

Quantized Impedance Networks of Dark Matter and Energy

PETER CAMERON

Michigan/MIT/Brookhaven (retired)
electronGaugeGroup@gmail.com

ABSTRACT

Dark matter has two independent origins in the impedance model:

Geometrically, extending two-component Dirac spinors to the full 3D Pauli algebra eight-component wavefunction permits calculating quantum impedance networks of wavefunction interactions. Impedance matching governs amplitude and phase of energy flow. While vacuum wavefunction is the same at all scales, flux quantization of wavefunction components yields different energies and physics as scale changes, with corresponding enormous impedance mismatches when moving far from Compton wavelengths, decoupling the dynamics.

Topologically, extending wavefunctions to the full eight components introduces magnetic charge, pseudoscalar dual of scalar electric charge. Coupling to the photon is reciprocal of electric, inverting fundamental lengths - Rydberg, Bohr, classical, and Higgs - about the charge-free Compton wavelength $\lambda = h/mc$. To radiate a photon, Bohr cannot be inside Compton, Rydberg inside Bohr,... Topological inversion renders magnetic charge ‘dark’.

Dark energy mixes geometry and topology, translation and rotation gauge fields. Impedance matching to the Planck length event horizon exposes an identity between gravitation and mismatched electromagnetism. Fields of wavefunction components propagate away from confinement scale, are reflected back by vacuum wavefunction mismatches they excite. This attenuation of the ‘Hawking graviton’ wavefunction results in exponentially increasing wavelengths, ultimately greater than radius of the observable universe. Graviton oscillation between translation and rotation gauge fields exchanges linear and angular momentum, is an invitation to modified Newtonian dynamics.

“The hard part will be getting physicists to think in terms of impedances”
Richard Talman, walking to lunch at Brookhaven cafeteria (April 2012)

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2 executive summary

The phenomenological impedance model has string theory roots in the 1960s S-matrix bootstrap [1–8]. Naturalness comprises the consistency conditions [9–17]. There is no Lagrangian, no differential equations to solve for a wavefunction. Model starts with the wavefunction. Equations of motion calculate quantized impedance networks of wavefunction interactions [18]. These govern amplitude and phase of energy transmission, such that the S-matrix is also the gauge group, with direct interaction of matrix elements citizens of Chew’s nuclear democracy [3, 4]. There are no free parameters, just **three assumptions** [19].

Foremost is the vacuum wavefunction, the math, **geometry** and **topology**. The model works not in unintuitive Pauli and Dirac matrix representations of Clifford algebra, but rather in the easily visualized geometric representation, the algebra of geometric objects [20–28]. Topology requires invertibility. There exist only four normed division algebras, all Clifford - real, complex, quaternion, and octonion [29, 30]. The model extends two-component Dirac spinors to the largest division algebra, the eight-component 3D Pauli algebra of flat space. *Vacuum wavefunction is the same at all scales, Planck to cosmological.*

Physical manifestation requires **fields**, a coupling constant. Various combinations of the four fundamental constants that define $\alpha = e^2/4\pi\epsilon_0\hbar c \approx 1/137$ permit assigning electric and magnetic flux quanta to the eight wavefunction components, and calculating quantized impedance networks of wavefunction interactions [18]. *This is important.* Impedance matching governs amplitude and phase of energy flow, of information transmission [31–34].

Third, the model requires a **mass gap** [35], a lightest rest mass charged particle to couple to the photon, setting the scale of space at the electron Compton wavelength $\lambda_e = h/m_e c$. *Different physics at different energies arises from scale to which flux quanta are confined.*

Such a model is naturally gauge invariant, finite, confined, asymptotically free, background independent, and contains the four forces, dark matter and dark energy [16, 17]. For dark matter focus is on origins in both topological and geometric impedances, on rotation and translation gauge fields; for dark energy on their mixing in gravitation. Model phenomenology, the data, reconciles String Theory and Standard Model [36].

3 historical perspective - two things lost and bootstrap fails

3.1 lost - geometric representation

Figure 1 shows evolution of the algebra [37], illustrating an important point - geometric representation of Clifford algebra unifies the mathematical physics timeline [26, 38–40].

Hermann Grassman was “...a pivotal figure in the historical development of a universal geometric calculus for mathematics and physics... He formulated most of the basic ideas and... anticipated later developments. His influence is far more potent and pervasive than generally recognized.” [38] Among many accomplishments, he introduced [41, 42] the outer wedge product $a \wedge b$ shown in figure 2.

Grassman’s work lay fallow until Clifford [20] “...united the inner and outer products into a single *geometric* product. This is associative, like Grassman’s product, but has the crucial extra feature of being *invertible*, like Hamilton’s quaternion algebra”. [28] While Clifford algebra attracted interest, it was “...largely abandoned with the introduction of what people saw as a more straightforward and generally applicable algebra, the vector algebra of Heaviside.” [40]

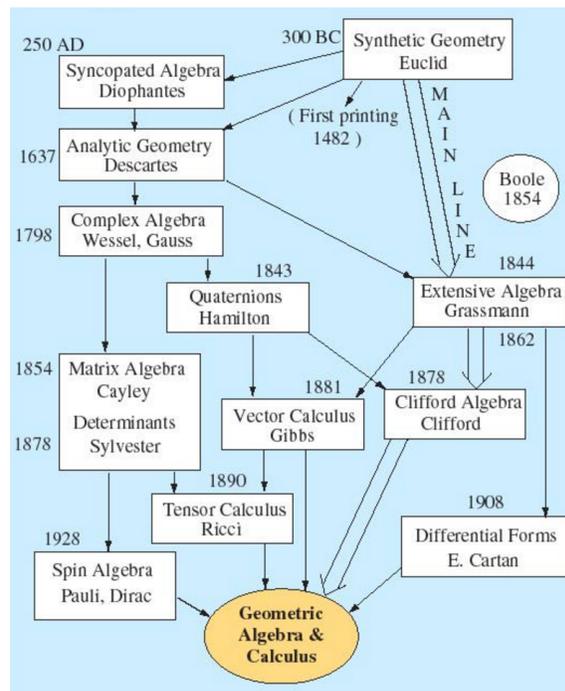


Figure 1: Geometric Algebra timeline [37]

With the early death of Clifford at age 33 in 1879, absence of advocates to balance the powerful Gibbs and Heaviside contributed to neglect of the algebra. “This was effectively the end of the search for a unifying mathematical language and beginning proliferation of novel algebraic systems...” [40]. The algebra resurfaced without geometric meaning in the 1920s Pauli and Dirac matrices, and with a few isolated exceptions remained dormant until rediscovered and extended by David Hestenes in the 1960s [23–25].

