Bell's theorem refuted as EPR and locality prevail

Gordon Stewart Watson[1]

Abstract: In a technical report under the auspices of The Nobel Committee for Physics, dated 4 October 2022, we find these claims: (i) Bell's first inequality was a spectacular theoretical discovery; (ii) Bell showed mathematically that no hidden variable theory would be able to reproduce all the results of quantum mechanics; (iii) Bell provided a proof ... that all attempts to construct a local realist model of quantum phenomena are doomed to fail. Against such claims, and with a special focus on EPR's criteria, this personal note shows: (i) that all such claims are flawed; (ii) that Bell's theorem and Bell's inequality fall to straightforward considerations; (iii) that, consequently, for their part in proving that EPR were right: we are indebted to those who develop the sources; hopefully en route to wholistic mechanics — a commonsense quantum mechanics — as we celebrate the birth of Olivier Costa de Beauregard, 111 years ago.

Keywords: beable, Bell's inequality, Bell's theorem, completeness, determinism, EPR, free choice, hidden variable, locality, reality, spin

[1] Correspondence welcomed: EPRB@me.com Ref: 2022Q.v1. 20221106

1. Introduction

1.1. On 2 June 1989 — reading about Bell's theorem,[2] Aspect's experiment,[3] and quantum mechanics (QM), in Mermin (1988)[4] — I saw that Bell's theorem was flawed.

1.2. So, with Mermin (p.52) open to surprises, and based on a thought experiment like that in Fig.1, I phoned the next day to offer this hypothesis: (i) Without mystery, correlated tests on correlated things produce correlated results; so Bell's theorem is flawed. (ii) For, typically, and also without mystery, similar tests on similar things produce similar results.

1.3. My hopes for the call were high. (i) Mermin would run with my thought experiment: for, to bring determinism to the fore, I could repeatedly restore any particle-pair to its pristine (post-source, pre-test) condition, and test it under new conditions. (ii) Alternatively, I'd obtain Bell's contact details. (iii) Alas, neither happened. (iv) Instead: ...

1.4. The Dean of Science at a nearby university assured me that John Bell had passed. He had not, and I missed a great opportunity: see §15. (Bell passed 16 months later.)

1.5. Years later, I shared my ideas with Reinhold Bertlmann. He kindly replied: "... you make a very thorough analysis of EPR-Bell. As you remain a 'realist' and refer to Bell's beables[5] when you resolve Bell's dilemma, Bell might have liked your approach, who knows," RB (2017), pers. comm. 26 June. "Who knows?" Let's see.

2. Motivation and boundary conditions

2.1. Based on Nobel (2022:4) — a report from The Nobel Committee for Physics — "(i) Bell formulated the first Bell inequality, which was a spectacular theoretical discovery. (ii) Using a special version of the Bohmian-EPR thought experiment, Bell showed mathematically that no hidden variable theory would be able to reproduce all the results of quantum mechanics. (iii) With this mathematical illustration, Bell provided a proof of the assertions made by Bohr and Schrödinger: Bell thus showed that all attempts to construct a local realist model

[2] To later find: Bell's theorem "is one of the most profound discoveries of science," Stapp (1975:271).
[3] Re Aspect's experiment, his later Aspect (2000) is related, and available online; see References.
[4] Re Mermin (1988), Mermin (1985c) is related, and available online.
[5] Note: We take it that a Bell (1975a) beable is an element of physical reality, and vice versa. After Bell (1984c:2): "their existence does not depend on 'observation'. Indeed ... observers must be made of beables."

of quantum phenomena are doomed to fail."

2.2. Also: "(i) Bell used the words *local* and *realist* here in a technical sense. (ii) *Local* indicates the impossibility of instantaneous signalling, limited by the finite speed of light. (iii) *Realist* means that the outcome of any test is fully determined by properties of the beables — often called hidden variables — that exist independently of any actual or potential ~~measurement~~[6] test/interaction." Further: in Mermin (1988) and Bell (1964), I found no reason to depart from the above understandings of *reality* and *locality*; see §6.2, §6.3.

2.3. Moreover, that correlative hypothesis, §1.2, was reinforced by this: (i) "It is the requirement of locality, or more precisely that the result of a test on one beable be unaffected by tests on a distant beable with which it has interacted in the past, *that creates the essential difficulty*," after Bell (1964:195); my italics. (ii) And by EPR's related criteria; see §6.

2.4. (i) So my analysis is bound by the principle of *true local realism*, the union of *true locality* and *true realism*: (ii) Under *true locality* (or *relativistic causality*), no influence propagates superluminally. (iii) Under *true realism* (or *non-naive realism*), some beables — and thus, some hidden properties of beables — *may change interactively*; see §13.8.(ii)-(iii). (iv) Thus: bound by §2.2 — with more bounds to come — we proceed.

3. Procedure

3.1. Our procedure is outlined in the *Table of contents*. Key points are: (i) Seeking to provide a more complete theory WRT Fig.1 — one in line with EPR's ideas — our analysis begins in §6 with EPR's key criteria. (ii) In §7, we sketch the idealized experiment that Bell (1964) studies. (iii) Then, in §8, arguing the EPR case, we find local realistic laws that attach to Fig.1. These provide the correct QM results.

3.2. A cascade follows: (i) In §9, Bell's theorem, and proof, are refuted. (ii) In §10, Bell's inequality is refuted via elementary algebra. (iii) In §11, tracking instances, Bell's errors are identified. (iv) In §12, we refute Bell's claim — in the paragraph above Bell-(15a) — that

[6] [Wary of the term *measurement* — and misleading connotations — we allow that "a 'measurement' may perturb both the 'measured' and the 'measuring' beable." See Bohr's insight at §13.8.(iii).]

the QM correlation, RHS Bell-(3), cannot be approximated via Bell-(2).

4. Abbreviations, edits, and goals under Bell (1964)

4.1. (i) Let Bell-(1) be short for Bell 1964:(1); etc. (ii) And after Bell-(14), label the unnumbered mathematical expressions: (14a)-(14c), (15a), (21a)-(21e), (23); respectively.

4.2. (i) We replace Bell's unit vectors $\vec{a}, \vec{b}, \vec{c}$, by a, b, c. (ii) The line after Bell-(14) should read, "It follows that if c is another unit vector."

4.3. (i) Using P for probabilities, we replace Bell's expectation $P(\vec{a}, \vec{b})$ over continuous variables by $E(a,b)$. (ii) We use this when we edit Bell's move from Bell-(14a)-(14b)-(15); see (42)-(52). (iii) And we use probabilistic $E(AB)$[7] over discrete results; see §8.3 and (12).

4.4. Generalizing — WRT Bell-(2), RHS Bell-(3), and Bell's related "not possible" claim — this next is, in our terms, Bell's theorem:

$$\text{FACT: } E(AB) = -a{\cdot}b \text{ under QM. BUT } E(AB) \neq -a{\cdot}b \text{ under EPR locality, §6.3.} \tag{1}$$

4.5. So, treating EPR correlations in a classical way, our goals are to prove: (i) that LHS (1) is also true under EPR and their classical locality; (ii) that RHS (1) is false. To these ends, LHS (1) — via §6.2.(ii) — is taken to be a fact relating to Fig.1.

