Visible matter asymmetric universe straightforwardly

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Abstract

We explore a minimal set of elements required in a phenomenological supersymmetric scenario describing the early phases of the evolution of the universe into the present matter-antimatter asymmetric form. The archeon level structure is key for the main result of this note, namely the mechanism which creates from C symmetric archeons the asymmetric standard model visible matter straightforwardly.

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Contents

1 Introduction

We explore and combine theoretical ingredients to build a simple phenomenological unified scenario of (ontic) particles that would be capable of describing the evolution of the universe during the earliest moments, and which leads to the present visible matter-antimatter asymmetric universe. The mechanism we propose for the asymmetry necessitates quark and lepton constituents, archeons[1], which above some energy scale Λ_{cr} adhere to 1+2 dimensional Chern-Simons (CS) equation.. This scale turns out to be close to the usual grand unified theory (GUT) scale, about 10^{16} GeV. Below Λ_{cr} our scenario becomes the standard model, or the minimal supersymmetric standard model (MSSM) depending on whether the MSSM supersymmetric partners are experimentally discovered or not.

We have for visible matter two archeons, and their antiparticles, obeying unbroken global supersymmetry (SUSY). The charged archeons have only gravitational and electromagnetic interactions. The neutral archeons carry color. The archeon baryon (B) and lepton (L) numbers are zero. Weak interactions works as in the standard model but they are not discussed in this introductory note. The dark sector is symmetric and may provide a new source for detecting gravitational waves.

In our proposal, the spectrum of fundamental fields gets smaller towards Planck scale. This has been typical of physics when going from larger to shorter distances: hundreds of nuclear states can be explained by two nucleons, and hundreds of hadronic states by six quarks.

The main time period considered is around the era of inflation when archeons form the standard model particles as bound states. The interaction binding the archeons has been missing from our model. In particular how to make an

[1] Synonym for preon and superon

electron from three each other repelling archeons. This requires strong screening of the Coulomb potential. Such a screening has been proposed in the literature [1].

We only briefly mention supersymmetry breaking, reheating and later phases ending to thermalization of matter, and quarks forming nucleons and nucleons making the nuclei of the three lightest elements. Supergravity scalar potential has been found for inflation by other people. Bouncing universe is not excluded.

The article is organized as follows. In section 2 we propose our model for the asymmetry for visible matter. Though we prefer unbroken SUSY we discuss briefly the broken SUSY case. Conclusions are given in section 4. Three appendices are provided for more detailed background.

2 Matter-antimatter asymmetry

The first observation for asymmetric creation of standard model matter in the early universe is that the all groups of twelve C symmetric archeons, each set consisting of four m^+, four m^- and four m^0, may form hydrogen H only, antihydrogen $\bar{\mathrm{H}}$ only only or any combination of H and $\bar{\mathrm{H}}$ atoms [2, 3].

This is achieved by organizing the archeons appropriately (mod 3) using table 1:

$$\begin{aligned} p + e^- &:= u^{2/3} + u^{2/3} + d^{-1/3} + e^- \\ &= u^{-2/3} + u^{-2/3} + d^{+1/3} + e^+ \\ &=: \bar{p} + e^+ \end{aligned} \tag{2.1}$$

where the superscript refers to the charge of the particle.

In this scenario neither baryon number B nor lepton number L is fundamental but their difference is, which can be seen from (2.1)

$$B - L = 0 \tag{2.2}$$

If (2.2) is elevated as a law of nature the proton decay $p \to e^+\pi^0$ is forbidden, unlike in the MSSM. One may consider B−L as a continuous gauge symmetry $U(1)_{B-L}$ [4, 5] above the energy scale Λ_{cr}. We call it $U(1)_{archeon}$ because archeons are available above Λ_{cr}, not baryons or leptons.

Independent of the details of the inflationary model or bounce, towards the end of rapid expansion a phase transition takes place from archeon gas directly after into standard model particles. The state $|\Psi\rangle$ of the universe is (leaving radiation aside)

$$|\Psi\rangle = c_1(t)|\Psi\rangle_{\text{matter}} + c_2(t)|\Psi\rangle_{\text{antimatter}} \tag{2.3}$$

where at time t_i archeon and antiarcheon numbers are equal: $c_1(t_i) = c_1(t_i) = \frac{1}{\sqrt{2}}$. However, nature has chosen the first line of (2.1), or $c_1 = 1$ and $c_2 = 0$ within current experimental accuracy.