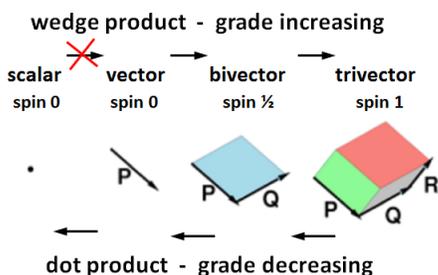


Figure 2: Pauli algebra of 3D space

Difference mode is ~ 10 GeV dominant bottomonium decay family. Higgs mass is absent.

The algebra has properties useful to physics. Geometric products mix spatial dimension (grade) of wavefunction components. Product of two grade 1 vectors $ab = a \cdot b + a \wedge b$ yields grade 0 scalar boson and grade 2 bivector fermion, $WZ = Higgs + top$. The four superheavies comprise a minimally complete 2D algebra - one scalar, two vectors, and one bivector (1,2,1). Sum mode of W and Z is top mass within .007, curiously close to the QED coupling constant lowest order perturbation.

Mixing of grades makes the algebra unique in handling geometric concepts in any dimension. In 3D geometric representation, the octonion algebra is comprised of 1 scalar point, 3 vector lines, 3 bivector areas, and 1 trivector volume element (1,3,3,1) [15, 43]. Geometric products of two wavefunctions mixes bosons and fermions, dynamic SUSY.

The algebra offers an origin of weak interaction chiral symmetry breaking in octonion algebra of the vacuum wavefunction, which is not three-component associative. Absence of three-component right-handed neutrinos has an origin in the math [30].

3.2 lost - impedance quantization

How are quantum impedance networks not already present in the mainstream? [44]

A pivotal oversight arose from theorists' habit of setting fundamental constants to dimensionless unity. Although $h=c=G = \dots = 1$ was not problematic for specialists, setting the $Z_0 = \sqrt{\mu_0/\epsilon_0} \simeq 377$ ohm free space impedance excited by the photon [45, 46] to dimensionless unity disappeared over the horizon, forgotten. While equating electric permittivity and magnetic permeability is in hindsight obvious QED folly, this arose in part as an historical accident, a consequence of the order in which experimentalists revealed the relevant phenomena. The scaffolding of QFT was erected on experimental discoveries of the first half of the twentieth century, on foundations of QED, set long before discovery of *exact* impedance quantization, a *new* fundamental constant of nature [47].

That discovery was greatly facilitated by scale *invariance*. This classically peculiar impedance is topological, the measured impedance being independent of Hall bar size or shape. Prior to that, impedance quantization was more implied than explicit in the literature [31, 48–55]. Scale-*dependent* quantum impedances are less obvious, much more elusive.

Bjorken's 1959 thesis [50] presented an approach summarized [51] as "...an analogy between Feynman diagrams and electrical circuits, with Feynman parameters playing the role of *resistance*, external momenta as current sources, and coordinate differences as voltage drops. Some of that found its way into section 18.4 of..." the canonical text (emphasis added) [52]. As presented there, Feynman renormalization parameter SI units are [sec/kg], units of mechanical *conductance* [56]. It is not difficult to understand what led Bjorken astray, as well as those who have made more recent similar attempts [31, 57–59]. Units of mechanical impedance are [kg/sec]. One would think that more [kg/sec] would mean more mass flow. However, the physical reality is more [kg/sec] means more impedance and *less* mass flow. With confusion that resulted from misinterpreting conductance as resistance, and lacking the concept of exact impedance quantization, the anticipated intuitive advantage of the circuit analogy [52] was lost. The jump from well-considered analogy to a naturally finite QED impedance model was not realized at that time.

The photon is our fiducial in measurements of the properties of space. Topological duality [60, 61] arises from the difference in coupling to the photon of magnetic and electric charge. If we take magnetic charge g to be defined by the Dirac relation $eg = \hbar$ and the electromagnetic coupling constant to be $\alpha = e^2/4\pi\epsilon_0\hbar c$, then e is proportional to $\sqrt{\alpha}$ whereas g varies as $1/\sqrt{\alpha}$ [62]. The characteristic coherence lengths of figure 3, precisely spaced in powers of α , are inverted for magnetic charge. The Compton wavelength $\lambda = h/mc$ is independent of charge.

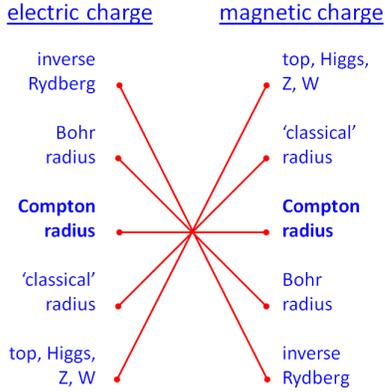


Figure 3: Topological inversion

With electric charge, fundamental lengths correspond to specific physical mechanisms of photon emission or absorption, matched in both quantized impedance and energy. Inversion results in mismatches in both. Magnetic charge g is ‘dark’, cannot couple to the photon, not despite its great strength, but rather because of it. The α -spaced lengths of figure 3 correspond to specific physical mechanisms of photon absorption and emission. Bohr radius cannot be inside Compton wavelength in the basic photon-charge coupling of QED, Rydberg cannot be inside Bohr,... Specific physical mechanisms of photon emission and absorption no longer work.

Like the first Rochester Conference on Coherence and Quantum Optics in 1960, the 1963 paper/thesis by Vernon and Feynman [53] on the “Interaction of Systems” was motivated by invention of the maser. The authors devoted a thesis to concepts needed for matching to the maser. However, again lacking was the explicit concept of impedance quantization. The path integral book mentions matching as well. [54]

Mechanical impedance quantization in both the hydrogen atom and gravitation was introduced in a 1975 unpublished note [31]. However, the quantity with units [kg/sec] was interpreted as mass flow in the de-Broglie wave, confusion arising again with inversion of units.

