Seeing the light: Relativity and Simultaneity

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The acclaimed Caltech series, "The Mechanical Universe", discusses the speed of light and relativity. Here we elaborate on their demonstration to show how one observer will determine that another observer will see that an event is simultaneous if light is followed back to the observer's eyes. Video demonstrations can help clarify some ideas about motion, light, and relativity, so we also refer to other video presentations that are available and accessible to the general public. We expand upon insights from a Veritasium video which shows that there is no way to measure the one-way speed of light, which Einstein took as *c* by *convention*. Since only a two-way speed of light must be *c*, we propose an *alternative relativity* in which the one-way speed of light includes the velocity of an inertial frame (*i.e.*, $c + v$ or $c - v$). The result is that all inertial observers determine that events happens simultaneously, when following the light. We show this using an example from a World Science Festival video . And there is no way to disprove this *alternative relativity* without establishing a one-way speed of light, which has been deemed impossible.

Part 1: Follow the light.

Caltech's acclaimed series, "The Mechanical Universe…And Beyond", by David Goodstein was released in the mid 1980s. It has 52 episodes for introductory college physics.

Episode 42 is titled "The Lorentz Transformation"[1] and won the 1987 Japan Prize for educational content.[2] The subsequent episode is titled "Velocity and Time"[3] and continues the discussion into Einstein's relativity. The insightful videos and animations show:

- The speed of light is the same for all inertial (non-accelerating) observers, no matter their speed.
- Space (length) and time are relative measures.
- Observers in uniform relative motion to each other will disagree on whether events occur simultaneously.

Both episodes show Figures 1 and 2 which capture a sequence of moving animations. Henry Lorentz is on a train moving to the right at 0.4*c* (*i.e.*, 0.4 the speed of light) relative to Albert Einstein, who stands on the ground. The following text is from Episode 42 (starting at 8:24):

> … at the exact place and time they pass each other, they observe a flash of light. A sphere of light expands outward from that point. Since each measures

the speed of light relative to himself, each believes correctly that he is always at the center of that expanding sphere - even though they themselves move farther and farther apart. How can two people in different places both be at the center of the same sphere?

To confirm his perception, each sets up light detectors an equal distance apart. However, while Albert's detectors register the light arriving simultaneously, he believes the light strikes Henry's detectors at two different times. Meanwhile, Henry sees the same thing - in reverse. They agree on the speed of light, but they disagree on whether events happen simultaneously or at different times.

Later, 3D like space-time diagrams are used (and offset) to show how each observer views their own light detectors to flash simultaneously. However, we will focus on Figures 1 and 2 here.

