side of the current is a retarded wave,

$$\boldsymbol{h}_1 = \frac{J_{10}}{2} \exp(j\omega t - jkx)\hat{y} \tag{307}$$

$$\boldsymbol{e}_1 = j\eta_0 \frac{J_{10}}{2} \exp(j\omega t - jkx)(-\hat{z}) \tag{308}$$

$$\boldsymbol{h}_{2r} = -\frac{J_{20}}{2} \exp(j\omega t - jk(x - L))\hat{y}$$

The negative sign is because the direction of current $\boldsymbol{J}_2$ is chosen at $(-\hat{z})$ direction

$$\boldsymbol{h}_{2r} = -\frac{j\eta_0 \frac{J_{10}}{2} \exp(-jkL)}{2Z_2} \exp(j\omega t - jk(x - L))\hat{y}$$

$$= -\frac{j\eta_0 J_{10}}{4Z_2} \exp(j\omega t - jkx)\hat{y} \tag{309}$$

$$\boldsymbol{e}_{2r} = -j\eta_0 \frac{J_{20}}{2Z_2} \exp(j\omega t - jk(x - L))(-\hat{z})$$

$$= -j\eta_0 \frac{j\eta_0 \frac{J_{10}}{2} \exp(-jkL)}{2Z_2} \exp(j\omega t - jk(x - L))(-\hat{z})$$

$$= -jj \frac{\eta_0^2 J_{10}}{4Z_2} \exp(j\omega t - jkx)(-\hat{z}) \tag{310}$$

$$\boldsymbol{e}_{2r} \times \boldsymbol{h}_{2r}^*$$

$$= ((-j)j\eta_0^2 \frac{J_{10}}{4Z_2} \exp(j\omega t - jkx)(-\hat{z})) \times (-\frac{j\eta_0 J_{10}}{4Z_2} \exp(j\omega t - jkx)\hat{y})^*$$

$$= j(\eta_0^2 \frac{J_{10}}{4Z_2} \exp)(\frac{\eta_0 J_{10}}{4Z_2})^* \hat{x}$$

$$= j\eta_0^3 \frac{J_{10} J_{10}^*}{16 Z_2 Z_2^*} \hat{x} \quad (311)$$

Hence,

$$\Re(\boldsymbol{e}_{2r} \times \boldsymbol{h}_{2r}^*) = 0 \quad (312)$$

2. The left side of current

Refer to the following formula

$$\boldsymbol{h}_{1l} = \frac{J_{10}}{2} \exp(j\omega t - jkx)(-\hat{y}) \quad (313)$$

$$\boldsymbol{e}_{1l} = j\eta_0 \frac{J_{10}}{2} \exp(j\omega t - jkx)(-\hat{z}) \quad (314)$$

We obtain,

$$\boldsymbol{h}_2 = -\frac{J_{20}}{2} \exp(j\omega t - jk(x - L))(-\hat{y})$$

$$= -\frac{j\eta_0 \frac{J_{10}}{2Z_2} \exp(-jkL)}{2} \exp(j\omega t - jk(x - L))(-\hat{y})$$

$$= -\frac{j\eta_0 J_{10}}{4Z_2} \exp(j\omega t - jkx)(-\hat{y}) \quad (315)$$

$$\boldsymbol{e}_2 = -j\eta_0 \frac{J_{20}}{2} \exp(j\omega t - jk(x - L))(-\hat{z})$$

$$= -j\eta_0 \frac{j\eta_0 \frac{J_{10}}{2Z_2} \exp(-jkL)}{2} \exp(j\omega t - jk(x - L))(-\hat{z})$$

$$= (-j)j\frac{\eta_0^2 I_{10}}{4Z_2} \exp(j\omega t - jkx)(-\hat{z}) \quad (316)$$

Hence,

$$\boldsymbol{e}_2 \times \boldsymbol{h}_2^* = ((-j)j\frac{\eta_0^2 I_{10}}{4Z_2} \exp(j\omega t - jkx)(-\hat{z}))$$

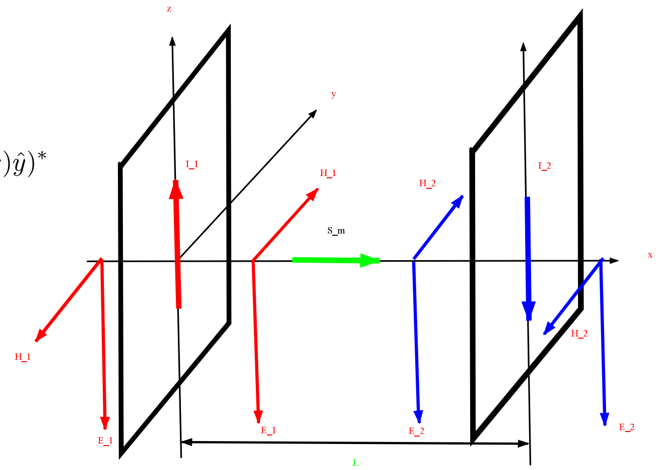

Figure 14. For the transformer system with two plane current sheets, the primary coil current is red, indicating energy release, and the secondary coil is blue, indicating energy absorption.

$$\times(-\frac{j\eta_0 I_{10}}{4Z_2^*} \exp(j\omega t - jkx)(-\hat{y}))^*$$

$$= (\frac{\eta_0^2 I_{10}}{4Z_2}(-\hat{z})) \times (-\frac{j\eta_0 I_{10}}{4Z_2}(-\hat{y}))^*$$

$$= (\frac{\eta_0^2 I_{10}}{4Z_2})(\frac{j\eta_0 I_{10}}{4Z_2})^* \hat{x}$$

$$= -j\frac{\eta_0^3 I_{10} I_{10}^*}{16 Z_2 Z_2^*} \hat{x} \quad (317)$$

hence,

$$\Re(\boldsymbol{e}_2 \times \boldsymbol{h}_2^*) = 0 \quad (318)$$

C. The calculation of the mutual energy flow

Figure 14 following shows a transformer with double plane current sheets. The primary coil has a currents $\boldsymbol{J}_1$. The secondary coil has a current $\boldsymbol{J}_2$. The distance between the current is L. Assume the two current produce all rightward waves. That means there is advanced wave on the left and there is retarded wave on right.