QFT permits defining only one fundamental length [63]. In models that aren’t naturally finite this is problematic. Either UV singularity or IR boundary at infinity cannot be removed. However, quantum impedances render QED finite without renormalization. Feynman’s regulators are the impedance mismatches [52]. Mismatches to both singularity and boundary are infinite. At the point singularity inductance is infinite and capacitance zero; at the boundary capacitance is infinite, inductance zero.

Had exact impedance quantization been discovered in 1950 rather than 1980, one wonders whether quantum impedance networks might have found their way into QED at that time, and how the distinction between geometric and topological wavefunction interactions might have informed and shaped our understanding of dark matter and energy.

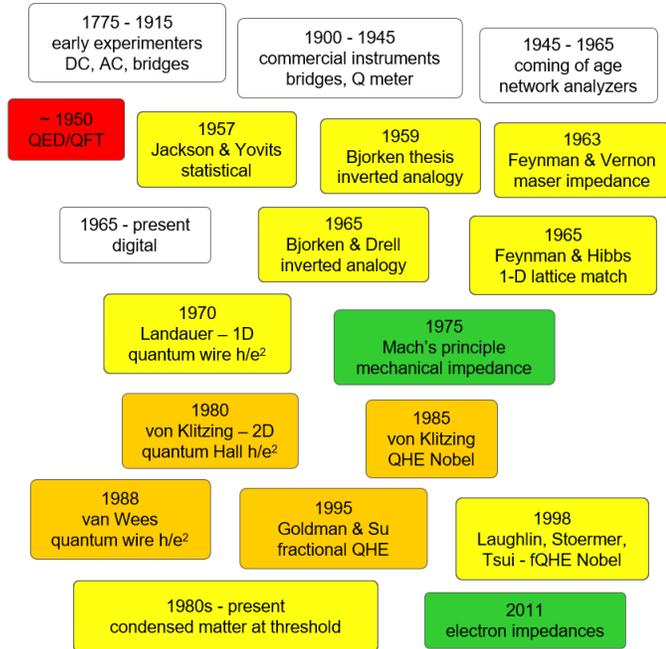


Figure 4: Impedance quantization timeline

3.3 ‘failure’ of the bootstrap

Beyond absence of geometric algebra and impedance matching, additional circumstances contributed to stall the 1960s bootstrap and its evolution into string theory (fig. 5).

Goal of 1960s S-matrix bootstrap program was to understand nucleon structure. Fundamental length was nucleon Compton wavelength, a reasonable choice given absence of geometric algebra and quantum impedance networks, and the consequent failure of QED to encompass all four forces. A successful-at-all-energies QED bootstrap has the photon-electron interaction at its foundation, requires that the mass gap be not nuclear mass, but rather the lightest charged particle, defining the electron Compton wavelength $\lambda_e = h/m_e c$. This of itself was sufficient to stall the bootstrap program, and eventually replicate it at the Planck length, where it found amazingly rich and productive new life as string theory.

Absence of closure in analytic continuation contributed to stalling the 1960s bootstrap. At that time it was not possible to sufficiently tie observed phases to the amplitudes. As shown in figure 9 in both model and data, unstable particle lifetimes are structured in powers of the coupling constant α [34, 64–68]. Time is the integral of phase. The essential phases present themselves in correlation of measured unstable particle lifetimes with impedance nodes of the network. It is there at the nodes that modes are matched in both amplitude and phase, as required for propagation of energy during wavefunction decoherence.

Here we find the essential non-linear mechanisms for frequency domain translation of energy during wavefunction interactions. Scale dependence in figure 9 is logarithmic, with spin 1 bosons the perfect pumps for spin 1/2 fermions in noiseless nonlinear parametric amplification and frequency conversion of the resulting Mexican hat potential [69–71].

Bootstrap philosophy suggests that understanding the emergent S-matrix of observables might be guided and constrained by global symmetries, local gauge invariance and the related analyticity, unitarity, locality, causality, conformality, Lorentz invariance, crossing symmetry of Mandelstam variables,... and nuclear democracy; all S-matrix modes are on an equal footing, and none is more elementary than others. This is a powerful and comprehensive collection of consistency conditions, subtle and rich with nuance, convoluted

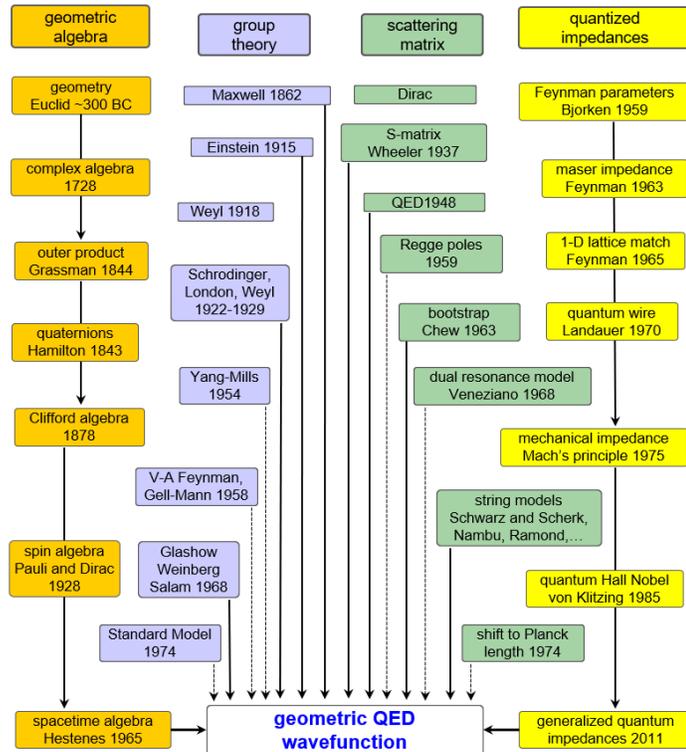


Figure 5: Merging Four Timelines

with much mathematical abstraction; a refined and sophisticated top-down approach to understanding emergence from such constraints. A complementary bottom-up approach employs the constraint of naturalness [16,17]. The former has four forces, each comparatively old and deep and exquisitely detailed, abstract and complex. The latter has but one force, reborn in a form both minimally and maximally complete, yet so young and callow, shallow and simple while deep and profound. They have much to inform each other.