5. A colloquial issue

5.1. (i) In Bohm-Aharonov contexts, I seek to treat photons (QM, spin $s = 1$) and spin-$\frac{1}{2}$ particles (QM, $s = \frac{1}{2}$) on a similar footing. (ii) Thus, with photonics — e.g., Aspect (2000), GHZ (1989) — I idealistically allow that, with certainty, an interaction between a photon and a two-channel linear polarizer produces a linearly polarized particle.

5.2. (i) Similarly, from Fig.1, and colloquially here: ${}_p(a^+)$ is a spin-$\frac{1}{2}$ particle linearly polarized in the a^+ direction via the interaction of ${}_p(\lambda_i^\alpha)$ with a linear polarizer $\Phi_a^\pm$. (ii)

[7] AB is the set of all possible paired results under the $[a,b]$ setting in Fig.1; see (4).

Whereas, after Bell (1964:175): the spin component $\vec{\sigma}_1 \cdot \vec{a}$ was measured [?] via Stern-Gerlach magnets [*polarizers* in my terms], and the particle was found [?] to be spin-up in the *a* direction. (iii) The point is: my colloquialisms are not intended to dispute QM facts.

5.3. In this regard, the *Introduction* in Bohm and Aharonov (1957) is a fine antidote. For while they talk QM-wise about *spin,* I talk colloquially about *angular momentum*: hoping to find *spin* in the consequent equations; all bound by EPR's key criteria. See next.

6. EPR's three key criteria and supplementary beables

6.1. Completeness criterion: "Whatever the meaning assigned to the term complete ... every relevant beable must have a counterpart in the physical theory," after EPR (1935:777).

6.2. Reality criterion: (i) "The beables cannot be determined by *a priori* philosophical considerations, but must be found by an appeal to the results of experiments and measurements." [(ii) Note: relatedly, in analyzing thought experiments — seeking to interpret and explain the associated reality in a classically intuitive way; see Fröhner (1998) — we *also* appeal to QM facts; e.g., see LHS (1) in the context of §13.10.] (iii) "If, without in any way disturbing a beable, we can predict with certainty the value of a physical quantity, then there exists a λ-beable corresponding to this physical quantity. It seems to us that this criterion, while far from exhausting all possible ways of recognizing a physical reality, at least provides us with one such way, whenever the conditions set down in it occur. Regarded not as a necessary, but merely as a sufficient, condition of reality, this criterion is in agreement with classical as well as quantum-mechanical ideas of reality," after EPR (1935:777).

6.3. Locality criterion: "... since at the time of measurement the two beables no longer interact, no real change can take place in the second beable in consequence of anything that may be done to the first beable. This is, of course, merely a statement of what is meant by the absence of an interaction between the two beables," after EPR (1935:779). Also, after Einstein — see Bell (1964: 200) — "on one supposition, we should, in my opinion, absolutely hold fast: the real factual situation of the beable $\mathbf{B_1}$ is independent of what is done with the beable $\mathbf{B_2}$, which is spatially separated from the former."

6.4. Supplementary beables, after Bell (1964:195): (i) "EPR argued that QM could not be a complete theory but should be supplemented by λ-beables that restore causality and locality to the theory." (ii) We share Bell's indifference as to whether the λ-beables are continuous or discrete, etc: we seek to handle all; see §§11-12.

6.5. (i) Relatedly, after Bohm & Aharonov (1957:1070): "EPR's work involves a kind of correlation of the properties of distant non-interacting beables, which is quite different from previously known kinds of correlation." (ii) So we now idealize the experiment — advocated by Bohm and Aharonov — under which Bell (1964) studies the EPR argument.

7. A thought experiment as one with EPR's key propositions

$$\pm 1 = A_i^{\pm} \Leftarrow \mathscr{A} \leftarrow {}_p(a^{\pm}) \leftarrow \Phi_a^{\pm} \leftarrow {}_p(\lambda_i^{\alpha}) \leftarrow \bullet \rightarrow {}_p(\lambda_i^{\beta}) \rightarrow \Phi_b^{\pm} \rightarrow {}_p(b^{\mp}) \rightarrow \mathscr{A} \Rightarrow B_i^{\mp} = \mp 1$$

Alice's locale α, local laws, ... $\rfloor$ $\lfloor$Source$\rfloor$ $\lfloor$ Bob's locale β, local laws, ...

Figure 1: After Bell (1964), but in our terms, the idealized experiment Ω.

7.1. (i) Under *completeness* (§6.1) — i.e., every relevant beable is included — Fig.1 depicts the experiment under which Bell (1964) studies the EPR argument. (ii) After Bell (2004:86): "Einstein argued that EPR correlations could be made intelligible only by completing the quantum mechanical account in a classical way." (iii) So that's what we do.

7.2. (i) Each particle ${}_p(\lambda_i^{\alpha})$ $\left[{}_p(\lambda_i^{\beta})\right]$ — from the i-th pair of spin-$\frac{1}{2}$ particles — is uniquely identified via the locale α $[\beta]$ and the instance, $i = 1, 2, .., n$, in which it is tested; n being adequate. (ii) The particles move freely in opposite directions, as arrows $\leftarrow$ $[\rightarrow]$ map their flight from source $\bullet$ to interaction with a 2-channel linear-polarizer $\Phi_a^{\pm}$ $[\Phi_b^{\pm}]$. (iii) The subscript unit vector a $[b]$ denotes the orientation of the polarizer's principal axis, freely and independently chosen by the (out of frame) autonomous robot Alice [Bob]. (iv) The superscripts $(\pm)$ show the orientation of the polarizer's output channels, $a^{\pm}$ $[b^{\pm}]$.

7.3. Thus, in the i-th run of the experiment, polarizer $\Phi_a^{\pm}$ $[\Phi_b^{\pm}]$ transforms the pristine incoming particle ${}_p(\lambda_i^{\alpha})$ $\left[{}_p(\lambda_i^{\beta})\right]$ to a polarized output-particle ${}_p(a^{\pm})\,[{}_p(b^{\mp})]$. That is: polarized parallel a^+ $[b^+]$ or antiparallel a^- $[b^-]$, to the related principal axis a $[b]$.