Considering

$$0 \le x \le L$$

$$\boldsymbol{h}_1 = \frac{I_{10}}{2} \exp(j\omega t - jkx)\hat{y} \quad (319)$$

$$\boldsymbol{e}_1 = j\eta_0 \frac{I_{10}}{2} \exp(j\omega t - jkx)(-\hat{z}) \quad (320)$$

$$\boldsymbol{h}_2 = -\frac{j\eta_0 I_{10}}{4Z_2} \exp(j\omega t - jkx)(-\hat{y}) \quad (321)$$

$$\boldsymbol{e}_2 == (-j)j\frac{\eta_0^2 I_{10}}{4Z_2} \exp(j\omega t - jkx)(-\hat{z}) \quad (322)$$

$$\mathrm{S}_m = \boldsymbol{e}_1 \times \boldsymbol{h}_2^* + \boldsymbol{e}_2^* \times \boldsymbol{h}_1$$

$$= (j\eta_0 \frac{I_{10}}{2} \exp(j\omega t - jkx)(-\hat{z})) \times (-\frac{j\eta_0 I_{10}}{4Z_2} \exp(j\omega t - jkx)(-\hat{y}))^*$$

$$+((-j)j\frac{\eta_0^2 I_{10}}{4Z_2} \exp(j\omega t - jkx)(-\hat{z}))^* \times (\frac{I_{10}}{2} \exp(j\omega t - jkx)\hat{y})$$

$$= (j\eta_0 \frac{I_{10}}{2}(-\hat{z})) \times (-\frac{j\eta_0 I_{10}}{4}(-\hat{y}))^* + ((-j)j\frac{\eta_0^2 I_{10}}{4}(-\hat{z}))^* \times (\frac{I_{10}}{2}\hat{y})$$

$$= ((j\eta_0 \frac{I_{10}}{2}) \times (\frac{j\eta_0 I_{10}}{4Z_2})^* + ((-j)j\frac{\eta_0^2 I_{10}}{4Z_2})^*(\frac{I_{10}}{2}))\hat{x}$$

$$= (jj^*\eta_0^2 \frac{I_{10}I_{10}^*}{8Z_2^*} + (-j)j\eta_0^2 \frac{I_{10}I_{10}^*}{8Z_2^*})\hat{x}$$

$$= \eta_0^2 \frac{I_{10}^2}{4Z_2^*}\hat{x} \quad (323)$$

In the region,

$$x < 0 \quad (324)$$

$$\boldsymbol{h}_{1l} = H_{10} \exp(j\omega t - jkx)(-\hat{y}) \quad (325)$$

$$\boldsymbol{e}_{1l} = j\eta_0 H_{10} \exp(j\omega t - jkx)(-\hat{z}) \quad (326)$$

$$\boldsymbol{h}_2 = -\frac{j\eta_0 I_{10}}{4Z_2} \exp(j\omega t - jkx)(-\hat{y}) \quad (327)$$

$$\boldsymbol{e}_2 == (-j)j\frac{\eta_0^2 I_{10}}{4Z_2} \exp(j\omega t - jkx)(-\hat{z}) \quad (328)$$

$$\mathrm{S}_m = \boldsymbol{e}_{1l} \times \boldsymbol{h}_2^* + \boldsymbol{e}_2^* \times \boldsymbol{h}_{1l}$$

$$= (j\eta_0 H_{10} \exp(j\omega t - jkx)(-\hat{z})) \times (-\frac{j\eta_0 J_{10}}{4Z_2} \exp(j\omega t - jkx)(-\hat{y}))^*$$

$$+((-j)j\frac{\eta_0^2 J_{10}}{4Z_2} \exp(j\omega t - jkx)(-\hat{z}))^* \times (H_{10} \exp(j\omega t - jkx)(-\hat{y}))$$

$$= (j\eta_0 H_{10}(-\hat{z})) \times (-\frac{j\eta_0 J_{10}}{4Z_2}(-\hat{y}))^* + ((-j)j\frac{\eta_0^2 J_{10}}{4Z_2}(-\hat{z}))^* \times (H_{10}(-\hat{y}))$$

$$= (\eta_0 H_{10}(-\hat{z})) \times (-\frac{\eta_0 J_{10}}{4Z_2}(-\hat{y}))^* + (\frac{\eta_0^2 J_{10}}{4Z_2}(-\hat{z}))^* \times (H_{10}(-\hat{y}))$$

$$= ((\eta_0 H_{10})(-\frac{\eta_0 J_{10}}{4Z_2}))^* + (\frac{\eta_0^2 J_{10}}{4Z_2})^*(H_{10}))(-\hat{x})$$

$$= ((\eta_0 \frac{J_{10}}{2})(\frac{\eta_0 J_{10}}{4Z_2}))^* - (\frac{\eta_0^2 J_{10}}{4Z_2})^* \frac{J_{10}}{2})\hat{x}$$

$$= (\frac{\eta_0^2 J_{10} J_{10}^*}{8Z_2^*} - \frac{\eta_0^2 J_{10} J_{10}^*}{8Z_2^*})\hat{x}$$

