SIMPLEST APPROACH TO QUANTUM GRAVITY HYPOTHESIS

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Abstract. In this short paper I will explore idea of quantazing gravity by using complex space-time and operators acting on wave vector field.

Contents

1. Complex space-time

Space-time in this paper is complex [1] it means that I can write a vector field of that space-time:

$$\psi^{\mu}(z) = \begin{pmatrix} \psi^{0}(z) \\ \psi^{1}(z) \\ \psi^{2}(z) \\ \psi^{3}(z) \end{pmatrix} \tag{1.1}$$

Where z is a complex coordinate that can be expressed:

$$(z) = (x + i\chi) = \left(x^{0} + i\chi^{0}, x^{1} + i\chi^{1}, x^{2} + i\chi^{2}, x^{3} + i\chi^{3}\right) \tag{1.2}$$

This space-time has to obey a field equation [2] [3] [4]:

$$\partial_{\mu} U^{\mu}_{\alpha} \psi^{\alpha}(z)\, \eta^{\mu\kappa} \partial_{\kappa} \left(U^{\dagger}\right)^{\alpha}_{\mu} \left(\psi^{*}(z)\right)_{\alpha} = g_{\mu\kappa} \delta^{\mu\kappa} \psi^{\mu}(z) \left(\psi^{*}(z)\right)_{\mu} \tag{1.3}$$

Where $g_{\mu\kappa}$ is metric tensor and U^{μ}_{α} [5] is matrix acting of wave vector field. When there is measurement done it changes from probability of all possible states to just one position. Probability function needs to be normalized so:

$$\int \psi^{\mu}(z) \left(\psi^{*}(z)\right)_{\mu} d^{3}x = 1 \tag{1.4}$$

In this hypothesis, I use $SU(4)$ matrix. Field equation states that change of complex vector field that can be rotated in complex space-time is equal to that complex vector field times sum of diagonal elements of metric tensor, it restricts all possible metric tensors to only those one where $SU(4)$ matrix [6] [7] acting on complex vector field are solutions. This matrix has properties:

$$U^{\mu}_{\alpha} \left(U^{\dagger}\right)^{\alpha}_{\mu} = \mathbf{I} \tag{1.5}$$

$$\det U^{\mu}_{\alpha} = 1 \tag{1.6}$$

Base space-time is Minkowski space-time from field equation comes solutions that modify it. How fast field changes in complex plane or more precise how it rotates in that plane gives metric tensor components.

2. Quantum clock and ruler- space-time from field equation

Complex vector field and all possible rotations of that complex vector field in four dimensions are key components of field equation. From it follows that how field rotates in given direction or more precise how it changes in complex space-time where only action possible is rotation of that field is equal to metric tensor components. But how to define space-time out of this information? Quantum clock and ruler works close to just a normal space-time interval but interpretation is not the same. Let's say I have two events with two possible metric tensor values of time-time component g_{00}, g'_{00}, and primed one has bigger value so wave vector complex field changes faster in time direction. It means that first object is at time $g_{00}dt^2$ and second one $g'_{00}dt^2$ for simplest case of wave vector complex field, so if first object wants to get to event second object is currently at there need to pass time equal to that second object time so $g'_{00}dt^2$ but for first object time that passes is equal to $g'_{00}dt^2$ so i need to divide first object time by second one to get what is ratio of passage of time:

$$dt' = \sqrt{\frac{g_{00}}{g'_{00}}}dt \tag{2.1}$$

It means that this clock does not work like in relativity, metric component of time-time does not say how much time did pass for an observer but how many units of time is needed to go to event other observer is at. So clock is defined this way: **Coordinate time is number of ticks of a clock of any observer, where metric time-time component is equal to scale of that clock relative to flat space-time. Where scale means how many ticks of a clock it takes for a clock of flat space-time to make one tick of clock of any observer.** Now I can move to ruler so space distance, it is same thing sa with time-but now I have how much distance in space i need to travel to get to a point where observer is from perspective of flat space. So in general I can define a property of how much distance i need to travel to get an event any observer is from perspective of flat space-time:

$$d\Psi^2 = g_{\mu\kappa}\delta^{\mu\kappa}d\psi^{\mu}(z)\left(d\psi^*(z)\right)_{\mu} \tag{2.2}$$

Where what is left is probability but first I used $d\Psi^2$ notation not ds^2 notation to not confuse it with space-time interval from relativity. This interval says only how much distance there is need to travel for given wave complex field of flat space-time to get to event this wave field is at where for each part of wave field I have probability of that happening.

3. Casual structure of events and gravity

Events are connected by casual structure, in this hypothesis there is main change from relativity. If I write interval as $d\Psi^2$ defined in previous section, for trajectory of a massless particle it always gives zero. That can be written:

$$g_{\mu\kappa}\delta^{\mu\kappa}d\psi^{\mu}\left(z\right)\left(d\psi^{*}\left(z\right)\right)_{\mu}=0 \tag{3.1}$$

That means that distance needed to travel to reach an event massless particle is at is always zero for all observers. From it follows that massless particles don't move at all at space-time. What doest propagate is change of energy of those fields. Now real question is what is gravity? Objects in gravity field follow shortest path, geodesic- but here geodesic is not defined as in relativity. Shortest path is defined as path with lowest coordinate time and length, where I go from one point of space-time to another (x_A) to (x_B):

$$P\left(x\right)=\sqrt{\min\left(\frac{1}{g_{\mu\kappa}\eta^{\mu\kappa}\psi^{\mu}\left(z\right)\left(\psi^{*}\left(z\right)\right)_{\mu}}\int_{(x_A)}^{(x_B)}g_{\mu\kappa}\eta^{\mu\kappa}d\psi^{\mu}\left(z\right)\left(d\psi^{*}\left(z\right)\right)_{\mu}\right)} \tag{3.2}$$