7.4. Consistent with Fig.1 and its $[a,b]$ setting — via the transformative particle-polarizer interactions there — the set of all possible polarized output-particle pairs is:

$$ {}_p(a^*)_p(b^*) \equiv \left\{ {}_p(a^+)_p(b^+),\, {}_p(a^+)_p(b^-),\, {}_p(a^-)_p(b^+),\, {}_p(a^-)_p(b^-) \right\}. \tag{2} $$

$$ \therefore P({}_p(a^+)_p(b^+)) + P({}_p(a^+)_p(b^-)) + P({}_p(a^-)_p(b^+)) + P({}_p(a^-)_p(b^-)) = 1. \tag{3} $$

7.5. Further re Fig.1: (i) Identical analyzers, $\mathscr{A}$, absorb the polarized inputs and identify the direction of polarization $a^{\pm}$ $[b^{\mp}]$. (ii) An arrow $\Leftarrow$ $[\Rightarrow]$ shows that each $\mathscr{A}$ consequently prints the related (single) result from $A_i^{\pm} = \pm 1$ $\left[B_i^{\mp} = \mp 1\right]$. (iii) So — under the $[a,b]$ setting of the polarizers in Fig.1 — the set of all possible paired results is:

$$ AB \equiv \{A^+B^+, A^+B^-, A^-B^+, A^-B^-\};\ \text{cf. (2)}. \tag{4} $$

$$ \therefore P(A^+B^+) + P(A^+B^-) + P(A^-B^+) + P(A^-B^-) = 1;\ \text{cf. (3)}. \tag{5} $$

7.6. (i) Then, under our hypothesis, §1.2, let our pristine λ-beables be orientations (unit vectors), pairwise anti-correlated via the pairwise conservation of total angular momentum. (ii) And otherwise random: i.e., *uncorrelated* from pair to pair; for, given the infinity of λ-orientations in Fig.1, it is probability zero that any two particle pairs are the same.

$$ \therefore \lambda_i^{\alpha} \neq \lambda_j^{\alpha} \neq \lambda_k^{\beta};\ \lambda_i^{\alpha} + \lambda_i^{\beta} = \lambda_j^{\alpha} + \lambda_j^{\beta} = 0;\ \text{or, pairwise in general, } \lambda^{\alpha} + \lambda^{\beta} = 0;\ \text{etc.} $$

With Bell's single continuous λ, a compensatory minus-sign suffices: see (54). (6)

7.7. Then, since each λ-beable is an unknown random variable prior to a pair-related test, here are the marginal probabilities: $P(A^+) = P(A^-) = P(B^+) = P(B^-) = \frac{1}{2}$.

7.8. (i) From §7.6 it follows that $P(A^+B^+)$ and $P(A^-B^-)$ are equiprobable, as are $P(A^+B^-)$ and $P(A^-B^+)$. (ii) Further, we have this related feature, as also in Bell-(13): if $a = b$, then

A^+B^- and A^-B^+ arise equiprobably, while $A^+B^+$and A^-B^- do not occur at all. (iii) Hence the mnemonic sign convention $[\pm,\mp]$ in Fig.1, just in case $a=b$.

7.9. Finally, we now complete the specification of Fig.1 under *reality* (§6.2) and *locality* (§6.3): (i) To that end, we allow that each off-the-shelf black-box polarizer-analyzer operates as a single function — $\mathbf{A}$[8] — say, $[\![\Phi^{\pm}_{\circ}\otimes\mathscr{A}]\!]=\mathbf{A}$. (ii) In that way, and implicit in Fig.1, we next provide our version of Bell-(1), *with its vital assumption* about locality.

7.10. (i) Thus, *in the same instance*: A_i is determined by $\mathbf{A},a$ and λ_i^{α} alone, and B_i is determined by $\mathbf{A},b$ and λ_i^{β} alone. (ii) Which is similar to Bell-(1), *but with a vital qualification*:

$$A_i^{\pm}=\mathbf{A}\left(a,\lambda_i^{\alpha}\right)=\pm1,\, B_i^{\mp}=\mathbf{A}\left(b,\lambda_i^{\beta}\right)=\mp1, \text{ and } \left\{\mathbf{A}\left(a,\lambda_i^{\alpha}\right)\mathbf{A}\left(b,\lambda_i^{\beta}\right)\right\}=\mp1.$$

(Implicitly, $\mathbf{A}$ is conditioned on Ω, the conditions attaching to Fig.1; see Ω at §12.) (7)

Note: $\mathbf{A}\left(a,\lambda_i^{\alpha}\right)\mathbf{A}\left(b,\lambda_j^{\beta}\right)$ [sic] $=\pm1$ is *inadmissible* here. For — since λ_i^{α} and λ_j^{β} are uncorrelated under §7.6-7.7 — some of its outputs breach §7.8 when $a=b$! (8)

7.11. (i) Under Fig.1, and for us, the above inadmissibility is *instance-busting*, the meaningless coupling of beables from different instances.[9] (ii) So now, *bound also by* (7)-(8) — and taking mathematics to be the best logic — we next advance the EPR case.

8. Advancing the EPR case against Bell's theorem

8.1. (i) From Fig.1: $A_i^{\pm}$ $[B_i^{\mp}]$ are directly linked to $_p(a^{\pm})$ $[_p(b^{\mp})]$ via "capitalization" of the related polarization $a^{\pm}$ $[b^{\pm}]$ and linking it to the related instance; here, i. (ii) So, given the correlational hypothesis in §1.2, we now seek the transformative laws that deliver those expected correlations. (iii) To that end: on the elements $_p(\lambda_i^{\alpha})$ of Alice's and $\Phi_a^{\pm}$'s domain [each paired with its twin $_p(\lambda_i^{\beta})$ in Bob's and $\Phi_b^{\pm}$'s domain], let $\overset{a}{\sim}$ $\left[\overset{b}{\sim}\right]$ denote the equivalence relation *has the same output under* $\Phi_a^{\pm}$ $[\Phi_b^{\pm}]$. (iv) Then, *in the same instance from*

[8] Consistent with its arguments — as we also seek laws that relate determinative local causes to local effects (e.g., polarizer inputs to outputs) — function $\mathbf{A}$ is provided in (24) below.

[9] In (7), the (apparently superfluous) bracketing, $\{...\}$, helps us remain *instance-true* and *trusting*; see (36)-(40). And we use *[sic]* when Bell is not *instance-true*; see (47)-(52). In other words: when a helpfully superfluous $\{...\}$ drops from a line of Bellian reasoning, we add *[sic]s* to the consequent chain of Bellian errors: such chains flowing from just one inadmissible *instance-bust*; e.g., see (46)-(48).