$$= (0)\hat{x} \quad (329)$$

In the region,

$$x > L$$

$$\boldsymbol{h}_1 = \frac{J_{10}}{2} \exp(j\omega t - jkx)\hat{y} \quad (330)$$

$$\boldsymbol{e}_1 = j\eta_0 \frac{J_{10}}{2} \exp(j\omega t - jkx)(-\hat{z}) \quad (331)$$

$$\boldsymbol{h}_{2r} = -\frac{j\eta_0 J_{10}}{4Z_2} \exp(j\omega t - jkx)\hat{y} \quad (332)$$

$$\boldsymbol{e}_{2r} = (-j)j\eta_0^2 \frac{J_{10}}{4Z_2} \exp(j\omega t - jkx)(-\hat{z}) \quad (333)$$

$$\mathrm{S}_m = \boldsymbol{e}_1 \times \boldsymbol{h}_{2r}^* + \boldsymbol{e}_{2r}^* \times \boldsymbol{h}_1$$

$$= (j\eta_0 \frac{J_{10}}{2} \exp(j\omega t - jkx)(-\hat{z})) \times (-\frac{j\eta_0 J_{10}}{4Z_2} \exp(j\omega t - jkx)\hat{y})^*$$

$$+((-j)j\eta_0^2 \frac{J_{10}}{4Z_2} \exp(j\omega t - jkx)(-\hat{z}))^* \times (\frac{J_{10}}{2} \exp(j\omega t - jkx)\hat{y})$$

$$= (j\eta_0 \frac{J_{10}}{2}(-\hat{z})) \times (-\frac{j\eta_0 J_{10}}{4Z_2}\hat{y})^* + ((-j)j\eta_0^2 \frac{J_{10}}{4Z_2}(-\hat{z}))^* \times (\frac{J_{10}}{2}\hat{y})$$

$$= ((j\eta_0 \frac{J_{10}}{2})(-\frac{j\eta_0 J_{10}}{4Z_2})^* + ((-j)j\eta_0^2 \frac{J_{10}}{4Z_2})^*(\frac{J_{10}}{2}))\hat{x}$$

$$= ((\eta_0 \frac{J_{10}}{2})(-\frac{\eta_0 J_{10}}{4Z_2})^* + (\eta_0^2 \frac{J_{10}}{4Z_2})^*(\frac{J_{10}}{2}))\hat{x}$$

$$= (-\frac{\eta_0^2 J_{10} J_{10}^*}{8Z_2^*} + \frac{\eta_0^2 J_{10} J_{10}^*}{8Z_2^*})\hat{x}$$

$$= (0)\hat{x} \tag{334}$$

Hence,there is,

$$S_m = \begin{cases} (0)\hat{x} & x < 0 \\ \eta_0^2 \frac{J_{10} J_{10}^*}{4R_2^*}\hat{x} & 0 \le x \le L \\ (0)\hat{x} & x > L \end{cases} \tag{335}$$

We have assume Z_2 is like a resistance $Z_2 = R_2$. This means that the mutual energy flow calculated according to the mutual energy theory is generated on the primary coil and annihilated on the secondary coil. The nature of the mutual energy flow is the same as that of photons.

D. The relationship between mutual energy flow interpretation and Cramer's quantum mechanics transaction interpretation

The above formula (331) shows that the mutual energy flow generate at the plate current $\boldsymbol{J}_1$ and annihilate at $\boldsymbol{J}_2$ place. When we use Poynting vector, we can sometimes get the correct direction of energy flow, but we can never describe the nature of energy flow generated at the source and annihilated at the sink. So the mutual energy flow is really much more powerful than the Poynting vector. The mutual energy flow in the above example is very similar to the photon model [5, 6] given in John Cramer's quantum mechanics transactional interpretation. In Cramer's model, the amount of superposition is the two waves, the advanced waves and the retarded waves. In the author's example, the amount of superposition is two components of the mutual energy flow,

$$\boldsymbol{S}_{12} = \boldsymbol{e}_1 \times \boldsymbol{h}_2^* \tag{336}$$

$$\boldsymbol{S}_{21} = \boldsymbol{e}_2^* \times \boldsymbol{h}_1 \tag{337}$$

In Cramer's example, the phase difference between the advanced wave generated by the light source and the advanced wave generated by the light sink is exactly 180 degrees, so it is just offset. There is a phase difference of exactly 180 degrees between the retarded wave emitted by the light sink and the retarded wave emitted by the light source, so they are just offset. But this 180 degree phase difference does not give any explanation.

But in the above example of the author, when the wave running to the right passes through the current $\boldsymbol{J}_1$ the magnetic field is reversed, the reversal of the magnetic field makes $\boldsymbol{S}_{21}$ The direction of magnetic filed $\boldsymbol{H}_1$ has changed. When the wave in the running to the right passes through the current $\boldsymbol{J}_2$, the magnetic field $\boldsymbol{H}_2$ backward, and $\boldsymbol{S}_{12}$ reverse, which offsets $\boldsymbol{S}_{21}$. In the author's mutual energy theory, the 180 degree phase difference is obviously caused by the opposite direction of the magnetic field on both sides of the plane current sheet. So 180 degrees is very natural. Cramer's model cannot do this.

In addition, in the author's model, the radiation power of the retarded wave and the advanced wave is reactive power, so they do not transfer energy. Cramer's model is only valid in one dimension. The author's annotation model will be extended to 3D in the next chapter.

