A Novel Quantum Belief Entropy for Uncertainty Measure in Complex Evidence Theory

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Abstract

In this paper, a new quantum representation of CBBA is proposed. In addition, a novel quantum belief entropy is proposed to measure the uncertainty of CBBA in complex evidence theory.

Keywords: Complex Evidence Theory, Uncertainty Measurement, Quantum Interference, Quantum Belief Entropy.

1. The proposed QB entropy

Definition 1. The quantum form of P_k in \mathbb{M} is represented as

$$|P_i\rangle = \sum_{e \in P_i} z_e |e\rangle,\tag{1}$$

A FOD can be represented through an associated density matrix $\rho = \sum_{i} p_i \rho_i$ where $\rho_i = |P_i\rangle \langle P_i|$.

Definition 2. QB entropy of CBBA is defined as follows:

$$E_Q(\mathbb{M}) = S(\rho) + \sum_{i \neq j} QI_{ij}, \qquad (2)$$

$$S(\rho) = \sum_{\theta_k \in \Theta} |P(\theta_k) \log_2 P(\theta_k)|, \qquad (3)$$

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where $P(\theta_k) = \text{Tr}(\mathbb{M}_{\theta_k} + \mathbb{M}_{\theta_k} \rho)$. Specificially, QI_{ij} in QB entropy is defined by the following formula:

$$QI_{ij} = 2 |P(\theta_i)| |P(\theta_j)| \cos \alpha, \tag{4}$$

Specifically, QB entropy can be used to deal with decision making problems with multiple sources of complex evidence, etc.

References

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