$i = 1, 2, .., n$ — i.e., for twins, our pairwise-correlated beables — by way of example:

$$\text{If } {}_p(\lambda_i^\alpha) \to \Phi_a^\pm \to {}_p(a^+) \text{ then } {}_p(\lambda_i^\alpha) \overset{a}{\sim} {}_p(a^+) \text{ for } {}_p(a^+) \to \Phi_a^\pm \to {}_p(a^+). \tag{9}$$

$$\text{If } {}_p(\lambda_i^\beta) \to \Phi_b^\pm \to {}_p(b^+) \text{ then } {}_p(\lambda_i^\beta) \overset{b}{\sim} {}_p(b^+) \text{ for } {}_p(b^+) \to \Phi_b^\pm \to {}_p(b^+). \tag{10}$$

8.2. So, heuristically: (i) The laws that we seek are bound by §6, (3), (5), §7.8. (ii) And by LHS (1), a fact that is accepted by us under §6.2.(ii). (iii) From (9)-(10), the laws *are heuristically akin to* relations between polarized particles; e.g., there are Malus' $\cos^2$ and $\sin^2$ Laws WRT the transformation of polarized light beams. (iv) In QM, photons are said to be spin-1 particles $[s = 1]$, with spin-$\frac{1}{2}$ particles having $s = \frac{1}{2}$: so it's possible that a related "$\frac{1}{2}$" will be found in the laws. (v) Then, from (9)-(10), again: ${}_p(\lambda_i^\alpha)$ and ${}_p(\lambda_i^\beta)$ are logically — thus law-like — anticorrelated via their λ-relations under (6). (vi) So the general product rule (here, for the probability of correlated results), applies:

$$P(XY) = P(X)P(Y\,|\,X) = P(Y)P(X\,|\,Y) \neq P(X)P(Y), \text{ except via supporting facts.} \tag{11}$$

8.3. (i) WRT the particle-pair in (9)-(10): since each output polarized particle is directly related to an associated result — and seeking to maximize clarity — we often work with the results; e.g., A^+ instead of ${}_p(a^+)$, etc. (ii) Relatedly, seeking laws of nature, we now solve equations given by the heuristics in §8.2 as follows: (iii) Given AB, the set of all possible paired results — cf. (4) — we follow Whittle (1976) and axiomatize the concept of expectation: i.e., we define $E(AB)$ probabilistically, with each paired result (± 1) weighted according to its probability. (iv) Relatedly — via §6.2.(ii) — LHS (1) provides the overall result for the thought experiment in Fig.1. So:

$$\begin{aligned}
E(AB) &\equiv P(A^+B^+) - P(A^+B^-) - P(A^-B^+) + P(A^-B^-) && (12)\\
-\cos(a,b) &= E(AB), \text{ from LHS (1). Then, next, from (5):} && (13)\\
1 &= P(A^+B^+) + P(A^+B^-) + P(A^-B^+) + P(A^-B^-). && (14)\\
\therefore 1 - \cos(a,b) &= 4P(A^+B^+) \because P(A^-B^-) = P(A^+B^+), \text{ via §7.8.(i).} && (15)
\end{aligned}$$

$$\therefore P(A^+B^+) \;=\; \tfrac{1}{2}\sin^2\tfrac{1}{2}(a,b);\text{ etc.} \tag{16}$$

$$=\; P(B^+)P(A^+\,|\,B^+) = \tfrac{1}{2}P(A^+\,|\,B^+) \because P(B^+) = \tfrac{1}{2};\text{ cf. §7.7.} \tag{17}$$

$$\therefore P(A^+\,|\,B^+) \;=\; \sin^2\tfrac{1}{2}(a,b).\ \therefore P(A^-\,|\,B^+) = P(A^+\,|\,B^-) = \cos^2\tfrac{1}{2}(a,b);\text{ etc.} \tag{18}$$

8.4. Here, then — via §6.2.(ii) — are the laws that associate the local transformation of one twin with that of the spatially-separated (and anticorrelated) other:

$$P\left[{}_p(\lambda^\alpha) \overset{a}{\sim} {}_p(a^+)\,|\,{}_p(\lambda^\beta) \overset{b}{\sim} {}_p(b^+)\right] = P\,(A^+\,|\,B^+) = \sin^2\tfrac{1}{2}(a,b);\text{ etc.} \tag{19}$$

$$\therefore\ P\left[{}_p(\lambda^\alpha) \overset{a}{\sim} {}_p(a^-)\,|\,{}_p(\lambda^\beta) \overset{b}{\sim} {}_p(b^+)\right] = P\,(A^-\,|\,B^+) = \cos^2\tfrac{1}{2}(a,b);\text{ etc.} \tag{20}$$

8.5. (i) Thus, as it must, our local realistic theory delivers the QM result in (1) as follows:

$$E(AB) \;\equiv\; P(A^+B^+) - P(A^+B^-) - P(A^-B^+) + P(A^-B^-),\text{ as in (12).} \tag{21}$$

$$=\; \tfrac{1}{2}\left[\sin^2\tfrac{1}{2}(a,b) - \cos^2\tfrac{1}{2}(a,b) - \cos^2\tfrac{1}{2}(a,b) + \sin^2\tfrac{1}{2}(a,b)\right]\ \text{via (16).} \tag{22}$$

$$=\; -\cos(a,b) = -a\cdot b\text{: the QM result; see LHS (1). QED.} \tag{23}$$

8.6. (i) So, WRT our goals, ex §4.5: Bell's theorem, our (1), is refuted! For — via §6.2.(ii) and our classically intuitive way — (23) is derived under the EPR criteria, §6. (ii) Then, relatedly WRT (7) and by observation, here's a law that relates local determinative causes to local effects — i.e., polarizer inputs to outputs — the function $\mathbf{A}$; as in Bell-(1) and our (7):

$$A_i^\pm = \mathbf{A}\,(a,\lambda_i^\alpha) = a\cdot\left(\lambda_i^\alpha \overset{a}{\sim} a^\pm\right) = \pm 1,\ B_i^\mp = \mathbf{A}\left(b,\lambda_i^\beta\right) = b\cdot\left(\lambda_i^\beta \overset{b}{\sim} b^\mp\right) = \mp 1. \tag{24}$$

9. With Bell's theorem (BT) refuted, Bell's proof falls similarly

9.1. (i) Via §8.6, Bell's theorem at (1) is refuted. (ii) So, auditing Bell's (1964:196) "quite simple proof" — ranging from Bell-(12)-(23) — we note that Bell-(12)-(14a) is fine.

9.2. (i) However, by observation,[10] Bell-(14b) $\neq$ Bell-(14a): so (14b) is a false step. (ii) So Bell's proof fails, and any reliance thereon is unsafe: yet we find that reliance in Bell's move

[10] Bell's move from Bell-(14a) to Bell-(14b) — "using Bell-(1)" — is the *Bellian error* of *instance-busting*; see §7.11. In the broader context of discrete and continuous beables, we say more about this error at §11.

from his (21a) to his (21b): and we see the aftermath of falsity in Bell-(21d), and Bell-(15). (iii) Since the last is Bell's famous inequality (BI), we continue with it as the exemplar.

10. Bell's famous inequality (BI) refuted algebraically

10.1. (i) Via §9, Bell's proof of his theorem, Bell-(12)-(23), relies on his inadmissible move from (14a) to (14b). (ii) So we now reformat BI for easy comparison with our similarly-formatted (but irrefutable) WI. (iii) In this way, since BI and WI will have the same LHS, relevant departures of BI's RHS from WI's RHS will signal a Bellian error.