For instance, the requirement for Lorentz invariance appears moot in the fundamental two-body interaction. As shown in figure 8, the S-matrix generated by geometric products of two eight-component wavefunctions is comprised exclusively of two-body modes. Yet figure 9 spans the full range of unstable particle lifetimes, both sides of the Compton wavelength. There is no observer in the background-independent impedance model, just two interacting wavefunctions [31]. Special relativity is three-body, Lorentz transform the Pythagorean theorem. Interaction of two wavefunctions is fundamental. Special relativity is emergent. Three-body problem enters the model first with failure of three-component associativity in the algebra and consequent parity violation of the three-component neutrino [72–75].

4 photon-electron interaction - Rosetta Stone of QED

Electromagnetism includes three kinds of impedances - inductive, resistive, and capacitive. Resistance is the most familiar, is dissipative, turns coherent information into incoherent heat. Quantum inductors and capacitors have no resistance, are classically ‘ideal’. Their effect is to retard (capacitive) or advance (inductive) phase of oscillations.

Figure 6 shows the photon near-field impedance match to the Hydrogen atom, Rosetta stone of atomic physics. Here the physics community is lost. Neither photon [76] nor electron [33] near-field impedances can be found in textbooks, curricula, or journals of the physicist, are for the most part absent from our education and practice. What governs the flow of energy in the basic photon-electron interaction of QED was lost in physics [44].

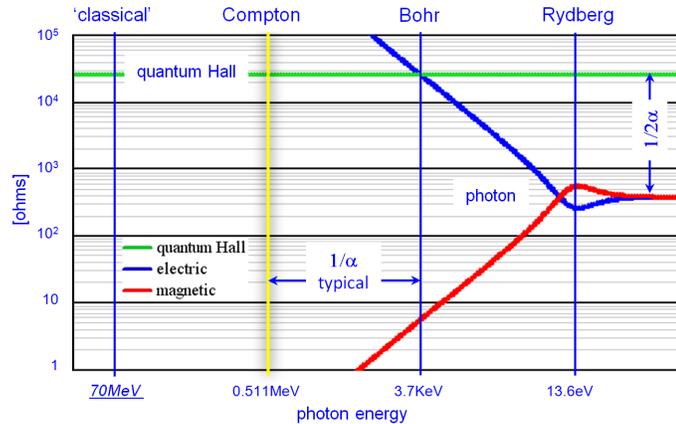


Figure 6: H atom impedance match [32]

There are two essential points:

First, what matters are not absolute impedances, but rather the matching, their relative values. In this they are like the energy whose transmission they govern.

The second point distinguishes between scale-dependent and invariant impedances.

Scale-dependent impedances are geometric, include Coulomb, scalar Lorentz, and dipole-dipole, with $1/r$ and $1/r^3$ potentials. They are causal and local, communicate both amplitude and phase. Scale dependence renders them parametric [69–71], nonlinear, permitting essential noiseless quantum amplification and frequency domain transformation of energy. They are the translation gauge fields of Gauge Theory Gravity [43, 77–81]

Scale-invariant impedances are topological, include vector Lorentz of quantum Hall and Aharonov-Bohm effects, centrifugal, chiral, Coriolis, and three-body. Associated potentials are inverse square, those of anomalies [82]. Resulting motion is perpendicular to applied force. They cannot do work, communicate only relative phase, not a single measurement observable. They cannot be shielded, are the acausal channels of non-local entanglement, the rotation gauge fields of Gauge Theory Gravity.

Photon appears unique in having both a non-local topological far-field impedance and local geometric near-field impedances, as shown in figure 6. Photon excitation of the Dirac spinor virtual wavefunction permits calculation of the far-field 377 ohm vacuum impedance it sees [45, 46]. We seek to extend this to dark matter, dark energy...

Mass is quantized. Rest mass particles have easily calculated mechanical impedances [31]. Electromagnetic transformation is straightforward via the electromechanical oscillator [18]. What this lacks is phase information. Mechanical impedances are of a single field, a ‘matter field’, electromagnetic of two - E and B. Nonetheless, this is a tremendous calculational simplification. And phase information is partially recovered via the experimentalist, in phase correlation of measured unstable particle lifetimes with network nodes of figure 9.

5 the S-matrix - stable, unstable, and dark modes

Foremost is vacuum wavefunction, the **geometry**, that of the math. The model works not in unintuitive Pauli and Dirac matrix representations, but rather the easily visualized geometric representation, the algebra of geometric objects [20–23, 26–28]. Geometric product operation is shown in figure 2, and expanded upon there with top/Higgs/Z/W dynamics. Product of two minimally complete vacuum wavefunctions generates the geometric S-matrix of figure 7. Topology requires invertibility. There exist only four normed division algebras, all Clifford - real, complex, quaternion, and octonion [29, 30]. The model extends two-component Dirac spinors to the largest division algebra, eight-component flat space 3D Pauli algebra.

	scalar	vector	bivector	trivector
scalar	scalar	vector	bivector	trivector
vector	vector	scalar + bivector	vector + trivector	bivector + quad
bivector	bivector	vector + trivector	scalar + quadvector	vector + penta
trivector	trivector	bivector + quadvector	vector + pentavector	scalar + sexta

Figure 7: geometric S-matrix

Vacuum wavefunction is the same at all scales, Planck to cosmological.

Clifford products change spatial dimensionality, making geometric algebra unique in the ability to handle dynamics in all dimensions. With introduction of the coupling constant and assignment of of geometrically and topologically appropriate flux quanta to the wavefunction components of figure 7, products of minimally complete Dirac wavefunctions at top and left of figure 8 generate the S-matrix in 6D phase space, three each space and phase. Each of the three orientational degrees of freedom requires its own relative phase. Time is the integral of phase, excepting factors of two for fermions *and* bosons the same for all three, collapsing phase space to flat 4D Minkowski spacetime [17].