Within an electron the archeons repel each other, so how can this dramatic change for $|\psi\rangle$ and $c_i(t)$ happen? In nutshell, it can be done as follows [1]: introduce a massive photon and a complex scalar with the action (C.1) and (C.2). The MCS model provides a new mass term to the topological gauge field, the Proca term $m^2 A_\mu A_\mu$. The spontaneous symmetry breaking is caused by the sixth power self-interaction potential (C.2). Choose the parameters such that the Yukawa attractive force between like charge archeons is the strongest force.

To obey condition (2.2) on baryon-lepton balance, for one electron with three archeons to obtain a charge balancing proton nine archeons have to be created. A neutrino requires a neutron to be created. The m^0 carries color helping neutrino formation with $\alpha_{QCD} \sim \alpha_{QED}$. Note that archeon-antiarcheon mesons also have a chance to form though α_{QCD} is close to zero under the conditions considered.

The electron is more tightly Yukawa bound than the other SM particles. Therefore it is assumed to form first. Once the neutrinos and quarks have formed, at the same time spacetime has been created from 'nothingness'. We make now a novel proposal that the spacetime has the 'seed' or immutable 'genome' property meaning that *the after the first charged lepton* e^- *all the next charged leptons carry the charge of this first lepton.* Consequently, in this framework the universe is exactly matter-antimatter asymmetric from the very beginning.

The second and third generation SM particles, including top quark mass, are treated elsewhere [6].

Consider now some consequences for inflation. The inflaton decay takes place after the inflaton has reached the minimum of its potential and it couples to the quarks and leptons while vibrating in its ground state causing reheating. Visible matter fields loose their original quantum fluctuations and their distribution is remodeled by reheating towards lesser uniform distribution in space. All dark matter is smoothly distributed in the universe, apart from quantum fluctuations of the corresponding fields, because they were unaffected by reheating. Quantum fluctuations in the dark fields during inflation may lead to formation of primordial black holes in the universe. These density variations of dark matter (DM) provide attractive gravitational potential regions for visible matter to accumulate in the various formations we observe.

Fermionic dark matter has in this scenario no mechanism to become 'baryon' asymmetric like visible matter. Therefore we expect that part of dark matter has annihilated into bosonic dark matter. Secondly, there should exist both dark matter and anti-dark matter clumps in the universe. Collisions of anti-dark matter and dark matter celestial bodies would give us a new source for wide spectrum gravitational wave production (the lunar mass alone is $\sim 10^{49}$ GeV). High dark matter density.

We expect roughly twice as much visible matter from the m^+ and m^0 than fermionic dark matter from the n. The fraction of n of all matter today is about 2.5%. Therefore there should be about ten times more bosonic dark matter and e.g. primordial black holes than fermionic dark matter.

3 Broken supersymmetry

There are several ways supersymmetry may get broken - though we prefer it unbroken. They are described extensively in a number of articles, reviews and textbooks, e.g. [7, 8, 9]. The first and to us the relevant method is the gravitationally mediated scenario. Supersymmetry is unbroken in the archeon sector. If the MSSM superpartners are experimentally established SUSY breaking can be mediated by gravitational interaction to the visible MSSM by soft term contributions, which means that the Lagrangian has two terms: symmetric and symmetry breaking

$$\mathcal{L} = \mathcal{L}_{susy} + \mathcal{L}_{soft} \tag{3.1}$$

where $\mathcal{L}_{soft}$ violates supersymmetry but only by mass terms and coupling constants having positive mass dimension.

If needed, the MSSM superpartners can be thought of in terms of archeons by adding an m^0 to the three m composites. This adds color to the superpartners making it possibly heavier. Color neutrality in turn requires one or two other such particles.