10.2. (i) This in turn signals that Bell-(15) is erroneous and refuted. (ii) Thus, preparing for comparison with our *irrefutable* WI, here's BI in our terms:

$$\text{BI} \equiv |E(AB) - E(AC)| - 1 \leq E(BC) \text{ [sic] from Bell 1964:(15).} \tag{25}$$

10.3. Then, given the limits (± 1) for the results in (7), we see these expectations in (25):

$$-1 \leq E(AB) \leq 1,\ -1 \leq E(AC) \leq 1,\ -1 \leq E(BC) \leq 1. \tag{26}$$

$$\therefore E(AB)[1 + E(AC)] \leq 1 + E(AC);\ \text{for, if } V \leq 1, \text{ and } 0 \leq W, \text{ then } VW \leq W. \tag{27}$$

$$\therefore E(AB) - E(AC) - 1 \leq -E(AB)E(AC) \text{ from (27).} \tag{28}$$

$$\text{Similarly: } E(AC) - E(AB) - 1 \leq -E(AB)E(AC). \tag{29}$$

So: irrefutably via (28)-(29), here's WI, our never-false inequality;

$$\text{the irrefutable WI} \equiv |E(AB) - E(AC)| - 1 \leq -E(AB)E(AC).\ \text{QED.} \tag{30}$$

10.4. (i) Thus, comparing (25) and (26)-(30), it is evident that Bell has erred: for BI is thereby refuted algebraically. (ii) What's more, based on §6.2.(ii): (23) provides a simple way to examine the *extent* of Bell's error. (iii) Let a, b, c be co-planar: and not necessarily orthogonal to the particles' line of flight. (iv) Let $(a,b) = (b,c) = \frac{(a,c)}{2}$, with $0 < (a,c) < \pi$. (v) Then, in this example — with WI as the standard, because it is everywhere irrefutable — BI is *everywhere false*.

10.5. (i) So now, to understand the *nature* of Bell's error, we proceed from the irrefutable to the flawed in a few short steps. (ii) Since all beables commute *within* instances — i.e., *pairwise, and not otherwise* — this next exhausts *all the admissible variants* in RHS (30):

$$\{E(AB)\}\{E(AC)\}=\{E(AB)\}\{E(CA)\}=\{E(BA)\}\{E(AC)\}=\{E(BA)\}\{E(CA)\}. \tag{31}$$

10.6. Let $\overset{!?}{=}$ or $\overset{!?}{\leq}$ signal, "This relation is false!" And the reason will be, "Unique beables commute pairwise, not otherwise!" So here's the essence of Bell's related error:

$$|\{E(AB)\}-\{E(AC)\}|-1 \quad \leq \quad -\{E(AB)\}\{E(AC)\} = \text{ irrefutable WI from (30).} \tag{32}$$

$$\overset{!?}{\leq} \quad -E(AA)E(BC) = -(-1)E(BC) = E(BC) \text{ [sic].} \tag{33}$$

10.7. (i) So, with (33) as an exemplar of Bell's key error, we next study this error in situ: instance-by-instance, discrete or continuous, etc.

11. Key results: via discrete or continuous beables

11.1. We now derive three key results: (i) We derive irrefutable WI, (30), instance-by-instance via discrete beables. (ii) Via continuous beables and Bell's inadmissible *instance-busting* (cf. §7.11), we expose the error in (the already refuted) BI, (25). (iii) In §12, we refute Bell's claimed impossibility: that RHS Bell-(3) cannot be approximated via Bell-(2). (iv) We begin via (7)-(8) — i.e., via our equivalent to Bell-(1) — with placeholders (Z) for clarity in presentation:

$$\underline{\text{WI: via our } \textit{instance-trusting} \text{ analysis with discrete } \lambda\text{-beables.}} \tag{34}$$

$$Z_1 = \text{LHS Bell-(14a)} = E(AB) - E(AC) \text{ via Bell (1964:198).} \tag{35}$$

$$= \tfrac{1}{n}\sum_{i=1}^{n}\left\{\mathbf{A}(a,\lambda_i^{\alpha})\mathbf{A}\left(b,\lambda_i^{\beta}\right)\right\} - \tfrac{1}{n}\sum_{j=1}^{n}\left\{\mathbf{A}(a,\lambda_j^{\alpha})\mathbf{A}\left(c,\lambda_j^{\beta}\right)\right\} \tag{36}$$

$$= \tfrac{1}{n}\sum_{i=1}^{n}\left\{\mathbf{A}(a,\lambda_i^{\alpha})\mathbf{A}\left(b,\lambda_i^{\beta}\right)\right\}\cdot \tag{37}$$

$$\left[1-\tfrac{1}{n}\sum_{i=1}^{n}\left\{\mathbf{A}(a,\lambda_i^{\alpha})\mathbf{A}\left(b,\lambda_i^{\beta}\right)\right\}\cdot\tfrac{1}{n}\sum_{j=1}^{n}\left\{\mathbf{A}(a,\lambda_j^{\alpha})\mathbf{A}\left(c,\lambda_j^{\beta}\right)\right\}\right]. \text{ And} \tag{38}$$

$$\text{we can now eliminate that external } \tfrac{1}{n}\sum_{i=1}^{n}\left\{\mathbf{A}(a,\lambda_i^{\alpha})\mathbf{A}\left(b,\lambda_i^{\beta}\right)\right\} \leq 1. \tag{39}$$

$$\therefore |E(AB) - E(AC)|$$

$$\leq \quad 1 - \tfrac{1}{n}\sum_{i=1}^{n}\left\{\mathbf{A}(a,\lambda_i^{\alpha})\,\mathbf{A}\left(b,\lambda_i^{\beta}\right)\right\}\cdot\tfrac{1}{n}\sum_{j=1}^{n}\left\{\mathbf{A}(a,\lambda_j^{\alpha})\,\mathbf{A}\left(c,\lambda_j^{\beta}\right)\right\} \tag{40}$$

$$\leq \quad 1 - E(AB)E(AC) = \text{irrefutable WI again; see algebra-based (30). QED.} \tag{41}$$

Cf. BI (25) and (next) Bell's defective derivation thereof.