E. Choose whether to adjust the magnetic field phase or the electric field phase

We said earlier that the phase of the magnetic field should be adjusted, but in the example in this section, we kept the phase of the magnetic field and adjusted the phase of the electric field. In doing so, we can also prove our theory. For the above example, we can add a phase to all electric fields and magnetic fields, which does not affect the calculation of self energy flow and mutual energy flow. After doing so, we will change from adjusting the phase of the electric field to adjusting the phase of the magnetic field. So my above theory is still valid.

One reason why we chose the above approach is that we are studying photons. We assume that the electromagnetic field in the photon is a plane wave, because the retarded wave in the photon is led by the advanced wave, and the advanced wave is led by the retarded wave. Finally, they all seem to live in a waveguide. We call this kind of waveguide natural waveguide. Even at the beginning of the photon phenomenon, both the retarded wave and the advanced wave are spherical waves, which are in the so-called broadcast mode. Once the two waves are synchronized, the electromagnetic field in the photon changes from broadcast mode to a single mode of point-to-point energy propagation. At this time, the electromagnetic field in the photon becomes a quasi-plane wave. Because the energy flow beam of photons is very narrow, the light source and sink of photons produce quasi-plane waves. The dimensions of light source and sink are very small, so they can not be regarded as current on the infinite plane. It should be seen as the electromagnetic field of dipole antenna. But dipole cannot produce plane waves. We need to use infinite plate current sheet to produce the plane wave. However this plane wave actually

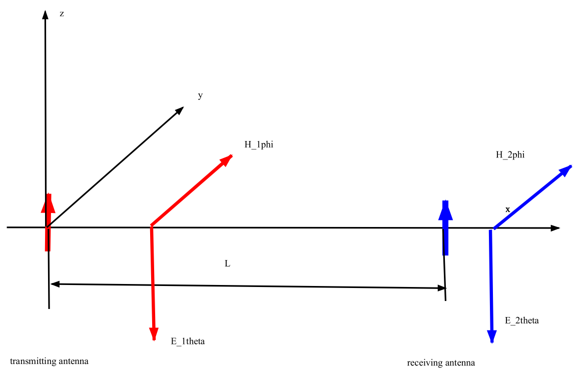

Figure 15. Double dipole antenna system, the red is the transmitting antenna, and the blue is the receiving antenna.

is produced by finite plane sheet current like dipoles. For this reason, we choose the initial phase of electromagnetic wave which is similar to that of dipoles.

VII. MUTUAL ENERGY FLOW FROM DIPOLE TRANSMITTING ANTENNA TO DIPOLE RECEIVING ANTENNA

Considering that both the transmitting antenna and the receiving antenna are dipole antennas, the distance between them is L, as shown in figure 15. The author uses the computer program language Julia to calculate the Poynting vector and the mutual energy flow of the double dipole antenna. The author mainly uses symbolic operation, and uses machine language to complete the substitution operation, simplification, the vector cross multiplication and conversion from spherical coordinates to rectangle coordinates.

Note that during manual calculation, in order to facilitate the selection of the downward direction of the dipole of the receiving antenna, since this chapter is a computer operation, the dipole direction of the receiving antenna is still oriented in the upward direction, which is more convenient for computer calculation.

We new that dipole 1 sends the retarded wave, dipole 2 sends the advanced wave, only far field is considered.

A. Electromagnetic field of dipole transmitting antenna.

The transmitting antenna is at the position,

$$x = 0, y = 0, z = 0 \tag{338}$$

Assume the current is,

$$\boldsymbol{I}_1 = I_{10}\hat{z} \tag{339}$$

The length of the dipole is,

$$\Delta z \tag{340}$$

the far field of the magnetic field is,

$$h_{\phi_1} = H_{10}\exp(-jkr_1)\frac{\sin\theta_1}{r_1} \tag{341}$$

the far field of the electric field is,

$$e_{\theta_1} = j\eta_0 H_{\phi_1} \tag{342}$$

among them

$$H_{10} = \frac{I_{10}\Delta z}{2\lambda} \tag{343}$$

Note in the above formula, the phase factor j of e_{θ_1} is determined according to the magnetic quasi-static field, not according to the Maxwell's equations. Magnetic quasi-static field

$$\boldsymbol{E} = -j\omega\boldsymbol{A} \sim j(-\hat{z}) \tag{344}$$

The phase of Magnetic field h_{ϕ_1} is also determined according to the magnetic quasi-static field, not by Maxwell's equations.

In the case of $\theta = 0$,

$$\hat{\theta} = (-\hat{z}) \tag{345}$$

We get the ϕ component of the magnetic field and θ component of the electric field, and the other components are zero, so we can form the electric field vector and the magnetic field vector, and represent the electromagnetic field in spherical coordinates,

$$\boldsymbol{h}_1 = [h_r \leftarrow 0, h_\theta \leftarrow 0, h_{\phi_1}] \tag{346}$$

$$\boldsymbol{e}_1 = [e_r \leftarrow 0, e_{\theta_1}, e_{\phi_1} \leftarrow 0] \tag{347}$$

Calculate Poynting vector,

$$\boldsymbol{S}_1 = \boldsymbol{e}_1 \times \boldsymbol{h}_1 \tag{348}$$

Then symbolic calculation result is,

$$\boldsymbol{S}_1 \cdot \hat{r} = j\eta_0 H_{10}H_{10}^*\frac{\sin^2(\theta_1)}{r_1} \tag{349}$$

It is found that Poynting vector is a pure imaginary number and reactive power.