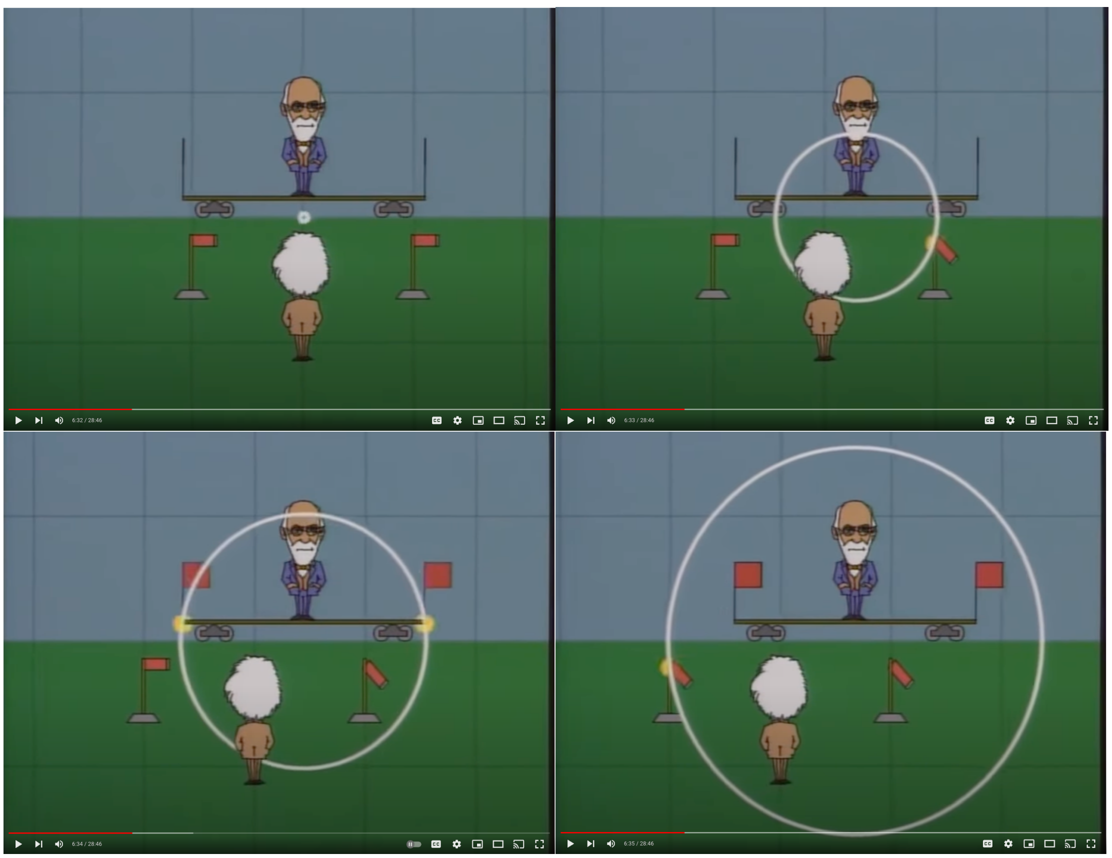

Figure 1: Henry's observation that he is at the center of the expanding sphere of light while riding on a train at a speed of 0.4c relative to the ground. His light detectors flash simultaneously, while those on the ground flash at different times.

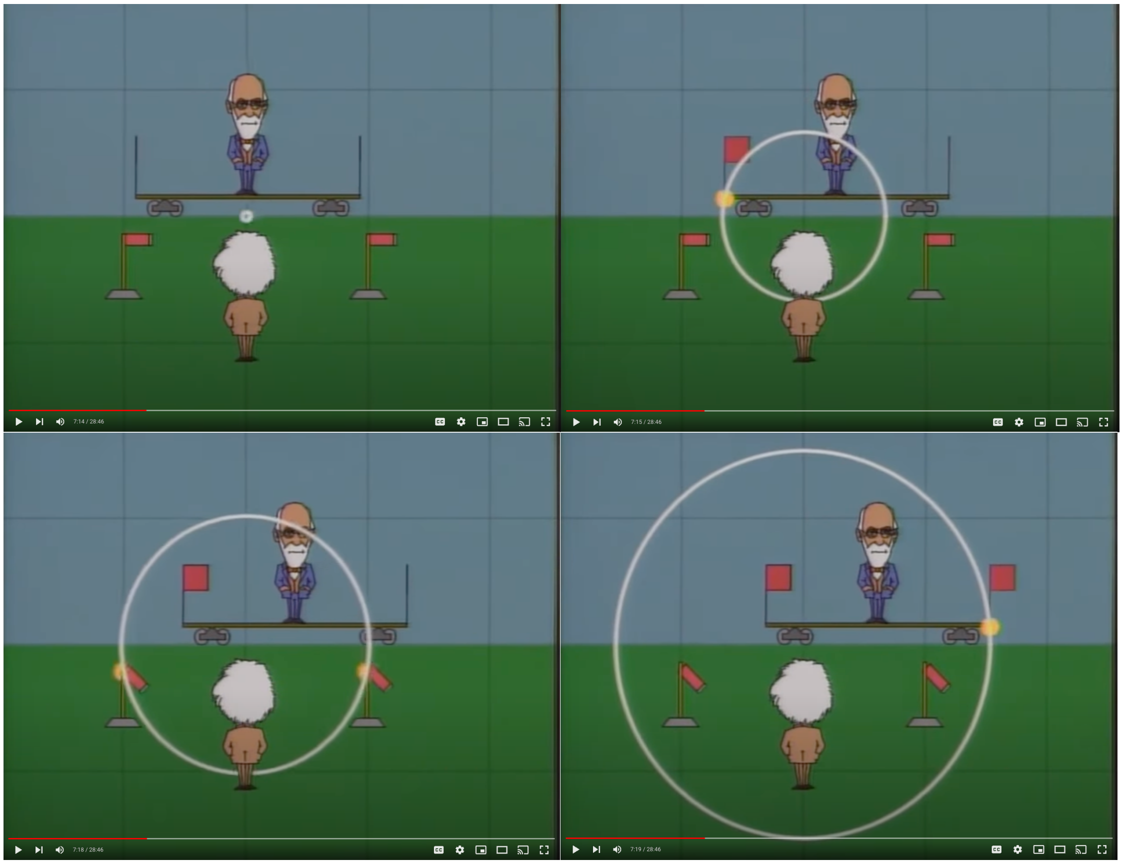

Figure 2: Albert's observation that he is at the center of the expanding sphere of light on the ground. His light detectors flash simultaneously, while those on the train flash at different times.