The brief description is that if supersymmetry is broken in the archeon sector by a vev $\langle F\rangle$ then the soft terms in the visible sector are expected to be approximately $M_{soft} \sim \langle F\rangle/M_{\rm Pl}$. For $M_{soft} \sim 200$ GeV one would estimate that the scale associated with supersymmetry breaking in the archeon sector is about $\sqrt{\langle F\rangle} \sim 10^{10}$ or 10^{11} GeV, which must be below Λ_{cr} for consistency. This way the MSSM soft terms arise indirectly or radiatively, instead of tree-level renormalizable couplings to the supersymmetry breaking parameters. The gravitino mass is of the order of the masses of the MSSM sparticles. The gravitino in turn mediates the symmetry breaking with gravitational coupling to the MSSM. A gravitino mass of the order of TeV gives a lifetime 10^5 s, long enough not to disturb nucleosynthesis by decay products.

4 Conclusions

By defining the fundamental fields as archeons in (A.1) and (A.2) in section A it has been possible to develop a scenario for asymmetric visible matter as well as for the symmetric dark sector. The latter includes both fermionic and bosonic fields. The bosonic sector of (A.1) contains axion-like particles, a string theory concept. They are obvious candidates for bosonic dark matter are axions when $M_a \gtrapprox 10^{-25}$ eV and dark energy when $M_a \lessapprox 10^{-32}$ eV.

The matter-antimatter asymmetry is, according to our proposal, created from C symmetric, baryon and lepton neutral archeons in a direct way. Below the transition energy Λ_{cr} fractional charge three archeon composites form quarks while charge zero and one states are leptons. These composite states are to a good approximation point-like, radius between 10^{-18} cm and the electron Cartan radius. Baryons and electrons are produced towards the end of inflation in equal amounts (B−L=0) by the matter production process described in section 2. Dark matter is insensitive to reheating. It provides in the universe a background gravitational potential for visible matter to form the astronomical objects we observe. Anti-dark matter celestial body annihilation phenomena would provide a new source for observing gravitational waves.

In nutshell, starting from the Wess-Zumino supergravity Lagrangians with three fermions (m^+, m^0, n), the mSUGRA potential [10] driving inflation, and the Chern-Simons model for archeon binding we have constructed a unified picture of quarks, leptons and the dark sector. The main point, the creation of the matter-antimatter asymmetric universe has been made possible. The dark sector, instead, is predicted to be C symmetric.

In this analysis the role of superstring theory remains tenuous, see however [20]. This is not surprising since we have discussed a non-GUT 1+2 and 1+3 low energy models. One would have to start from 10D or 11D.[2] Torsion is a high energy density spacetime property in general relativity and string theory. We conjecture that the asymmetry caused by torsion to fermions is valid in general in higher dimensional theories. For decisive experimental tests one may have to wait for the next generation neutrino and gravitational wave detection experiments that are able to measure the energy range above EeV.

To prove or disprove the scenario presented above, extensive simulations are be done, more detailed Lagrangians be written and calculated. Phenomenological work is to be carried out with current data for supersymmetry breaking and particle masses while waiting for future precision experiments to be carried out in the years to come. A necessary step is to find the theory of gluing the fermionic archeons back into standard model particles.

Acknowledgemet

I thank Robert Brandenberger for advice about inflationary processes. All errors are naturally on my responsibility.

[2]It is claimed in [?] that regions of 4-dimensional spacetime, in which extra compact dimensions are sufficiently rich to be observed, must be trapped behind black holes.

A Particle archeon correspondence

We briefly recap the archeon scenario of [2, 3], which turned out to have close resemblance to the simplest N=1 globally supersymmetric 4D model, namely the free, massless Wess-Zumino model [11, 12] with the kinetic Lagrangian including three neutral fields m, s, and p with $J^P = \frac{1}{2}^+, 0^+$, and 0^-, respectively

$$\mathcal{L}_{\rm WZ} = -\frac{1}{2}\bar{m}\partial\!\!\!/ m - \frac{1}{2}(\partial s)^2 - \frac{1}{2}(\partial p)^2 \tag{A.1}$$

where m is a Majorana spinor, s and p are real fields (metric is mostly plus).

We assume that the pseudoscalar p is the axion [?], and denote it below as a. It has a fermionic superparther, the axino n, and a bosonic superpartner, the saxion s^0.

In order to have visible matter we assume the following charged chiral field Lagrangian

$$\mathcal{L}_{-} = -\frac{1}{2}m^{-}\partial\!\!\!/ m^{-} - \frac{1}{2}(\partial s_i^{-})^2, \quad i = 1, 2 \tag{A.2}$$

The table below gives the archeon content of SM matter and a proposal for dark matter.

