Unusual Boards and Meeples

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Abstract

We introduce boards others than the usual chessboard. Further we define meeples which can move in other ways than the usual chess meeples. We ask whether these meeples can reach every field, like a knight can reach every field on the chessboard.

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1 Introduction

We ask whether any figure on a board can reach all fields by valid moves. We assume that the reader is familiar with chess.

Definition 1. We use the term *board* as a synonym for a *polyomino*. For the definition of a polyomino see [1].

Definition 2. A *meeple* or a *figure* moves on the board by an action. An action is: 'Sway the meeple k squares horizontally and then l squares vertically', where k and l are natural numbers or zero. For a meeple there may be more than one possible action. For a move of a meeple we choose one of the admissible actions. The admissible actions of a meeple are determined by a rule.

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Definition 3. We say that a *way* of a meeple is a sequence of moves, such that every field is visted by the meeple a single time. With the final move it returns to the starting field. We call the set of all ways *Ways*.
We say that a meeple is *reaching* if and only if starting on an arbitrary starting field there exists a way for the meeple.

Proposition 1. A meeple is reaching if and only if there exists one way.

Proof. $\Longrightarrow$: If a meeple is reaching, the other claim is trivial.
$\Longleftarrow$: We call the starting field of the way xxx. We take another field yyy . There is one way of the meeple. Sometime it passes *yyy*. We take *yyy* as the new starting field. The meeple can go the way in the same direction as before. It passes *xxx*, and after some moves it comes back to *yyy*.
The proposition is proven. □

Proposition 2. On the usual chessboard the usual meeples king, queen, rook and knight are reaching. The bishop and the pawn are not reaching.

Proof. For bishop and pawn the claims are trivial. If the king or the queen or a rook is on a2 it goes to b2, c2, d2, e2, f2, g2, g3, f3, e3, d3, c3, b3, a3, a4, b4, c4, d4, e4, f4, g4, g5, f5, e5, d5, c5, b5, a5, a6, b6, c6, d6, e6, f6, g6, g7, f7, e7, d7, c7, b7, a7, a8, b8, c8, d8 e8, f8, g8, h8, h7, h6, h5, h4, h3, h2, h1, g1, f1, e1, d1, c1, b1, a1, and with one final move it returns to a2. By Proposition 1 the king and the queen and the rook are reaching.
If a knight is on a2, it moves to b4, d5, e7, g8, h6, g4, h2, f3, g1, h3, f2, h1, g3, h5, f6, e4, g5, h7, f8, g6, h8, f7, e5, d7, b8, a6, c5, a4, b6, a8, c7, e8, g7, e6, d4, c6, d8, b7, a5, b3, a1, c2, e3, f1, d2, c4, b2, d1, c3, b1, a3, b5, a7, c8, d6, f5, h4, g2, e1, d3, f4, e2 and c1.
Alternatively from a2 the knight goes to b4, a6, b8, c6, a7, c8, b6, a8, c7, e6, d8, b7, a5, c4, b2, d1, e3, g4, e5, d7, f8, h7, g5, e4, d6, b5, d4, f5, e7, d5, f4, e2, c3, a4, c5, d3, e1, g2, h4, g6, h8, f7, h6, g8, f6, e8, g7, h5, g3, h1, f2, h3, g1, f3, h2, f1, d2, b1, a3, c2, a1, b3 and c1. By Proposition 1 the knight is reaching. □

On *Ways* we define an equivalence. We say that two ways way_1 and way_2 are equivalent if and only if both ways have the same order. They may have different starting fields. The equivalent classes of *Ways* we call $[Ways]_{\cong}$, i.e.we get $[way_1]_{\cong} = [way_2]_{\cong}$.

We define for each meeple the number of different ways.

Definition 4. *Lat M be a meeple on any board. We define the natural number $W(M)$ as the number of eqivalent classes in $[Ways]_{\cong}$.*

Proposition 3. *A meeple M is reaching if and only if $W(M)$ is positive*

Proposition 4. *For a bishop or a pawn $W(M)$ is 0, while for a knight $W(M)$ is at least 2.*

Questions 1. *Let M be a meeple on any board. We ask whether M has a way. We ask for the value of $W(M)$.*

Remark 1. *The entire concept can easily be generalized into higher dimensions.*

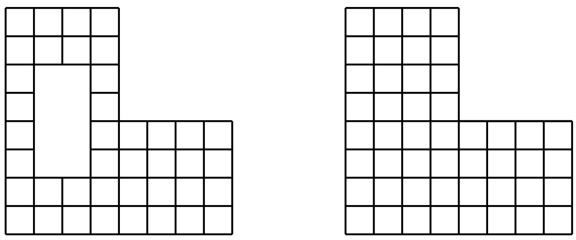

Figure 1:

On the left hand side

we see two boards.

They have 40 and 48 fields, respectively.

Please see the picture below. We show two boards. They have 60 and 64 fields, respectively. We call the right a *chessboard*.

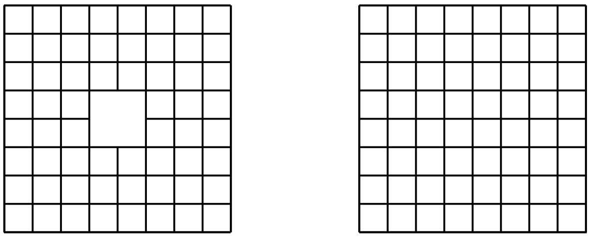

Figure 2:

On the left hand side

we show two boards.

They have 60 and 64 fields, respectively.

There is another possibility to generate boards. Instead the usual squares we can use other r-gons as fields.
Assume that r is an even natural number larger than 4. We take r-gons as fields. Even there is no complete covering of the plane with even numbers except with 4-gons and 6-gons, we can form boards with them. The fact that r is an even number ensures that there is an unique direction, while the meeple comes from the other side.
We will not continue this concept.

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References

[1] Anthony J. Guttmann: *Polygons, Polyominoes and Polycubes*, Springer 2009