B. Conversion from spherical coordinates to rectangular coordinates

$$\begin{cases} x_1 = r_1\sin\theta_1\cos\phi_1 \\ y_1 = r_1\sin\theta_1\cos\phi_1 \\ z_1 = r_1\cos\theta_1 \end{cases} \tag{350}$$

$$\begin{cases} r_1 = \sqrt{x_1^2 + y_2^2 + z_1^2} \\ \theta_1 = \arccos \frac{z_1}{r_1} \\ \phi_1 = \arctan(y_1, x_1) = atan2(y_1, x_1) \end{cases} \tag{351}$$

$$\begin{cases} \hat{x} & = \sin\theta_1 \cos\phi_1 \hat{r} + \cos\theta_1 \cos\phi\hat{\theta} - \sin\phi\hat{\phi} \\ \hat{y} & = \sin\theta \sin\phi\hat{r} + \cos\theta \sin\phi\hat{\theta} + \cos\phi\hat{\phi} \\ \hat{z} & = \cos\theta\hat{r} - \sin\theta\hat{\theta} \end{cases} \tag{352}$$

Hence,

$$\begin{cases} \hat{r}_1 & = \sin\theta_1 \cos\phi_1 \hat{x} + \sin\theta_1 \sin\phi_1 \hat{y} + \cos\theta_1 \hat{z} \\ \hat{\theta}_1 & = \cos\theta_1 \cos\phi_1 \hat{x} + \cos\theta_1 \sin\phi_1 \hat{y} - \sin\theta_1 \hat{z} \\ \hat{\phi}_1 & = -\sin\phi\hat{x} + \cos\phi\hat{y} \end{cases} \tag{353}$$

define:

$$M = \begin{bmatrix} \sin\theta\cos\phi, & \sin\theta\sin\phi, & \cos\theta \\ \cos\theta\cos\phi, & \cos\theta\sin\phi, & -\sin\theta \\ -\sin\phi, & -\cos\phi & 0 \end{bmatrix} \tag{354}$$

In M, $\theta = \theta_1$, $\phi = \phi_1$

$$\begin{bmatrix} \hat{r} \\ \hat{\theta} \\ \hat{\phi} \end{bmatrix} = M \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} \tag{355}$$

$$\boldsymbol{h}_1 = [h_{1r_1}, h_{1\theta_1}, h_{1\phi_1}] \begin{bmatrix} \hat{r} \\ \hat{\theta} \\ \hat{\phi} \end{bmatrix} = [h_{1r_1}, h_{1\theta_1}, h_{1\phi_1}] M \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} \tag{356}$$

$$\boldsymbol{e}_1 = [e_{1r_1}, e_{1\theta_1}, e_{1\phi_1}] \begin{bmatrix} \hat{r} \\ \hat{\theta} \\ \hat{\phi} \end{bmatrix} = [e_{1r_1}, e_{1\theta_1}, e_{1\phi_1}] M \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} \tag{357}$$

Calculate the modulus of Poynting vector in rectangular coordinate system, the computer symbolic calculation result is,

$$||S_1|| = \boldsymbol{e}_1 \times \boldsymbol{h}_1 = \eta_0 H_{10}^2 \frac{\sin^2(\theta_1)}{r_1} \tag{358}$$

The above formula is the same as the value in spherical coordinates, so it is correct. This step is used for checking calculation.

C. Calculation current I_2

Receiving antenna is at the position of,

$$x = L, y = 0, z = 0 \tag{359}$$

We assume that the current of the receiving antenna dipole is proportional to the field of the transmitting antenna dipole,

$$\theta_1 = \frac{\pi}{2}, \phi_1 = 0 \tag{360}$$

$$\boldsymbol{I}_{20} \sim \boldsymbol{e}_1(x_1 = L, y_1 = 0, z_1 = 0) \tag{361}$$

$$\boldsymbol{I}_{20} = \frac{1}{R_2} e_{1\theta_1}(x_1 = L, y_1 = 0, z_1 = 0)\hat{\theta}$$

$$= e_{1\theta_1}(x_1 = L, y_1 = 0, z_1 = 0)(-\hat{z})$$

$$= \frac{1}{R_2}\eta_0 H_{10} j \frac{\exp(-jkL)}{L} \sin(\theta_1 = \frac{\pi}{2})(-\hat{z})$$

$$= \frac{1}{R_2}\eta_0 H_{10} j \frac{\exp(-jkL)}{L}(-\hat{z}) \tag{362}$$

Where $\frac{1}{R_2}$ is the ratio$\boldsymbol{I}_{20}$ and $e_{1\theta_1}$. The ratio is related to the circuit of the receiving dipole antenna. We assume that it is a pure real number. We can always adjust the impedance of the receiving antenna to make it a pure real number, so the problem can be simplified. Therefore,

$$I_{20} = -\frac{1}{R_2}\eta_0 H_{10} j \frac{\exp(-jkL)}{L} \tag{363}$$

D. Calculation of H_{20}

Where R_2 Is the impedance of the receiving antenna. We assume that the inductive reactance of the receiving antenna is approximately zero. Hence,the impedance receiving antenna is $Z_2 = R_2$. First calculate the

$$x_1 = L, y_1 = 0, z_1 = 0 \tag{364}$$

Considering,

$$H_{20} = \frac{I_{20}\Delta z}{2\lambda} \tag{365}$$

There is,

$$H_{20} = \frac{\Delta z}{2\lambda} I_{20} = \frac{\Delta z}{2\lambda}(-\frac{1}{R_2}\eta_0 h_{10} j \frac{\exp(-jkL)}{L})$$

$$= \frac{\Delta z}{2\lambda}(-\frac{1}{R_2}\eta_0 \frac{\Delta z}{2\lambda} I_{10} j \frac{\exp(-jkL)}{L})$$

$$= -j\frac{(\Delta z)^2}{4\lambda^2}\frac{\eta_0 I_{10}}{R_2}\frac{\exp(-jkL)}{L} \tag{366}$$