It is important to distinguish between wavefunction components and particles of the S-matrix. Wavefunction components are not particles, but rather geometrically and topologically appropriate electric and magnetic flux quanta. Particles are comprised of one or more modes of the S-matrix generated by geometric products of wavefunction components, coupled by Maxwell's equations mixed with topology.

	electric charge e scalar	elec dipole moment 1 d_{E1} vector	elec dipole moment 2 d_{E2} vector	mag flux quantum ϕ_B vector	elec flux quantum 1 ϕ_{E1} bivector	elec flux quantum 2 ϕ_{E2} bivector	magnetic moment μ_{Bohr} bivector	magnetic charge g trivector
e	ee scalar	ed_{E1}	ed_{E2} vector	$e\phi_B$	$e\phi_{E1}$	$e\phi_{E2}$ bivector	$e\mu_B$	eg trivector
d_{E1}	$d_{E1}e$	$d_{E1}d_{E1}$	$d_{E1}d_{E2}$	$d_{E1}\phi_B$	$d_{E1}\phi_{E1}$	$d_{E1}\phi_{E2}$	$d_{E1}\mu_B$	$d_{E1}g$
d_{E2}	$d_{E2}e$	$d_{E2}d_{E1}$	$d_{E2}d_{E2}$	$d_{E2}\phi_B$	$d_{E2}\phi_{E1}$	$d_{E2}\phi_{E2}$	$d_{E2}\mu_B$	$d_{E2}g$
ϕ_B	$\phi_B e$ vector	$\phi_B d_{E1}$	$\phi_B d_{E2}$ scalar + bivector	$\phi_B \phi_B$	$\phi_B \phi_{E1}$	$\phi_B \phi_{E2}$ vector + trivector	$\phi_B \mu_B$	$\phi_B g$ $bv + qv$
ϕ_{E1}	$\phi_{E1}e$	$\phi_{E1}d_{E1}$	$\phi_{E1}d_{E2}$	$\phi_{E1}\phi_B$	$\phi_{E1}\phi_{E1}$	$\phi_{E1}\phi_{E2}$	$\phi_{E1}\mu_B$	$\phi_{E1}g$
ϕ_{E2}	$\phi_{E2}e$	$\phi_{E2}d_{E1}$	$\phi_{E2}d_{E2}$	$\phi_{E2}\phi_B$	$\phi_{E2}\phi_{E1}$	$\phi_{E2}\phi_{E2}$	$\phi_{E2}\mu_B$	$\phi_{E2}g$
μ_B	$\mu_B e$ bivector	$\mu_B d_{E1}$	$\mu_B d_{E2}$ vector + trivector	$\mu_B \phi_B$	$\mu_B \phi_{E1}$	$\mu_B \phi_{E2}$ scalar + quadvector	$\mu_B \mu_B$	$\mu_B g$ vector + pv
g	ge trivector	gd_{E1}	gd_{E2} bivector + quadvector	$g\phi_B$	$g\phi_{E1}$	$g\phi_{E2}$ vector + pentavector	$g\mu_B$	gg scalar + sv

Figure 8: Impedance representation of the S-matrix, arranged in even flavor eigenmodes (blue) and odd color transition modes (yellow) by geometric grade. Modes indicated by colored symbols (diamond, tringle,...) are plotted in figure 9.

Of the eight wavefunction components, we see only three - electric charge e , magnetic flux quantum ϕ_B , and the Bohr magneton magnetic moment μ_{Bohr} . Modes containing only these ‘visible’ components have green backgrounds in figure 8, are modes of the stable proton [83]. Phase shifts of the vacuum modes they excite remain phase stable, are coherent far beyond present lifetime of the universe. Modes containing one each visible and dark components are unstable, excite differential phase shifts from the vacuum and decohere. Remaining modes have no visible components, don’t couple to the photon, are dark.

The model assigns flux quanta to the eight wavefunction components via the coupling constant, removing degeneracy of the three each vector and bivector orientational degrees of freedom. E and B flux quanta add two additional DOFs, raising wavefunction ‘dimensionality’ to that of 10D string theory. A consequence of this increase from 8D to 10D is that the S-matrix increases dimensionality as well, from 8×8 to $8 \times 8 \times 8$, a cube.

Whereas three-component wavefunction modes cannot be had in the 8×8 S-matrix of figure 8, they far outnumber two-component surface modes in the cube. One might conjecture that $3 \times 3 \times 3$ PMNS and CKM matrices can be found there. Rubik's cube is perhaps a helpful hands-on tool for visualizing analytic continuation at the edges [84].

