ON EXTENSION OF EINSTEIN FIELD EQUATIONS

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Abstract. In this paper I will explore how extension of Einstein field equation can lead to new equation that can be important way of looking at gravity.

Contents

1. Field equation

In this paper I will present an extension of Einstein field equations [1] [2] [3]. This extension can be reduced to Einstein field equation by contraction and it's form they are equal to:

$$R^{\rho}_{\sigma\mu\nu} - \frac{1}{2} R_{\mu\kappa} g^{\kappa\rho} g_{\sigma\nu} = \kappa T_{\sigma\kappa} g^{\kappa\rho} g_{\mu\nu} \tag{1.1}$$

First thing that can be deduced from those equation is that they do not have vacuum solutions, if I set energy tensor to zero so does Ricci tensor go to zero so i just flat space-time:

$$R^{\rho}_{\sigma\mu\nu} = 0 \tag{1.2}$$

It seems like an big problem for this model to work but I will explore what if that model really does explain gravity, so there is no true vacuum empty of energy. First off there is need for all space to be filled with energy or matter and this seems with common sense, objects are well located in space, but I will assume that this view is not correct. That leads to key principle of this whole idea:

All physical objects extend in space and focus at some point, flow of energy from all space to a point where object is localized is responsible for gravity effects.

This is from point of view of energy, but it could be looked other way around. Space-time normally is empty that gives impression of it being flat- it's same everywhere it means that it's not focused in any particle region of space-time, or more simple terms in space. When space-time is curved it's focus at some point of space-time, it's not same everywhere, there is more of it at one point than other that leads to flow like behaviour. So putting in in terms of only space-time:

Curvature of space-time is equal to it's amount being not same in every point. It means that where curvature is changing there is a flow of it, from regions of lower curvature to regions to higher curvature. That flow represents all gravity effects.

It means there is no change between space-time curvature and energy. Energy is effect of space-time curvature, where it focus is where there is most energy so it means we find there matter, but if this idea is true matter extends in whole space and is just focus in some region.

2. Mathematical details

Now before I will move further into this idea let's put some parts that are key to make this idea work. First of this tensor I did use on left side of equation needs to have same property as Einstein tensor so it has to vanish with respect to covariant derivative, I will start by Einstein tensor and adding only metric tensors and expanding element of Einstein I will get this result [4] [5]:

$$\nabla_\delta \left(R^{\gamma\delta} - \frac{1}{2} R g^{\gamma\delta} \right) = 0 \tag{2.1}$$

$$\nabla_\delta \left(g_{\sigma\gamma} g_{\nu\delta} R^{\gamma\delta} - \frac{1}{2} R g_{\sigma\gamma} g_{\nu\delta} g^{\gamma\delta} \right) = 0 \tag{2.2}$$

$$\nabla_\delta \left(R_{\sigma\nu} - \frac{1}{2} R g_{\sigma\nu} \right) = 0 \tag{2.3}$$

$$\nabla_\delta \left(g_{\mu\alpha} g^{\alpha\rho} g^{\mu\alpha} g_{\alpha\rho} R^{\rho}_{\sigma\mu\nu} - \frac{1}{2} g_{\mu\alpha} g^{\alpha\rho} R_{\mu\kappa} g^{\kappa\mu} g_{\sigma\nu} \right) = 0 \tag{2.4}$$

$$\nabla_\delta \left(\delta^{\rho}_{\mu} \delta^{\mu}_{\rho} R^{\rho}_{\sigma\mu\nu} - \frac{1}{2} \delta^{\rho}_{\mu} R_{\mu\kappa} g^{\kappa\mu} g_{\sigma\nu} \right) = 0 \tag{2.5}$$

$$\nabla_\delta \left(R^{\rho}_{\sigma\mu\nu} - \frac{1}{2} R_{\mu\kappa} g^{\kappa\rho} g_{\sigma\nu} \right) = 0 \tag{2.6}$$

It means that like Einstein tensor it does vanish so gravity field is conserved. Next important part is that this equation has only equal amount of unknowns only in four dimensions it means that it will only work correctly in four dimensional space-time. And at last I can prove that this equation reduces to Einstein field equations I do it by contraction of all indexes from μ to ρ:

$$R^{\rho}_{\sigma\mu\nu} - \frac{1}{2} R_{\mu\kappa} g^{\kappa\rho} g_{\sigma\nu} = \kappa T_{\sigma\kappa} g^{\kappa\rho} g_{\mu\nu} \tag{2.7}$$

$$R^{\rho}_{\sigma\rho\nu} - \frac{1}{2} R_{\rho\kappa} g^{\kappa\rho} g_{\sigma\nu} = \kappa T_{\sigma\kappa} g^{\kappa\rho} g_{\rho\nu} \tag{2.8}$$

$$R_{\sigma\nu} - \frac{1}{2} R g_{\sigma\nu} = \kappa T_{\sigma\kappa} \delta^{\kappa}_{\nu} \tag{2.9}$$

$$R_{\sigma\nu} - \frac{1}{2} R g_{\sigma\nu} = \kappa T_{\sigma\nu} \tag{2.10}$$

Same procedure can be using two metric tensor that will lead to delta that will change index from μ to ρ and even without writing this it comes from equation before as this is just reversing that equation. But there it's done opposite way I change ρ to index μ both are equivalent.