E. Electromagnetic field of dipole receiving antenna

the magnetic field is, (note the advanced wave)

$$h_{2\phi_2} = H_{20}\exp(+jkr_2)\frac{\sin\theta_2}{r_2} \tag{367}$$

The electric field is

$$e_{2\theta_2} = j\eta_0 h_{2\phi_2} \tag{368}$$

Note that the phase factor j in the above formula is determined according to the magnetic quasi-static field

$$\boldsymbol{E} = -j\omega\boldsymbol{A} \sim j(-\hat{z}) \tag{369}$$

In the case $\theta = 0$

$$\hat{\theta} = (-\hat{z}) \tag{370}$$

Represent the electromagnetic field vector in spherical coordinates,

$$\boldsymbol{h}_2 = [h_{2r_2} \leftarrow 0, h_{2\theta_2} \leftarrow 0, h_{2\phi_2}] \tag{371}$$

$$\boldsymbol{e}_2 = [e_{2r_2} \leftarrow 0, e_{2\theta_2}, e_{2\phi_2} \leftarrow 0] \tag{372}$$

Calculate Poynting vector

$$\boldsymbol{S}_2 = \boldsymbol{E}_2 \times \boldsymbol{H}_2 \tag{373}$$

Calculated results,

$$\boldsymbol{S}_2 \cdot \hat{r} = j\eta_0 H_{20}H_{20}^*\frac{\sin^2(\theta_2)}{r_2} \tag{374}$$

In this way, Poynting vector is a pure imaginary number, which is reactive power. Switch to rectangular coordinate system

$$\boldsymbol{h}_2 = [h_{2r}, h_{2\theta}, h_{2\phi}]M\begin{bmatrix}\hat{x}\\ \hat{y}\\ \hat{z}\end{bmatrix} \tag{375}$$

$$\boldsymbol{e}_2 = [e_{2r}, e_{2\theta}, e_{2\phi}]M\begin{bmatrix}\hat{x}\\ \hat{y}\\ \hat{z}\end{bmatrix} \tag{376}$$

Calculate the Poynting vector modulus, the computer symbolic calculation result is,

$$||\boldsymbol{S}_2|| = ||\boldsymbol{e}_2 \times \boldsymbol{h}_2|| = \frac{\eta_0}{r_2^2}\sqrt{G}\sin(\theta_2) \tag{377}$$

where

$$G = (H_{20}H_{20}^*)^2$$

$$\cdot(\sin^2(\theta_2)|\sin^2(\phi_2)|^2 + \sin^2(\theta_2)|\cos^2\phi_2|^2 + |\cos(\theta_2)|^2)$$

The above formula verifies that the calculation of Poynting vector of dipole 2 is correct.

F. Calculation of special value of mutual energy flow between two dipole antennas

Now the fields of the two dipole antennas are converted to the rectangular coordinate system, so the mutual energy flow can be calculated in the rectangular coordinate system,

$$\boldsymbol{S}_m = \boldsymbol{e}_1 \times \boldsymbol{h}_2^* + \boldsymbol{e}_2^* \times \boldsymbol{h}_1 \tag{378}$$

This formula is too long to write, so we put in the special value

$$||\boldsymbol{S}_m||(x_1 \leftarrow x, y_1 \leftarrow 0, z_1 \leftarrow 0, x_2 \leftarrow L-x, y_2 \leftarrow 0, z_2 \leftarrow 0, x = 0.5)$$

$$= \frac{5.656854249}{L^2}\sqrt{2}\eta_0\sqrt{H_{20}H_{20}^*}|H_{10}| \tag{379}$$

Where $x_1 \leftarrow x$ means to replace x_1 with x. Let's consider

$$\boldsymbol{I}_{20} = E_1(x = L)\cdot(-\hat{z}) \tag{380}$$

$$H_{20} = \frac{I_{20}}{R_2} = -j\frac{(\Delta z)^2}{4\lambda^2}\frac{\eta_0 I_{10}}{R_2}\frac{\exp(-jkL)}{L} \tag{381}$$

Consider

$$k = \frac{2\pi}{\lambda} \tag{382}$$

$$r_1 = \sqrt{x_1^2 + y_1^2 + z_1^2} \tag{383}$$

$$r_2 = \sqrt{x_2^2 + y_2^2 + z_2^2} \tag{384}$$

$$\theta_1 = \arccos(\frac{z_1}{r_1}) \tag{385}$$

$$\theta_2 = \arccos(\frac{z_2}{r_2}) \tag{386}$$

$$\phi_1 = \arctan(y_1, x_1) = atan2(y_1, x_1) \tag{387}$$

$$\phi_2 = \arctan(y_2, x_2) = atan2(y_2, x_2) \tag{388}$$

This formula is too long to write, so it is substituted with a special value

$$\boldsymbol{S}_m = \boldsymbol{e}_1 \times \boldsymbol{h}_2^* + \boldsymbol{e}_2^* \times \boldsymbol{h}_1 \tag{389}$$

$$(\boldsymbol{S}_m \cdot \hat{y})(x_1 \leftarrow x, y_1 \leftarrow 0, z_1 \leftarrow 0, x_2 \leftarrow L - x, y_2 \leftarrow 0, z_2 \leftarrow 0) \tag{390}$$

$$= 0$$

$$(\boldsymbol{S}_m \cdot \hat{z})(x_1 \leftarrow x, y_1 \leftarrow 0, z_1 \leftarrow 0, x_2 \leftarrow L - x, y_2 \leftarrow 0, z_2 \leftarrow 0) \tag{391}$$

$$= 0$$

$$(\boldsymbol{S}_m \cdot \hat{x})(x_1 \leftarrow x, y_1 \leftarrow 0, z_1 \leftarrow 0, x_2 \leftarrow L - x, y_2 \leftarrow 0, z_2 \leftarrow 0, x \leftarrow 0.5)$$

$$= 1.0$$

In the last equation, we assume that

$$\frac{I_{10}^2 \eta_0^2}{R_2} (\frac{\Delta z}{\lambda})^3 \frac{1}{L^3} = 1 \tag{392}$$

These are also to verify the correctness of the program. Because the formula is too long, we can only judge its correctness with special values.