$$\underline{\text{BI: via Bell's } \textit{instance-busting} \text{ analysis with his continuous } \lambda.} \tag{42}$$

$$Z_2 \quad = \quad \text{LHS Bell-(14a)} = E(a,b) - E(a,c) \text{ via Bell (1964:198) and our §4.3.(i).} \tag{43}$$

$$= \quad -\int \rho(\lambda)\,[\{\mathbf{A}(a,\lambda)\mathbf{A}(b,\lambda)\} - \{\mathbf{A}(a,\lambda)\mathbf{A}(c,\lambda)\}]\,d\lambda. \text{ Then, in tracking} \tag{44}$$

$$\textit{Bell's unexplained use of Bell-(1)}\text{, we find an OK chain of instances:} \tag{45}$$

$$\{\mathbf{A}(a,\lambda)\mathbf{A}(b,\lambda)\}\,\{\mathbf{A}(a,\lambda)\mathbf{A}(b,\lambda)\} = 1. \text{ But this next chain is not OK:} \tag{46}$$

$$\{\mathbf{A}(a,\lambda)\mathbf{A}(b,\lambda)\}\,\{\mathbf{A}(a,\lambda)\mathbf{A}(c,\lambda)\} = \mathbf{A}(b,\lambda)\mathbf{A}(c,\lambda)\,\text{[sic]. For it is} \tag{47}$$

$$\text{an inadmissible reduction under instance-busting; see §7.11. Thus:} \tag{48}$$

$$\overset{!?}{=} \quad \int \rho(\lambda)\,\{\mathbf{A}(a,\lambda)\mathbf{A}(b,\lambda)\}\,[\mathbf{A}(b,\lambda)\mathbf{A}(c,\lambda) - 1]\,d\lambda\,\text{[sic]} = \text{Bell-(14b).} \tag{49}$$

$$\therefore |E(a,b) - E(a,c)| \tag{50}$$

$$\overset{!?}{\leq} \quad \int \rho(\lambda)\,[1 - \mathbf{A}(b,\lambda)\mathbf{A}(c,\lambda)]\,d\lambda\,\text{[sic]} = \text{Bell-(14c), and thus [sic] BI,} \tag{51}$$

$$\leq \quad 1 + E(b,c)\,\text{[sic]: which is BI, Bell-(15), as refuted algebraically in §10.} \tag{52}$$

11.2. (i) With key lines identified by number (above) to facilitate discussion, we now see that Bell — similar to the illustrative (33) — introduces *an inadmissible combination* via (47)-(49). (ii) That is: inadmissible because —via (7)-(8) — such beables do not commute *between* instances. (iii) On the other hand, however, we now refute Bell's theorem under §6, (7)-(8) and (24).

12. $\underline{E(AB) \equiv -\int \rho(\lambda)\mathbf{A}(a,\lambda)\mathbf{A}(b,\lambda)d\lambda = -a{\cdot}b}$, as in QM

12.1. We now refute Bell's claim — in the paragraph above Bell-(15a) — that the QM correlation [expectation], RHS Bell-(3), cannot be approximated via Bell-(2). As can be seen in the above section-heading: our counter-claim shows them to be identical.

Here's the valid variant of Bell-(2) that Bell uses at Bell-(14)-(14a): (53)

$$E(a,b) = -\int \rho(\lambda)A(a,\lambda)A(b,\lambda)d\lambda. \text{ And here's our equivalent:} \tag{54}$$

$$E(A,B) = -\int \rho(\lambda)\mathbf{A}(a,\lambda)\mathbf{A}(b,\lambda)d\lambda. \text{ [See (6) for a note on the minus-sign.]} \tag{55}$$

$$\begin{aligned} &= -\int \rho(\lambda)\left(a\cdot\left[\lambda \overset{a}{\sim} a^{\pm} \mid \Omega\right]\right)\left(b\cdot\left[\lambda \overset{b}{\sim} b^{\pm} \mid \Omega\right]\right)d\lambda \text{ via (7), (24),} \\ &\qquad \text{and over the pairings } a^+b^+, a^+b^-, a^-b^+, a^-b^-. \end{aligned} \tag{56}$$

$$\begin{aligned} &= -\tfrac{1}{2}\int \delta(\lambda-a^+)\left(a\cdot\left[\lambda \overset{a}{\sim} a^{\pm} \mid \Omega\right]\right)\left(b\cdot\left[\lambda \overset{b}{\sim} b^{\pm} \mid \Omega\right]\right)d\lambda \\ &\quad -\tfrac{1}{2}\int \delta(\lambda-a^-)\left(a\cdot\left[\lambda \overset{a}{\sim} a^{\pm} \mid \Omega\right]\right)\left(b\cdot\left[\lambda \overset{b}{\sim} b^{\pm} \mid \Omega\right]\right)d\lambda \end{aligned} \tag{57}$$

$$\begin{aligned} &\because \int \rho(\lambda)d\lambda = \tfrac{1}{2}\int\left[\delta(\lambda-a^+)+\int\delta(\lambda-a^-)\right]d\lambda = 1 \\ &\text{via the two values } \left(a^+,\, a^-\right) \text{ that } \lambda \text{ takes in (56) equiprobably.} \end{aligned} \tag{58}$$

$$\begin{aligned} &= -\tfrac{1}{2}\int\left[\left(b\cdot\left[\lambda \overset{b}{\sim} b^+ \,|\, \Omega, \lambda \overset{a}{\sim} a^+\right]\right)-\left(b\cdot\left[\lambda \overset{b}{\sim} b^- \,|\, \Omega, \lambda \overset{a}{\sim} a^+\right]\right)\right]d\lambda \\ &\quad +\tfrac{1}{2}\int\left[\left(b\cdot\left[\lambda \overset{b}{\sim} b^+ \,|\, \Omega, \lambda \overset{a}{\sim} a^-\right]\right)-\left(b\cdot\left[\lambda \overset{b}{\sim} b^- \,|\, \Omega, \lambda \overset{a}{\sim} a^-\right]\right)\right]d\lambda \end{aligned} \tag{59}$$

$$= -\tfrac{1}{2}\left[P(B^+ \mid A^+) - P(B^- \mid A^+) - P(B^+ \mid A^-) + P(B^- \mid A^-)\right] \tag{60}$$

$$= -\tfrac{1}{2}\left[\cos^2\tfrac{1}{2}(a,b) - \sin^2\tfrac{1}{2}(a,b) - \sin^2\tfrac{1}{2}(a,b) + \cos^2\tfrac{1}{2}(a,b)\right] \text{ via (18).} \tag{61}$$

$$\begin{aligned} &= -\cos(a,b) = -a\cdot b. \text{ QED. For Bell's theorem, see (1), is refuted} \\ &\quad \text{under EPR criteria, (§6), etc., and our goals in §4.5 are met.} \end{aligned} \tag{62}$$

12.2. This completes our analysis and refutation of Bell's key technical claims. We conclude with a series of specific "Claim-Conclusion" couplets.

13. Conclusions

13.1. Claim-1: from Nobel (2022:4): "Bell formulated the first Bell inequality, which was a spectacular theoretical discovery." **Conclusion-1:** We find this claim to be false. Bell's first inequality, (25)], is algebraically false, via §10. Further, it relies on inadmissible instance-busting [§7.11]: e.g., see (46)-(49).

13.2. Claim-2: from Nobel (2022:4): "Using a special version of the Bohmian-EPR thought experiment, Bell showed mathematically that no hidden variable theory would be able to reproduce all the results of QM." **Conclusion-2:** We find this claim to be false. Via Fig.1, which is our representation of the special Bohmian-EPR thought experiment — with boundary conditions that include hidden variables — we have shown mathematically that all the results of QM; e.g., see (55)-(62).