SM Matter	archeon state
ν_e	$m_R^0 m_G^0 m_B^0$
u_R	$m^+ m^+ m_R^0$
u_G	$m^+ m^+ m_G^0$
u_B	$m^+ m^+ m_B^0$
e^-	$m_R^- m_G^- m_B^-$
d_R	$m^- m_G^0 m_B^0$
d_G	$m^- m_B^0 m_R^0$
d_B	$m^- m_R^0 m_G^0$
Dark Matter	archeon state
boson (or BC)	axion(s), s^0
e'	axino n
meson, baryon o	$n\bar{n}, 3n$
nuclei (atoms with γ')	multi n
celestial bodies	any dark stuff
black hole	any archeon

Table 1: Visible and Dark Matter with corresponding particles. m^0 is color triplet, $m^{\pm}$ are color singlets. BC stands for Bose condensate. e' and γ' refer to dark electron and dark photon, respectively. Identical archeon state antisymmetrization not shown.

B Why Maxwell-Chern-Simons QED_3

Motivation for 1+2 dimensions can be found here [13] and [14], see also [15].

In $1 + 2$ dimensions the Coulomb interaction becomes a potential $V(r) \sim \ln r$, which does not provide bound states. Between two electrons the interaction is, of course, repulsive while we desire an attractive force. A short range screening effect is needed. A Chern-Simons (CS) term [3] can be introduced to give topological mass to the photon The Maxwell-Chern-Simons (MCS) model became a theoretical framework for providing an attractive but not confining electron-electron interaction. More exactly, the e^-e^- potential becomes attractive when the photon topological mass θ exceeds the electron mass m_e. In the perturbative regime $(1/k \gg 1)$, the authors of [10] found an attractive potential for fermions $(V_{\psi\psi} < 0)$, and also for scalar bosons $(V_{\phi\phi} < 0)$, in the non-relativistic approximation. The presence of the non-minimal coupling seems to be the key factor for the attainment of the attractive potential between charges with the same sign. In this case, the potential remains negative even in the limit of a small topological mass $(\theta \ll m_e)$, under a suitable choice of parameters. The non-renormalizability of this model due to the non-minimal coupling, however, implies a restriction to the validity of their results only at tree-level calculations. We will see that the introduction of a spontaneous symmetry breaking (SSB), the Higgs mechanism will bring out a negative contribution to the scattering potential that will allow a global attractive potential despite the condition $\vartheta > m_e$.

Once the spontaneous breaking of the local U(1)-symmetry has taken place, a neutral massive Higgs scalar remains and the gauge field becomes a Maxwell-Chern-Simons-Proca vector field. The physical mass of such a photon, that may assume two different values, will be written in terms of these two mass parameters, as explicitly given by the expressions read off from the poles of the gauge field propagator.

C Maxwell-Chern-Simons QED_3 action

The action for a QED_3 model [1] built up by two polarization fermionic fields (ψ_+, ψ_-), a gauge (A_μ) and a complex scalar field (φ), mutually coupled, and endowed with spontaneous breaking of a local U(1)-symmetry [16], [17], reads as

$$\begin{aligned} S_{QED-MCS} = \int d^3x \{ & -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\overline{\psi}_+\gamma^\mu D_\mu\psi_+ + i\overline{\psi}_-\gamma^\mu D_\mu\psi_- \\ & +\frac{1}{2}\theta\epsilon^{\mu v\alpha}A_\mu\partial_v A_\alpha - m_e(\overline{\psi}_+\psi_+ - \overline{\psi}_-\psi_-) \\ & -y(\overline{\psi}_+\psi_+ - \overline{\psi}_-\psi_-)\varphi^*\varphi + D^\mu\varphi^* D_\mu\varphi - V(\varphi^*\varphi)\}, \end{aligned} \tag{C.1}$$

where $V(\varphi^*\varphi)$ represents the sixth-power self-interaction potential,

$$V(\varphi^*\varphi) = \mu^2\varphi^*\varphi + \frac{\zeta}{2}(\varphi^*\varphi)^2 + \frac{\lambda}{3}(\varphi^*\varphi)^3 \tag{C.2}$$