G. Calculation of general value of mutual energy flow between two dipole antennas

Three components of mutual energy flow can be obtained,

$$S_{mx} = \boldsymbol{S}_m \cdot \hat{x} \tag{393}$$

$$S_{my} = \boldsymbol{S}_m \cdot \hat{y} \tag{394}$$

$$S_{mz} = \boldsymbol{S}_m \cdot \hat{z} \tag{395}$$

Considering the substitution relationship,

$$k = \frac{2\pi}{\lambda} \tag{396}$$

$$\theta_1 = \arccos(\frac{z_1}{r_1}) \tag{397}$$

$$\theta_2 = \arccos(\frac{z_2}{r_2}) \tag{398}$$

$$\phi_1 = \arctan 2(y_1, x_1) \tag{399}$$

$$\phi_2 = \arctan 2(y_2, x_2) \tag{400}$$

$$r_1 = \sqrt{x_1^2 + y_1^2 + z_1^2} \tag{401}$$

$$r_2 = \sqrt{x_2^2 + y_2^2 + z_2^2} \tag{402}$$

$$x_1 = x \tag{403}$$

$$y_1 = 0 \tag{404}$$

$$z_1 = z \tag{405}$$

$$x_2 = x - L \tag{406}$$

$$y_2 = 0 \tag{407}$$

$$z_2 = z \tag{408}$$

$$H_{10} = I_{10} \frac{\Delta z}{2\lambda} \tag{409}$$

$$H_{20} = -j(\frac{\Delta z}{2\lambda})^2 \eta_0 I_{10} \frac{1}{R_2} \exp(-j\frac{2\pi}{\lambda} L) \frac{1}{L} \tag{410}$$

$$x = x_r * L \tag{411}$$

$$z = z_r * L \tag{412}$$

$$\lambda = \lambda_r * L \tag{413}$$

In the above subscript r means relative value. Furthermore,

$$I_{10}^2 \eta_0^2 \Delta z^3 \frac{1}{L^6 R_2} = 1 \tag{414}$$

$$\lambda_r = 0.1 \tag{415}$$

The last formula is equivalent to that we take the wavelength as one tenth of the distance L between the two dipoles. After the above substitution, the mixed Poynting vector is only the relative coordinate x_r,z_r The relative coordinates are using L as the unit.

$$S_{mx} = S_{mx}(x_r, z_r) \tag{416}$$

$$S_{my} = S_{my}(x_r, z_r) \tag{417}$$

$$S_{mz} = S_{mz}(x_r, z_r) \tag{418}$$

We know that,

$$S_{my} = 0 \tag{419}$$

We need to draw a picture to show S_{mx},S_{mz} in $x_r z_r$ In order to facilitate the use of the following two-dimensional vector fields,

$$V_x = \Re S_{mx}(x_r \leftarrow x, z_r \leftarrow y) \tag{420}$$

$$V_y = \Re S_{my}(x_r \leftarrow x, z_r \leftarrow y) \tag{421}$$

The modulus of mutual energy flow is defined as

$$V(x, y) = \sqrt{V_x^2 + V_y^2} \tag{422}$$

The figure below shows the modular and vector values of the mutual energy flow. Note that the figure above is drawn under certain the conditions,

$$\lambda_r = 0.1 \tag{423}$$

But the author found that the following figure is actually the same as λ_r. It doesn't matter. As long as we take the L value as 1, no matter λ_r Is 0.1 or 0.01, and the following figures are basically the same. That is to say, as long as L is drawn on the graph as a unit, the shape of the photon of the mutual energy flow is basically unchanged. Of course, this actually means that if the distance L increases by N times, the mutual energy flow photons will extend N times along the connecting line of the two dipoles. After this extension, it can actually be regarded as a plane wave around the axis. It can be seen from the figure that the direction of mutual energy flow can be opposite at some places away from the axis. But only on the axis, the mutual energy flow has a large value. And the direction is from the source to the sink.

It can be seen from the figure that the mutual energy flow does maintain a relatively large value on the connecting line of the two dipoles, especially at the source and sink. These mutual energy flows in the two points are represented as particles. The mutual energy flow in other places is relatively small.

The previous figure does not give the value of the modulus of the mixed Poynting vector. On the other hand, sometimes we need to know the direction of the vector at each point. The following picture is quite clear. It shows the properties of particles at the source and sink.

The above two figures actually show the pattern of photons. This figure is consistent with the author's initial estimate. Compare figures 10 and 11.

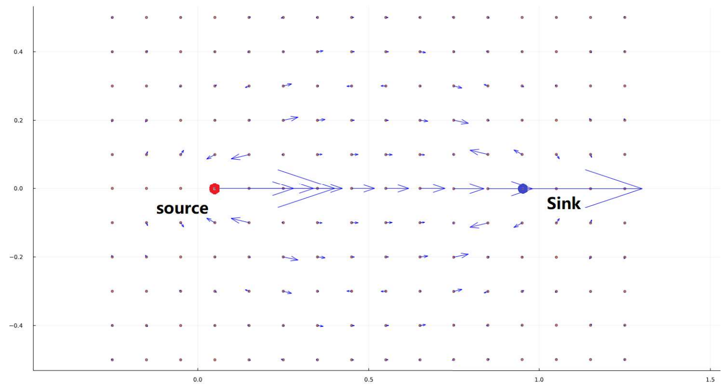

Figure 16. Vector value of mutual energy flow in (x_r, z_r) plane.