3. Gravity field

I did state in section Field equation that gravity field is equal to flow of energy or curvature of space-time. It means that both are same phenomenon just seen from two points of view. It can be easy proven by transformation of field equation. First i start by change in Ricci tensor to energy tensor [6] so I will get:

$$R^{\rho}_{\sigma\mu\nu} - \frac{1}{2}\kappa\left(T_{\mu\kappa} - \frac{1}{2}Tg_{\mu\kappa}\right)g^{\kappa\rho}g_{\sigma\nu} = \kappa T_{\sigma\kappa}g^{\kappa\rho}g_{\mu\nu} \tag{3.1}$$

$$R^{\rho}_{\sigma\mu\nu} = \kappa\left[T_{\sigma\kappa}g^{\kappa\rho}g_{\mu\nu} + \frac{1}{2}\left(T_{\mu\kappa} - \frac{1}{2}Tg_{\mu\kappa}\right)g^{\kappa\rho}g_{\sigma\nu}\right] \tag{3.2}$$

$$R^{\rho}_{\sigma\mu\nu} = \kappa\left[T_{\sigma\kappa}g^{\kappa\rho}g_{\mu\nu} + \frac{1}{2}T_{\mu\kappa}g^{\kappa\rho}g_{\sigma\nu} - \frac{1}{4}Tg_{\mu\kappa}g^{\kappa\rho}g_{\sigma\nu}\right] \tag{3.3}$$

$$R^{\rho}_{\sigma\mu\nu} = \kappa\left[T_{\sigma\kappa}g^{\kappa\rho}g_{\mu\nu} + \frac{1}{2}T_{\mu\kappa}g^{\kappa\rho}g_{\sigma\nu} - \frac{1}{4}T\delta^{\rho}_{\mu}g_{\sigma\nu}\right] \tag{3.4}$$

From it follows that curvature always has energy, so everywhere where there is curvature present there is energy present. Both can't be separated so in truth they are same phenomenon seen from two point of view, we do observe matter and gravity field but there is not say to be same thing, here they have to be same thing in order for this equation to work. This gives a clear definition of gravity field. Gravity field is curvature of space-time, from it follows that gravity field has energy at each point of space-time that is equal to matter itself. So there is matter field in every point of space-time and that matter field gives rise to curvature thus gravity. It means that if there is gravity present energy-momentum tensor is non zero at each point of space-time as follows from equation 3.4. That being said in first section of this paper, matter extends in whole space and time and it's only focus in some point of space for given instant of time. So gravity field is same as extended matter field and gravity field has energy in form of matter or energy. By extension of Einstein field equation result is more simple statement about gravity field, in Einstein field equation there is no energy tensor for gravity here matter extended in all space is a energy tensor of gravity. But in truth that energy tensor is just a extension of matter field, so there is equality between matter and space-time curvature. Gravity field is represented by energy and energy is represented by space-time curvature, so there is no vacuum solutions where point of space contains matter.

4. Space-time paths

From fact that matter is equal to gravity field comes that field itself does move toward focus where matter is concentrated at. It means that I must consider all possible space-time paths so that energy of central object that is focus stays same. Saying more precise where object is focus is just where all energy flows to- to region of highest curvature or most energy. First I will write geodesic equation for all possible paths [7]:

$$\left.\frac{d^2x^\mu(\mathbf{x})}{ds^2(\mathbf{x})}+\Gamma^\mu_{\alpha\beta}(\mathbf{x})\frac{dx^\alpha(\mathbf{x})}{ds(\mathbf{x})}\frac{dx^\beta(\mathbf{x})}{ds(\mathbf{x})}=0\right|_{\forall\mathbf{x}\in X} \tag{4.1}$$

Where I did denote set of all possible coordinates living on given manifold as X so now space-time paths are functions of space-time, for each point of space-time there is a solution to geodesic equation. Let me assume that object is stationary, it's always at point $(ct,\mathbf{0})$ flow of all paths tends towards that point of space where object is concentrated. So if I take point that is at some distance from where object is focus $\mathbf{r}$, after some time ct' object will eventually reach focus point if it's starting velocity is zero. Those lines are lines of how gravity field flows- at every instant of time volume of space contracts into focus point this flow is gravity field. If object does move so does move it's field- from it follows that for moving object volume that comes form paths will focus for given instant of time where object is localized. This still assumes that object is localized in just one point of space, it means that all physical objects are point like- but it does not have to be true, if for example object is not point like but has any internal structure then all does focus in that structure in space. It will be important part when there is need to deal with measurement, in general all information about focus point and object itself is encoded in energy tensor and Riemman curvature tensor form them follow is object a point like or has a shape. Key idea is that object extends itself in whole space and when time is added it focuses at it's current location in space for given instant of time. Now I can define proper time that again is defined for all possible points of space-time and does not differ otherwise from relativity:

$$\left.\tau^2(\mathbf{x})=\frac{1}{c^2}\int_{P(\mathbf{x})}ds^2(\mathbf{x})=\frac{1}{c^2}\int_{P(\mathbf{x})}g_{\mu\nu}(\mathbf{x})\,dx^\mu(\mathbf{x})\,dx^\nu(\mathbf{x})\right|_{\forall\mathbf{x}\in X} \tag{4.2}$$

Where $P(\mathbf{x})$ denotes path in space-time , where I do calculate proper time not for one path but for all paths for each point of space for given instant of time. This represents flow of proper time in gravity field.