6 impedance analysis of unstable particle lifetimes

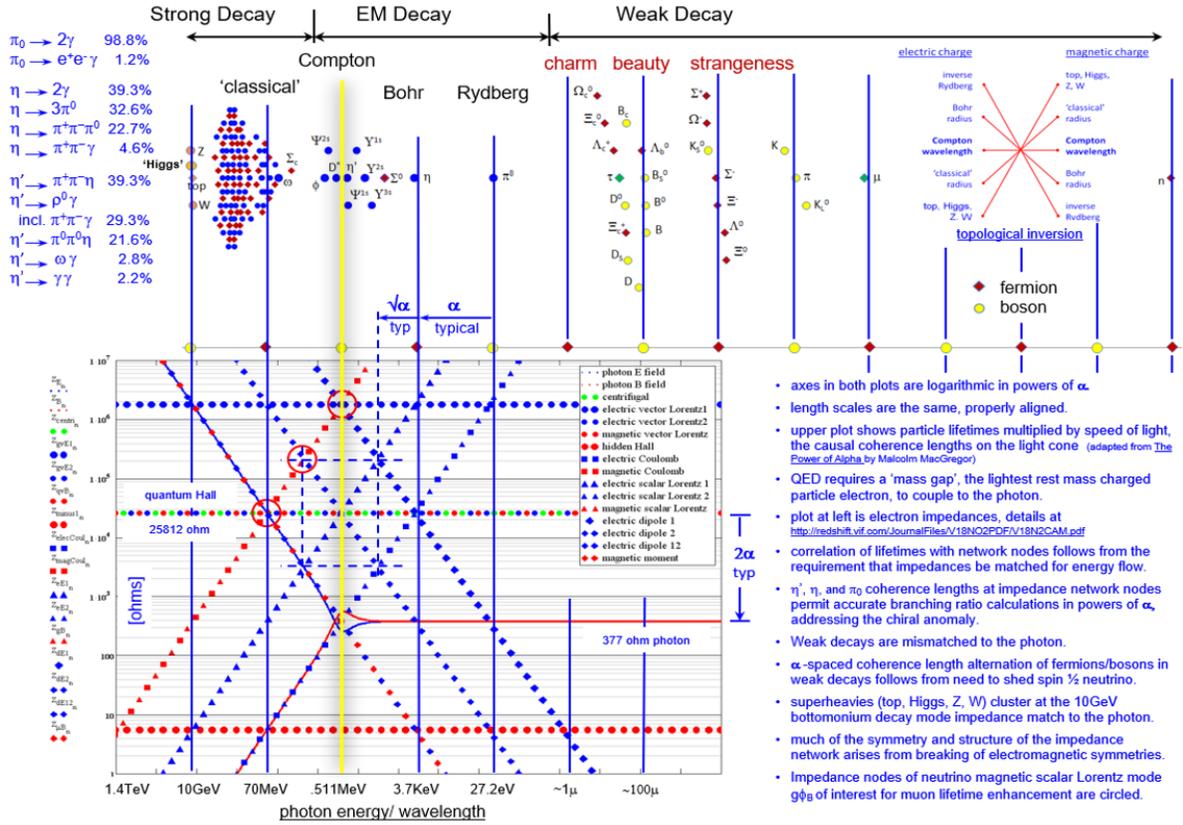


Figure 9: Correlation of lifetimes with α -spaced network nodes [34, 64–66]

A subset of S-matrix mode interaction impedances indicated by symbols (triangles, diamonds,...) in figure 8 are plotted in the network at lower left of figure 9 [85], revealing their causal role in coherence and decoherence of the unstable particle spectrum. Impedances must be matched for the energy transmission essential in decay, as illustrated by phase correlation of unstable particle lifetimes (light cone coherence lengths) with network nodes.

Vacuum wavefunction is the same at all scales. Different physics at different energies arises from scale-dependent field strength of flux quanta. Figure 9 can be extended in both UV and IR, beyond both Planck length and boundary of the observable universe [16].

7 Planck length impedance networks

Not all are in agreement that Einstein whole-heartedly endorsed curved space interpretations. He expressed this quite clearly in politically correct private communication:

“It is wrong to think that ‘geometrization’ is something essential. It is only a kind of crutch for finding of numerical laws. Whether one links ‘geometrical’ intuitions with a theory is a ... private matter.” [86,87]

Riemann’s curvature tensor preceded general relativity by six decades. It was Clifford who translated Riemann’s hypotheses into English, long before formulating his algebra of geometric objects [88]. Lacking Clifford’s flat space geometric representation [21, 44], Einstein’s adoption of Riemann’s formalism led to dominance of curved space interpretations. Equivalence of flat Minkowski spacetime Gauge Theory Gravity with curved space General Relativity was introduced by the Cambridge group and Professor Hestenes, and elaborated during following decades. [77, 78, 89–93]. What matters is not geometrization, but rather equivalence of gravitational and inertial mass, the equivalence principle [94].

Flat 4D Minkowski spacetime phase shifts of QED wavefunction interaction impedances are the GTG equivalent of GR’s spatial curvature. While strong classical arguments have been advanced against electromagnetic models of gravitation [95], preliminary examination suggests such arguments fail point-by-point when full consequences of wavefunction interaction impedance quantization is present in GTG [81].

As shown in figure 10, impedance mismatches between Compton and Planck wavefunctions reveal an identity¹. Gravitational force between two wavefunctions equals mismatch-attenuated electromagnetic force they share, at ppb accuracy of the five fundamental constants input by hand. Newton’s big G, by many orders of magnitude least accurate of the fundamental constants, cancels out in the ratio of ratios establishing the identity [96].

Inertial mass finds its origin in the Compton wavefunction, gravitational mass in the Planck wavefunction. Origin of inertial mass at Compton wavelengths of both electron and Planck particle is in wavefunction electromagnetic field energies. Origin of gravitational mass at the electron Compton wavelength is in the mismatch to the Planck length. They are numerically equal at sub-ppb accuracy of the four fundamental constants that define the coupling constant α . However to say they are equivalent (or not) requires the more refined understanding of ‘equivalence’ in section 8 [94].

Horizontal scales of figures 9 and 10 are logarithmic, respectively span $1/\alpha^{13} \simeq 6 \times 10^{27}$ and $1/\alpha^{14} \simeq 8 \times 10^{29}$ orders of magnitude. At the extreme left of figure 10, impedance network nodes are shifted from powers of the coupling constant by a mysterious offset of ~ 10.23 , by the lowest order of magnitude. A first thought might be to ultrafine tune α such that this offset disappears. Curiously, one finds the offset appears independent of the four fundamental constants’ numerical values that define the dimensionless coupling constant.

Vacuum wavefunction is the same at Planck and Compton scales. In the big bang bounce, a pure state ‘primordial photon’ entering from left of figure 10 is either left or right handed. It first encounters the heaviest rest mass S-matrix mode, the Planck-scale fermionic top quark. Chiral symmetry is broken at the outset - first in the photon polarization, then in the topological spin 1/2 fermion, with the vacuum wavefunction not yet fully manifested.

¹figures 10 and 11 lack one of the three electric dipole moments, previously unrecognized. They need updating to the more symmetric network of figure 9.