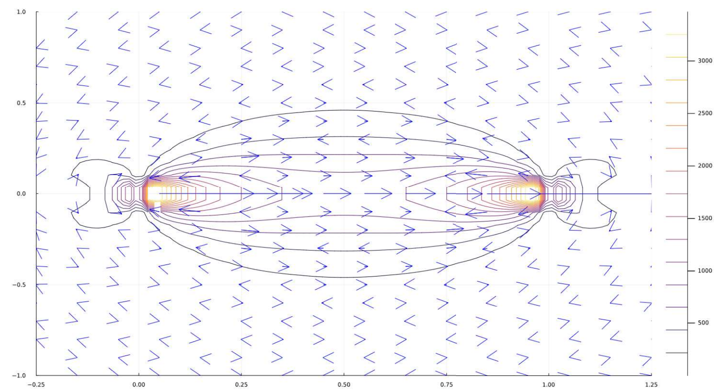

Figure 17. Modulus and vector values of mutual energy flow in x_r, z_r plane.

VIII. CONCLUSION

In this paper, the mutual energy theory of electromagnetic field is proposed, which comprehensively updates Maxwell's electromagnetic theory. Mutual energy theory does not believe that a changing current element can produce electromagnetic radiation. Because the radiation phenomenon is an interaction, the principle is essentially the same as that of a transformer. The transformer has a primary coil and a secondary coil. The energy flows from the primary coil of the transformer to the secondary coil. The same is true for electromagnetic radiation. There are transmitting antenna and receiving antenna, and the energy flow flows from the transmitting antenna to the receiving antenna. The antenna system consists of a transmitting antenna and a receiving antenna, which is equivalent to a transformer system, but the secondary coil and the primary coil are slightly far away. The source of light cannot emit light without the help of an environmental absorber.

This paper reviews the theory of mutual energy. Firstly, the author introduces the law of conservation of electromagnetic energy and applies it to the transformer system. Verify that this law is correct in the transformer system, and then consider moving the secondary coil of the transformer to a far place, which leads to the retarded potential. In order to make the law of energy conservation hold, the electromagnetic field of the secondary coil must be the advanced potential. This proves that the advanced wave exists. The wave equation can be obtained by using the retarded potential and the advanced potential, and then the Maxwell's equations can be derived by using the wave equation, from which the principle of mutual energy can be derived. The theorem of mutual energy flow is derived from the principle of mutual energy and the law of conservation of energy, and the conclusion that mutual energy flow does not overflow the universe is drawn. In addition, by comparing the principle of mutual energy with the Poynting theorem of N current elements, it is found that the conservation of energy can be satisfied only when the self energy flow is zero. Therefore, the self energy flow must be zero. However, according to the solution of Maxwell's equations, the self energy flow is not zero. For example, the Poynting vector of any antenna is indeed not zero. In order to make the self energy flow zero or not transfer energy, it is necessary to introduce a time reversal electromagnetic wave to counteract the self energy flow. This time reversal self energy flow is a modification of Maxwell's equation by mutual energy theory.

Furthermore, the author finds that if the self energy flow is reactive power, the self energy flow can be zero if the average value of the energy flow is zero. Therefore, an electromagnetic field and its time reversal electromagnetic field can be replaced by a wave of reactive power. This gives the author a new degree of freedom, that is, to properly adjust the phase between the electric field and the magnetic field so that Poynting's law is reactive power, so that the self energy flow does not radiate. For example, for a dipole antenna, the initial phase of the far field of the original electric field and magnetic field is j. The author finds that the phase of the magnetic field can be adjusted to 1 according to the quasi-static magnetic condition, so that the phase difference between the electric field and the magnetic field is 90 degrees. In this way, the Poynting vector of the dipole antenna is a pure imaginary number, so it is reactive power. In the same way, the advanced wave of the receiving antenna can also be adjusted to reactive power wave.

After the adjustment of the self energy flow, the mutual energy must still be the active power. We found that this was just possible. In this way, the mutual energy flow between the transmitting antenna and the receiving antenna is the active power. That is, the energy flow from the transmitting antenna dipole to the receiving antenna dipole is also active power. In fact, mutual energy flow is photon. Because it transmits energy from point to point. It is generated at the source and annihilated at the sink. In addition, the mutual energy theory also solves the problem of wave collapse. According to quantum mechanics, the wave collapses at the annihilation of the particle. The self energy flow in the collapse and mutual energy theory is reactive power, and the energy of photons transferred by mutual energy flow is exactly equivalent. Therefore, the mutual energy theory also well explains the wave collapse phenomenon.

The author has also described the mutual energy theory in the previous theory. The difference between this paper and the previous theoretical paper is that this paper gives two calculation examples. One is a double plane current sheets transformer system. One is a dual dipole antenna system including a transmitting antenna and a receiving antenna. From these two examples, it is clearly proved that the mutual energy theory is correct, and Maxwell's classical electromagnetic theory should indeed be revised. This correction not only adds the energy conservation law, the principle of mutual energy, the theorem of mutual energy flow and other theories, but also modifies the retarded wave and advanced wave obtained from Maxwell's equations to make them reactive power waves. In the above author's mutual energy flow theory, Maxwell's equations are still useful, but it become auxiliary equations instead of physical equations.

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