13.3. Claim-3: from Nobel (2022:4): "With a mathematical illustration, Bell provided a proof of the assertions made by Bohr and Schrödinger, and thus showed that all attempts to construct a local realist model of quantum phenomena are doomed to fail." **Conclusion-3:** We find this claim to be false. We have refuted all such mathematical illustrations; e.g., see (55)-(62) again.

13.4. Claim-4: from Bell (1964:196), below Bell-(3): "It is not possible that Bell-(2) equals RHS Bell-(3)." **Conclusion-4:** We find this claim to be false, being refuted in §12.

13.5. Claim-5: from Bell (1964:198), below Bell-(15): $E(,b,c)$ — our $E(BC)$ — cannot be stationary at the minimum value (-1 at $b=c$) and cannot equal the QM value in Bell-(23). **Conclusion-5:** We find this claim to be false. This claim — a consequence of Bell's instance-busting; see (46)-(48) — is refuted in §12 by the generality of (54)-(62).

13.6. Claim-6: from Bell (1964:199), below Bell-(23) *and at the end of his proof* [sic]: "The QM expectation value cannot be represented, either accurately or arbitrarily closely, in the form of Bell-(2)." **Conclusion-6:** We find this claim to be false. Since our §12 is successfully based on Bell-(2) and Fig.1's boundary conditions, we take this Bellian claim to be *proof* of the failure of his proof [sic]. Further, this failure can be traced to a single instance-bust: i.e., that in Bell's move from Bell-(21a) to (21b).

13.7. Claim-7: from Bell (1964:199), "In a theory in which parameters are added to quantum mechanics to determine the results of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the setting of one measuring device can influence the reading of another instrument, however remote. Moreover, the signal involved must propagate instantaneously, so that such a theory could not be Lorentz invariant." **Conclusion-7:** We find these claims to be false. In short, via the hypothesis in §1.2, with (7), etc: we have no need — and no back-door — for such mechanisms and signals.

13.8. Claim-8: after Mermin (1988:14),"Einstein maintained that certain properties had to have values whether or not they were measured, notwithstanding the fact that the quantum theory maintained that there was no meaning to such an assertion – that it was merely a 'misuse of language'." **Conclusion-8:** (i) Sounding like a long-rejected naive-realism, this claim is fascinating. (ii) "It seems incredible — but as the poet Nicolais Boileau wrote, 'Le vrai peut quelquefois n'être pas vraisemblable.'[11] — that Einstein (1927, 1935) and Bell (1964) overlooked that 'a measurement perturbs a system.' In the twenties this was a refrain among quantum physicists; and everybody knows that a thermometer does not retro-tell the temperature of the bath, but tells instead the one that it helped create," after Olivier Costa de Beauregard (2000), pers. comm. 21 February. (iii) Further, well-known to the founders, and to Bell (at least in his 1987 edition), here's Bohr's insight — and our view — after Bell (2004:xi-xii): "The result of a 'measurement' does not in general reveal some preexisting property of the 'system' but is a product of both 'system' and 'apparatus'." It seemed to Bell, "that full appreciation of this would have aborted most of the 'impossibility' proofs and most of 'quantum logic'."

13.9. Claim-9: after Bertlmann, §1.5, "Bell might have liked your approach, who knows." **Conclusion-9:** We are not aware of any other work that so clearly refutes Bellian claims (like those in §2.1). Nor one that so pointedly addresses the issues in §15.1. Thus, given §15.2: it is our view that Bell would have liked our approach.

13.10. Claim-10: "Using LHS (1) to find laws, then using those laws in (21)-(23) to find (1), is unhelpful circular reasoning." **Conclusion-10:** See §6.2.(ii): "It is the facts that matter, not the proofs. Physics can progress without the proofs, but we can't go on without the facts ... if the facts are right, then the proofs are a matter of playing around with the algebra correctly," Feynman (2002:137).

13.11. Claim-11: after Bell (1964:195), "... a hidden variable interpretation ... a grossly non-local structure ... of any such theory." **Conclusion-11:** We find this claim to be false, and proven so by our strict adherence to (7)-(8).

[11] "The truth can sometimes seem like untruth," GSW translation.

13.12. Claim-12: True realism, §2.4.(iii), is inconsistent with this: "Realism is a philosophical view, according to which external reality is assumed to exist and have definite properties, whether or not they are observed by someone," Clauser and Shimony (1978:1883). **Conclusion-12:** We need to clearly distinguish between our *true-realism* (which is valid in classical and quantum contexts), and the above *naive-realism* (which is not); see §13.8.

14. Acknowledgments

It's a pleasure to acknowledge beneficial exchanges with Lazar Mayants (late of Amherst), Olivier Costa de Beauregard (late of Paris), Fritz Fröhner (late of Karlsruhe), Roger Mc Murtrie (Canberra), Gerardus 't Hooft (Utrecht), Reinhold Bertlmann (Vienna), Phil Watson (Kariong), and with the late David Bohm's work. Further, though far from my dissident beliefs, I thank The Nobel Committee for Physics (2022) for so clearly presenting the beliefs of others. And for their part in enabling the dedicated Laureates to prove that EPR were right: I acknowledge our deep indebtedness to those who develop the sources.

15. Appendix A: Bell (1990) – dilemmas, feelings, classicality

15.1. After Bell (1990:82-88), re *dilemmas*: (i): "I cannot say that action at a distance [AAD] is required in physics. But I can say that you cannot get away with no AAD. You cannot separate off what happens in one place and what happens in another. Somehow they have to be described and explained jointly. Well, that's just the fact of the situation; the Einstein program fails, that's too bad for Einstein, but should we worry about that? ... it might be that we have to learn to accept not so much AAD, but the inadequacy of no AAD," p.82. (ii) "Conclusion: And that is the dilemma. We are led by analysing this situation to admit that in somehow distant things are connected, or at least not disconnected. And yet we do not feel that we are connected. So as a solution of this situation, I think we cannot just say 'Ohoh, nature is not like that'. I think you must find a picture in which perfect correlations are natural, without implying determinism, because that leads you back to nonlocality. And also in this independence as far as our individual experiences goes, our independence of the rest of the world is also natural. So the connections have to be very subtle, and I have told you all that I know about them," p.84. (iii) "I believe that quantum theory is compatible both with determinism and with indeterminism. ... If it's deterministic. Well I think it has to be nonlocal, whether it's deterministic or indeterministic. I think you're stuck with the nonlocality. I don't know any conception of locality which works with QM. So I think we're stuck with nonlocality ... There is no energy transfer and there is no information transfer either. That's why I am always embarrassed by the word action, and so I step back from asserting that there is AAD, and I say only that you cannot get away with locality. You cannot explain things by events in their neighbourhood. But, I am careful not to assert that there is AAD," p.88.

15.2. After Bell (1990:85), re *feelings*:[12] "Now, it's my feeling that all this AAD and no AAD business will go the same way [as the ether]. [That] someone will come up with the answer, with a reasonable way of looking at these things. If we are lucky it will be to some big new development like the theory of relativity. Maybe someone will just point out that we were being rather silly, and it won't lead to a big new development. But anyway, I believe the questions will be resolved."