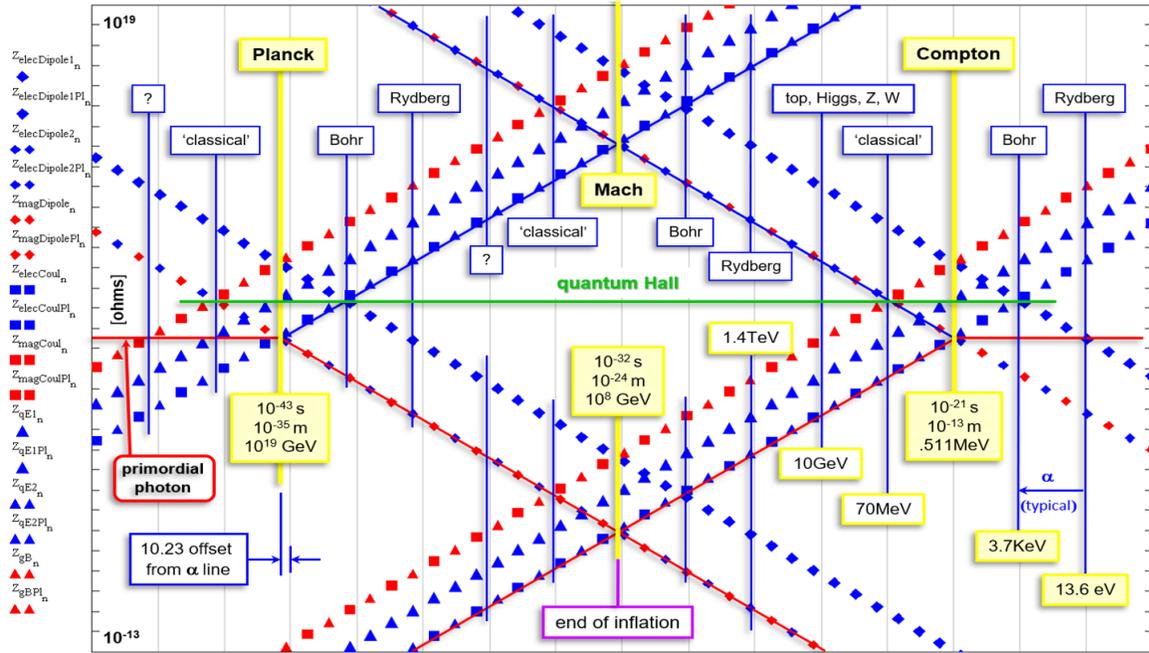


Figure 10: Interaction impedance networks defined at Compton and Planck lengths, showing a .511 Mev photon entering from right and ‘primordial photon’ from left. [96–99]

With chiral symmetry breaking comes the absence of antimatter at the primordial Planck scale. After *top*, next mode is the spin 0 *Higgs* scalar, the first essential gauge, the phase that couples *top* to *Z* and *W*, *Higgs + top = WZ* as shown by figure 2, transmitting energy from the primordial photon, ‘giving mass’ so to speak.

In that earliest instant there is \sim infinite energy but only one wavefunction, that of the infinitely massive Planck particle, whose event horizon is likewise at infinity. To set the scale of space requires a second wavefunction, ending inflation at $\sim 10^{-32}$ seconds with bifurcation of network nodes to extreme high and low ‘Mach scale’ impedances. Mach scale mismatch to the electron Compton scale is $\sim 10^9$, presently experimentally insurmountable. However, the 1.4 TeV coherence line adjacent to the 10 GeV bottomonium modes of figure 10 might be worth an LHC search if one could offer plausible triggers.

8 boundary of the universe - equivalence principle

The earth-moon system $1/r$ *gravitational* potential is associated with local geometric scale-dependent impedances, causally transmits both amplitude and phase. The $1/r^2$ *inertial* centrifugal potential is associated with non-local scale-invariant topological impedances, is acausal, communicates only phase, not a single measurement observable.

This distinction, potentially problematic for the equivalence principle, can be understood in terms of quantum phase. With topological impedances, phase shifts are unchanging with scale. Quantum Hall effects [100] are good examples. Phase shifts of geometric impedances vary with scale. The mismatch-attenuated long wavelength Hawking photon completes less than one cycle in present lifetime of the universe. Variation of phase with scale is extremely slow, almost scale-invariant, topological.

We take the graviton to be the full eight-component Planck particle wavefunction, the fields seeking to propagate to infinity, for the most part reflected back by the vacuum impedance mismatches they excite. What continues to propagate *in the near field* we define as the graviton, all eight components. Much of what follows is specific to the S-matrix photon transition modes, on the skew diagonal of figure 8, adjacent the main diagonal.

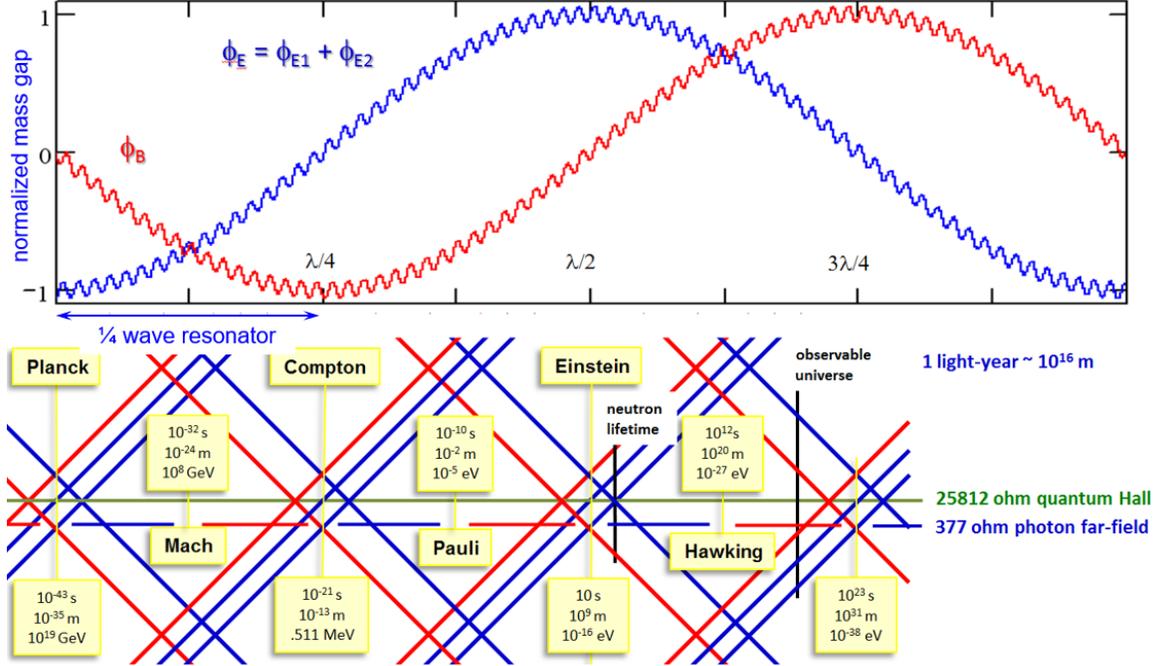


Figure 11: Correlation of physically manifested vacuum wavefunction impedance network nodes with Hawking photon attenuation by impedance mismatch reflections [34, 64–66]