Probability of travel along that path is equal to this path times integral along that path, that gives scalar wave field that is equal to probability times path:

$$\psi\left(x\right)=P\left(x\right)\int_{(x_A)}^{(x_B)}\psi^{\mu}\left(z\right)\left(\psi^{*}\left(z\right)\right)_{\mu}d^3x \tag{3.3}$$

When there is measurement done, all possible paths and all possible probabilities along that path reduce to one path and probability being equal to one:

$$P\left(x\right)\int_{(x_A)}^{(x_B)}\psi^{\mu}\left(z\right)\left(\psi^{*}\left(z\right)\right)_{\mu}d^3x\rightarrow P\left(x\right) \tag{3.4}$$

Where this space-time turns from complex space-time just normal space-time. Before measurement wave scalar field gives for each point of space-time a probability times path of object that it takes, when there is measurement done it turn just to one path object travels. That path is always shortest path. So gravity in this hypothesis is scaling of space-time, space-time scales with metric tensor and shortest path is always that path that takes least coordinate time and length to travel. All object travel the shortest path but their position in space is defined with probability at being at some point.

4. Energy and Lorentz Factor correction

Energy can't be more than Planck energy this is with conflict with relativity. That's why there is need for correction of Lorentz factor. For Planck energy object has to move with speed of light. I can write this correction:

$$\frac{1}{\sqrt{\frac{E_0^2}{E^2}-\frac{E_0^2}{E_P^2}}}=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{4.1}$$

Where total energy is equal to $E^2 = m^2c^4 + p^2c^2$ and zero energy is just mass term $E_0^2 = m^2c^4$. Now I can move to field equation. Energy of field equation is defined by energy tensor [8]. First I will write this equation then explain its meaning:

$$\hbar^2 \partial_\mu U^\mu_\alpha \psi^\alpha\left(z\right)\eta^{\mu\kappa}\partial_\kappa \left(U^\dagger\right)^\alpha_\mu \left(\psi^*\left(z\right)\right)_\alpha = T_{\mu\kappa}\eta^{\mu\kappa}\psi^\mu\left(z\right)\left(\psi^*\left(z\right)\right)_\mu \tag{4.2}$$

Space-time is a complex field, it's rotation generates energy. This equation states that energy of gravity field is equal to rotation of complex space-time. Gravity field is generated by energy tensor, that is second order tensor so it means that is has spin two [9]. In field equation I use only diagonal elements of that tensor like with metric tensor. Energy tensor general definition is how many radians of rotation in complex plane in direction $\mu\kappa$ per unit of time or per unit of space depending on direction of rotation. So again it's not energy momentum tensor from general relativity. Going back to Lorentz factor and energy, it can be rewritten:

$$\sqrt{\frac{E_0^2}{E^2}-\frac{E_0^2}{E_P^2}}=\sqrt{1-\frac{v^2}{c^2}} \tag{4.3}$$

From it there can be figured out general dependence of energy on speed [10]:

$$E=\pm\frac{cE_PE_0}{\sqrt{c^2\left(E_P^2+E_0^2\right)-E_P^2v^2}} \tag{4.4}$$

Energy can't be more than Planck energy same goes for energy tensor it can have value more than one Planck energy or one radian of rotation per Planck time or Planck length.

5. Speed of light invariance and interval

In relativity speed of light is kept constant by preforming Lorentz Transformations [11]. I will write first interval for flat space-time:

$$d\Psi^2 = \eta_{\mu\kappa}\delta^{\mu\kappa} d\psi^{\mu}(z)\left(d\psi^{*}(z)\right)_{\mu} \tag{5.1}$$

$$d\Psi^2 = \rho_{tt}c^2dt^2 - \rho_{xx}dx^2 - \rho_{yy}dy^2 - \rho_{zz}dz^2 \tag{5.2}$$

Where ρ represents probability of that space-time distance. Interval represents distance object needs to travel to get to event. It means that time that passes for observer is shorten by this amount- this interval says how much time separation there is between events. So when observer moves with speed of light there is no time between events all events happen at same moment. But what about constant speed of light? Speed of light is constant for all observers- it's well know experimental fact. Here quantum effects come into play, normally if time slows down only one photon that is emitted from same place and moves with same direction conserve speed of light but if it's a quantum object it moves along all possible trajectories of space- or saying more precise in all space directions. So it's wave function conserve speed of light as being constant- distance photon travels is equal to square root of interval and time that observer measures between events is same so if I want to calculate speed of light it's always constant from point of view of any observer. When measurement is done object can be only on one part of cone but still it creates new cone- so speed of light is always conserved. Now I can go back to space-time interval for flat space-time, all i written before consists of two facts:

1. Space-time interval is a quantum object , all trajectories that start from one point and move in any possible direction in space so they conserve that interval are possible and object moves along all of them.

2. Speed of light has to be constant for all observers independent of their velocity- it leads to that time between two events need to get smaller with faster speed so speed of light is kept constant for all observers. So interval of space-time is just how much distance in time separates two events that are only in space.

So for photons or any massless particle, that interval is zero. And those particles move always with constant speed from perspective of any observer:

$$0 = \rho_{tt}c^2dt^2 - \rho_{xx}dx^2 - \rho_{yy}dy^2 - \rho_{zz}dz^2 \tag{5.3}$$

References

[1] https://mathworld.wolfram.com/ComplexVectorSpace.html

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[5] https://mathworld.wolfram.com/SpecialUnitaryGroup.html

[6] https://mathworld.wolfram.com/UnitaryMatrix.html

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[11] https://mathworld.wolfram.com/LorentzTransformation.html