which is responsible for the SSB. It is the most general one renormalizable in $1+2$ dimensions [18]. The mass dimensions of the parameters μ, ζ, λ and y are respectively: 1,1,0 and 0. For the present purpose, we are interested only on stable vacuum, restriction satisfied by imposing some conditions on the potential parameters: $\lambda > 0, \zeta < 0$ and $\mu^2 \leq \frac{3\zeta^2}{16\lambda}$. The covariant derivatives are defined as: $D_\mu\psi_\pm = (\partial_\mu + ie_3A_\mu)\psi_\pm$ and $D_\mu\varphi = (\partial_\mu + ie_3A_\mu)\varphi$, where e_3 is the coupling constant of the $U(1)$-local gauge symmetry, here with dimension of $(\text{mass})^{1/2}$, particularity that will be more explored in the numerical analysis section. In $(1+2)\,-$dimensions, a fermionic field has its spin polarization fixed up by the mass sign [19]; however, in the action (C.1), it is manifest the presence of two spinor fields of opposite polarization. In this sense, it is necessary to stress that we have two positive-energy spinors (two spinor families), both solutions of the Dirac equation, each one with one polarization state according to the sign of the mass parameter, instead of the same spinor with two possibilities of spin-polarization.

Considering $\langle\varphi\rangle = v$, the vacuum expectation value for the scalar field product $\varphi^*\varphi$ is given by:

$$\langle\varphi^*\varphi\rangle = v^2 = -\zeta/\left(2\lambda\right) + \left[\left(\zeta/\left(2\lambda\right)\right)^2 - \mu^2/\lambda\right]^{1/2},$$

while the condition for minimum reads as: $\mu^2 + \frac{\zeta}{2}v^2 + \lambda v^4 = 0$. After the spontaneous symmetry breaking, the scalar complex field can be parametrized by $\varphi = v + H + i\theta$, where H represents the Higgs scalar field and θ the would-be Goldstone boson; the SSB will be manifest when this parametrization is replaced in the action (C.1). Thereafter, in order to preserve the manifest renormalizability of the model, one adopts the ´‘t Hooft gauge by adding the fixing gauge term $\left(S^{gt}_{R_\xi} = \int d^3x[-\frac{1}{2\xi}(\partial^\mu A_\mu - \sqrt{2}\xi M_A\theta)^2]\right)$ to the broken action; finally, by retaining only the bilinear and the Yukawa interaction terms, one has,

$$\begin{aligned} S^{\text{SSB}}_{\text{QED}} = \int d^3x\Big\{ & -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}M_A^2A^\mu A_\mu \\ & - \frac{1}{2\xi}(\partial^\mu A_\mu)^2 + \overline{\psi}_+(i\partial - m_{eff})\psi_+ \\ & + \overline{\psi}_-(i\partial + m_{eff})\psi_- + \theta\epsilon^{\mu v\alpha}A_\mu\partial_v A_\alpha \\ & + \partial^\mu H\partial_\mu H - M_H^2H^2 + \partial^\mu\theta\partial_\mu\theta - M_\theta^2\theta^2 \\ & - 2yv(\overline{\psi}_+\psi_+ - \overline{\psi}_-\psi_-)H - e_3\left(\overline{\psi}_+A\psi_+ + \overline{\psi}_-A\psi_-\right)\Big\} \end{aligned} \tag{C.3}$$

whose mass parameters,

$$M_A^2 = 2v^2e_3^2, \quad m_{eff} = m_e + yv^2, \quad M_H^2 = 2v^2(\zeta + 2\lambda v^2), \quad M_\theta^2 = \xi M_A^2 \tag{C.4}$$

are entirely or partially dependent on the SSB mechanism. The Proca mass, M_A^2, represents the mass acquired by the photon through the Higgs mechanism, while the Higgs mass, M_H^2, is the one associated with the real scalar field. The Higgs mechanism also corrects the mass of the electron, resulting in an effective electronic mass, m_{eff}. On the other hand, the would-be Goldstone mode, endowed with mass (M_θ^2), does not represent a physical excitation, since ξ is just a unphysical (dimensionless) gauge-fixing parameter. At this moment, it is instructive to point out the presence of two photon mass-terms in eq. (C.3): the Proca and the topological one. The physical mass of the gauge field will emerge as a function of two mass parameters.the next Section.

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