Hawking graviton mode energy at the quarter-wave electron Compton wavelength [96] of figure 11 is origin of gravitational mass, shared with several degenerate modes [33, 83, 102]. It precisely equals electromagnetic self-energy of electron wavefunction fields, the inertial mass. The progressively attenuated Hawking graviton resonates correspondingly smaller mass gaps at impedance nodes of successively greater wavelengths of figure 11. The graviton near-field extends beyond the limit of the observable universe. We are in the near-field of every rest mass particle in the universe. Gravitational potential as communicated by the near-field Hawking graviton is *almost* scale invariant, almost unshieldable,... just *barely* causal, a delicate mix of the topological and geometric.

Interpretation of figure 11 is dependent upon both initial phase of the Hawking photon and relative phases of E and B flux quanta (handedness) that comprise the photon. Initial and relative phases of the figure were quasi-randomly selected - pure electric field at the Planck length, left-handed. In terms of electric fields, it suggests gravitation was repulsive in the first quarter-wave of the first zeptosecond, peaks in attractive strength on solar system scale, and again becomes repulsive at a time far beyond present age of the universe.

In models where scalar electric charge is visible and pseudoscalar magnetic charge dark, oscillation of energy between translation and rotation gauge fields of figure 11 correlates with nodes of the impedance network. At bifurcated nodes photon energy is shared equally between electric and magnetic fields, between translation and rotation. At converged nodes photon energy is in one or the other. Here one might find modifications of Newtonian dynamics. Bifurcation of node networks suggests things are happening that are extremely difficult for us to observe due to extreme impedance mismatches at those scales. In particular, the 10^{-5} eV Pauli scale brings to mind difficulties of axion searches.

Timescale between Pauli and Einstein nodes is potentially interesting for CERN anti-matter experiments [101]. Model presented here suggests antimatter phase shift is opposite of matter, so anti-gravitation would be repulsive. It appears antimatter falls up on a matter planet. There is a second effect - energy flows from electric to magnetic fields in that interval, so that acceleration is time-dependent for brand-new accelerator antimatter, of interest in the first minute or so whether repulsive or attractive.

Timescale between Planck and Compton nodes is of potential interest to those who favor inflationary models, and at opposite end of the scale sharing of energy between translation and rotation at the $\sim 10^4$ lightyear Hawking scale calls to dark matter enthusiasts.

The observable universe is within the near-field first cycle of Hawking gravitons radiated from Planck lengths of every rest mass particle in the universe. In the impedance model, the consequent phase shift is what we call gravitation.

9 summary and conclusion

9.1 summary

abstract outlined differing roles of geometric and topological impedances in DM and DE.

executive summary touched on string theory roots of the impedance model via the 1960s S-matrix bootstrap, outlined the three model assumptions (geometry, fields, and mass gap), and made several important claims for naturalness.

historical perspective presented circumstances surrounding loss of geometric representation and impedance quantization from particle physics, and additional circumstances that stalled standard model and bootstrap/string theory. Dynamic SUSY and math origin of weak interaction chiral symmetry breaking were introduced. Topological inversion rendered magnetic charge ‘dark’.

Rosetta Stone of QED was extended to Hydrogen atom impedance matching, and two essential points (relative character of both impedance and energy, and differences between scale-dependent and invariant impedances) were clarified. Mass is quantized. Simplicity of calculating quantized impedances and converting to electromagnetic was emphasized.

S-matrices, both geometric and QED, were generated by Clifford products of minimally complete eight-component wavefunctions. S-matrix modes were identified as stable, unstable, or dark by presence or absence of dark wavefunction components in the modes. Model degrees-of-freedom increased from 8D vacuum wavefunction to 10D string theory via introduction of the coupling constant. This raised dimensionality of S-matrix from 8×8 to $8 \times 8 \times 8$, introducing three-component wavefunctions and chiral symmetry breaking.

unstable particle lifetime correlations with impedance network nodes offered BSM understanding of decoherence and decay.

Planck length impedance matching revealed an identity between gravitation and mismatched QED, and bade farewell to Newton's big G. Equivalence of spatial curvature and flat space phase shifts was presented, as were origins of inertial and gravitational mass. Higgs and inflation roles were discussed.

observable universe boundary and the path back to the Planck length revealed two additional enormously mismatched node bifurcations/dark networks. In the mismatched dark networks gravitational energy is shared equally between rotation and translation gauge fields. At the QED photon-matched networks, gravitational energy is found in either translation or rotation gauge fields.

9.2 conclusion

Greatest *strength* of the impedance model is that it is so different. It offers a new perspective, opens a new complementary window within both String Theory and the Standard Model. There is no Lagrangian, no differential equations to solve in search of the wavefunction. It starts with the wavefunction. Equations of motion calculate that which governs amplitude and phase of energy transmission, quantized impedance networks of wavefunction interactions. Greatest *weakness* of the model is it is so different. The hard part is getting physicists to think in impedance networks. The model presented here appears to contain plausible explicit wavefunctions and scattering matrices for both dark energy and dark matter. Time will tell.

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Endorsers/Collaborators

We take the view that endorsers have no responsibility for accomplishment of this white paper's goals, but rather only for being cognizant of the contents. Collaborators have intent to participate in evaluating and/or implementing the paper's goals. We seek endorsers and collaborators both within and beyond Snowmass, and particularly in condensed matter. Does the quantum computer require quantum impedance matching?

Endorsers:

Sastri Osks sastri.osks@hpcu.ac.in Himachal Pradesh Central University

Collaborators:

Sastri Osks sastri.osks@hpcu.ac.in Himachal Pradesh Central University

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