5. Spin

Spin is key property of quantum mechanics and it seems it's fully quantum property but what if it comes from geometry of manifold itself? That's idea I will hold on to. I will use curvature tensor integral for all possible paths that follow form one point x_A to opposite of that point $-x_A$ where this point is localized in space, it takes time from t_A to to t_B then from t_B to t_C get back to starting point of space. Then there is opposite way of spinning that starts with opposite point $-x_A$ and goes to point x_A then goes back to point $-x_A$ where it takes time t_D to time t_E then from it to finally time t_F so it does full rotation but in opposite way. Let's write it in line integral [8] form:

$$\int_{\forall P\left[(x_A,t_A)\to(-x_A,t_B)\right]\in X} R^{\rho}_{\sigma\mu\nu}ds = -\int_{\forall P\left[(-x_A,t_D)\to(-x_A,t_E)\right]\in X} R^{\rho}_{\sigma\mu\nu}ds \tag{5.1}$$

After half rotation, curvature tensor gives opposite vector- like in normal rotation but it's for all possible paths on manifold. So there are two possible spin states, one is positive rotation one is negative rotation they can be written:

$$\uparrow^{\rho}_{\sigma\mu\nu} = R^{\rho}_{\sigma\mu\nu}\Big|_{\forall P\left[(x_A,t_A)\to(-x_A,t_B)\to(x_A,t_C)\right]\in X} \tag{5.2}$$

$$\downarrow^{\rho}_{\sigma\mu\nu} = R^{\rho}_{\sigma\mu\nu}\Big|_{\forall P\left[(-x_A,t_D)\to(x_A,t_E)\to(-x_A,t_F)\right]\in X} \tag{5.3}$$

Where I did use notation for path that goes from those points, that denotes from those points to points where arrow points to, that was written: $P\left[(x_A,t_A)\to(-x_A,t_B)\right]$ in this case it's path from point of space x_A and time t_A to point $-x_A$ of space so opposite point and to time t_B. More precise spin state is for all possible paths that start from that point and end at it's opposite. Gravity field extends to all possible points of space, spin itself works only on closed manifolds it means that elementary particles like for example electron need to to be closed surface in space and they need to obey this property of doing two rotations to get to same point as spinors have. Or particles like photons are more simple they have spin one so they will behave just like ordinary sphere. So spin answers problem with point like particles, particles can't be point like in this model otherwise spin will not work. Spin in general can be used for any point on manifold that will represent all possible spin states, if I take all paths for given point I can chose another point and spin axis orientation will change. So there is need for a physical process of measurement to explain how spin state changes.

6. Measurement and summary

Measurement is key property of any physics theory. Quantum theory has proven that outcome of measurement has to be statistical in nature, it lead to so called measurement problem in quantum physics. Key question is what physical process does cause collapse of wave function of system? I will assume that gravity itself is responsible for collapse of wave function. All systems will tend to become concentrated by gravity itself, on small scale gravity field is weak compared to other forces so those effects are smaller than for larger objects but in both cases object focus at some region of space is was cause the wave function collapse. Same with all properties that look wave like - they are caused by un-focusing of object field. This view if is true would be explanation for quantum mechanics problem of measurement and it does come from field itself or rather it's ability to be focus and un-focus in some region of space. For spin that is last part of measurement quantum mechanics view is that it spin in both ways at once. I will assume that it does not spin at all it can spin only when measured, before measurement there is only geometric property of manifold we call spin, when we measure this property it can spin one way or they other but it's not determined. Spin is following all paths form one point of manifold to it's opposite and back to starting point, at least to assumptions of this model. There is no way to tell does spin goes one way or other from fact that flow of energy in field at first place is not focus on any point before measurement. When measured energy can flow only in specific way that gives spin states. Going back to wave function collapse, flow of let's say electron field (where I assume all fields not only gravity field) always focus at some volume of space, but it can be un-focus that will look like spreading of wave function of that electron. When measurement is done electron does focus in smaller volume of space and it leads to being localized in space. This is view I do present on measurement process that does not need quantum physics to explain it's effects. Direct calculations need solving of field equation that I do not present here. This view on measurement could be wrong and in truth quantum physical view of statistical nature of measurement could be true as it seems now, but this idea is in opposition to this view and it could be hard to prove it right but it's still idea worth considering in future.

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