15.3. After Bell (1990:86), re *classicality*: "But if there are hidden variables, somehow we are back in classical mechanics," speaker. "Well it depends on what you call classical mechanics. You won't be back with the mechanics of Einstein [sic]. With the light cone neatly separating what is relevant and what is irrelevant. But you would be back with classical mechanics in the sense that you would have some equations, presumably integral rather than differential equations, and the equations would tell you what is going to happen. Whereas in quantum mechanics it's perfectly obscure whether we have a theory like that. In fact we don't have a theory like that," John Bell.

[12] As for my own feelings (but for another time): wholistic mechanics (WM) — commonsense QM — begins with this hypothesis: "All beables are quantum in nature."

16. Appendix B: Glossary of key terms

Symbol	Description	See:
$\overset{!?}{=}$	Bell's false equality, or its equivalent in our terms	(49)
$\overset{!?}{\leq}$	Bell's false in equality, or its equivalent in our terms	(33)
$\{...\}$	A set, including beables in the same instance	(2), (4)
$\because$	because	(15)
α (β)	Alice's (Bob's) locale	Fig.1
λ	Supplementary beable	§6.4
λ^α	Generalized λ-beable in α, etc.	(6)
λ_i^α	Uniquely-defined supplementary beable; etc.	(6)
$\Phi_a^\pm$	Polarizer, principle axis a, output channels $a^\pm$; etc.	§7.2.(ii)
Ω	Our thought experiment, with all its conditions	(7)
a, b, c	Unit vectors	§4.2
(a, b)	Angle between unit vectors a and b; etc.	(13)
$[a, b]$	Defines an experiment by its polarizer settings	§7.4
$\overset{a}{\sim}$	Equivalence under $\Phi_a^\pm$; etc.	§8.1.(iii)
$a^\pm$	A unit vector such that $a^\pm = \pm a = +a$ or $-a$; etc.	§7.3
$A_i^\pm$	i-th result: $A_i^\pm = \pm 1$ in α-locale; $B_i^\mp = \mp 1$ in β-locale	§7.5.(ii)
$\mathbf{A}$	Polarizer-analyzer function	(24)
$\mathscr{A}$	Analyzer prints results: $A_i^\pm = \pm 1$ in α; $B_i^\mp = \mp 1$ in β	§7.5.(ii)
AB	The set of all possible paired results	(4)
BI	Bell's famous inequality, Bell-(15)	§10, (25)
BT	Bell's famous theorem, in our terms	(1)
EPRB	EPR-Bohm	p.1, fn.1
$E(a, b)$	Bell's expectation over his continuous beables; etc.	§4.3.(i), (43)
$E(AB)$	The expectation over AB; etc.	§4.3.(iii),(12)
i	The i-th instance, $i = 1, 2, .., n$; etc.	§7.2.(i)
n	Number of instances for adequate results	§7.2.(i)
p	Spin-$\frac{1}{2}$ particle	§7.2.(i)
$_p(a^\pm)$	Spin-$\frac{1}{2}$ particle, polarized in the $\pm a$ direction; etc.	§7.3
$P(...)$	Probability of ...	§4.3.(i)
s	Spin: $s = \frac{1}{2}$ for spin-$\frac{1}{2}$ particles; $s = 1$ for photons	§5.1.(i)
WI	Our inequality; it refutes BI algebraically	§10, (30)
Z_1, Z_2	Convenient placeholders	(35), (43)

17. References

• Aspect, A. (2000). "Bell's theorem: The naive view of an experimentalist."

http://arxiv.org/pdf/quant-ph/0402001v1.pdf

• Bell, J. S. (1964). "On the Einstein Podolsky Rosen paradox." Physics 1, 195-200.

http://cds.cern.ch/record/111654/files/vol1p195-200_001.pdf

• Bell, J. S. (1975a). "The theory of local beables." Geneva, CERN: TH. 2053, 0-13.

http://cds.cern.ch/record/980036/files/197508125.pdf

• Bell, J.S. (1984c). "Beables for quantum field theory." Geneva, CERN: TH.4035/84, 0-10.
https://cds.cern.ch/record/190753/files/198411046.pdf

• Bell, J. S. (1990). "Indeterminism and Nonlocality [with Discussion]" in "Mathematical Undecidability, Quantum Nonlocality and the Question of the Existence of God." A. Driessen and A. Suarez (Eds.), Springer (Kluwer) 1997: 78-89. https://www.academia.edu/25135051

• Bell, J. S. (1990a). "La nouvelle cuisine." In Between Science and Technology, A. Sarlemijn & P. Kroes (eds). Amsterdam, North-Holland: 97–115. In Bell (2004:232-248).

• Bell, J. S. (2004). Speakable and Unspeakable in Quantum Mechanics. Cambridge, CUP.

• Bohm, D. and Y. Aharonov (1957). "Discussion of experimental proof for the paradox of Einstein, Rosen, and Podolsky." Physical Review 108(4): 1070-1076.
https://www.tau.ac.il/~yakir/yahp/yh157.pdf

• Clauser, J. F. & A. Shimony (1978). "Bell's theorem: experimental tests and implications." Reports on Progress in Physics **41**: 1881-1927.

• EPR (1935). "Can quantum-mechanical description of physical reality be considered complete?" Physical Review 47(15 May): 777-780. http://journals.aps.org/pr/pdf/10.1103/PhysRev.47.777

• Feynman, R.P. (2002). "Lectures on Gravitation." Brian Hatfield (Ed.), 137. https://todayinsci.com

• Fröhner, F. H. (1998). "Missing link between probability theory and quantum mechanics: the Riesz-Fejér theorem." Z. Naturforsch. 53a, 637-654.
http://zfn.mpdl.mpg.de/data/Reihe_A/53/ZNA-1998-53a-0637.pdf

• GHZ (1989). "Going beyond Bell's theorem." in Bell's Theorem, Quantum Theory and Conceptions of the Universe. M. Kafatos. Dordrecht, Kluwer Academic: 69-72.
http://arxiv.org/pdf/0712.0921v1.pdf

• Mermin, N. D. (1985c). "Is the moon there when nobody looks? Reality and the quantum theory." Ex: Physics Today 38(4): 38-47. http://maltoni.web.cern.ch/maltoni/PHY1222/mermin_moon.pdf

• Mermin, N. D. (1988). "Spooky actions at a distance: Mysteries of the quantum theory." The Great Ideas Today 1988. M. J. Adler. Chicago, Encyclopædia Britannica Inc: 2-53.

• Nobel. (2022). "Scientific Background on the Nobel Prize in Physics 2022."
https://www.nobelprize.org/uploads/2022/10/advanced-physicsprize2022.pdf

• Stapp, H. P. (1975). "Bell's theorem and world process." Il Nuovo Cimento, 29B (2):270-276.

• Whittle, P. (1976). Probability. London, John Wiley.