This demonstration must use an exceptionally long train (and exaggerated dimensions for each man) since the speed of light is shown. Although the videos for Figures 1 and 2 show when the light detectors are triggered, they do not show when each observer sees the detector triggered. So, while watching the videos, we might ask when the light from each light detector actually reaches an observer.

Specifically, we will consider Figure 2 where the sphere of light expands in Albert's rest frame. In his own frame, Albert can trace the light from Henry's detectors to show that Henry sees them flash simultaneously.

A similar situation would arise if we considered Henry's frame – and it is assumed that Albert was next to the light source, for reasons that will become clear later.

At the exact time that Henry and Albert pass each other, the flash of light starts, and the time is set equal to 0. In Albert's frame, the expanding light sphere hits the left light detector, $D1$, on the train first, and give off Flash 1 (shown in the upper-right of Figure 2). When the light sphere hits the right light detector, $D2$, on the train, it gives off Flash

2 (shown in the lower-right of Figure 2). In order to quantify how long it takes for these light flashes to reach the center of the train where Henry is located (at a scale where he would appear to be microscopic relative to the wide length of the train) we must do some simple algebra for Albert's frame:

c is the speed of light
v is the speed of the train to the right ($0.4c$ in the video)
l is half the train's length
t_1 is the time for the expanding sphere of light to hit $D1$, causing Flash 1
t_2 is the time it takes for the light from Flash 1 to reach the center of the train
t_3 is the time for the expanding sphere of light to hit $D2$, causing Flash 2
t_4 is the time it takes for the light from Flash 2 to reach the center of the train

The following calculations are based on the speed of light c, the velocity of the train v, and the distance formula: $distance$ equals $velocity$ times $time$.

The time t_1, from the start until Flash 1, can be found from the distance formula:

$$ct_1 = l - vt_1, \quad \text{so } t_1 = \frac{l}{c+v}$$

The time t_2, for light Flash 1 to reach the center of the train, can be found from the distance formula:

$$ct_2 = l + vt_2, \quad \text{so } t_2 = \frac{l}{c-v}$$

The time t_3, from the start until Flash 2, can be found from the distance formula:

$$ct_3 = l + vt_3, \quad \text{so } t_3 = \frac{l}{c-v}$$

The time t_4 for light Flash 2 to reach the center of the train, can be found from the distance formula:

$$l = vt_4 + ct_4, \quad \text{so } t_4 = \frac{l}{c+v}$$

Therefore, the time for the initial flash of light to hit $D1$, and then the subsequent light from Flash 1 to reach the center of the train is:

$$t_1 + t_2 = \frac{l}{c+v} + \frac{l}{c-v}$$

The time for the initial flash of light to hit $D2$, and then for the subsequent light from Flash 2 to reach the center of the train is:

$$t_3 + t_4 = \frac{l}{c-v} + \frac{l}{c+v}$$

So $t_1 + t_2 = t_3 + t_4$.

Therefore, Albert views that Henry sees both Flash 1 and Flash 2 are triggered simultaneously, in Albert's frame. And, of course, Henry views these flashes to be simultaneous in his own frame. This seems to make sense after all, but it just required a little more thought, since it was not explicitly stated or shown in the video. Perhaps there is more agreement between observers after all.

Key Point 1. When inertial observers actually *see* light is a factor in accounting for some apparent discrepancies observed in their relatively moving frames.

Part 2: No one-way measurement of the speed of light. The two-way measurement of the speed of light back to its source. Light clocks. A question about simultaneity.

A Veritasium video[4] by Derek Muller discusses the possibilities for measuring the one-way and two-way speed of light. The two-way speed of light can be measurement by reflection a light off of a mirror back to its source. The velocity, c, can be calculated using the distance, d, between the light source and the mirror, and the roundtrip time, t. Here $c = 2d/t$. Where c is used in this paper, it refers to the specific velocity (in meters per second) of $c = 299{,}792{,}458\ m/s$.

The video discusses that the one-way speed of light has not, and cannot, be measured. At first, it may appear that this is merely "begging the point", and that no-one could really believe that the one-way speed of light is different from c. However, in a later section, we will see some important implications of this. The following text starts at 6:28 in the video:

> Now, you might think it is just simpler that light should travel at the same speed in all directions, but the truth is, that is a convention, rather than an experimentally verified fact. Einstein himself pointed this out in his famous 1905 paper, "On the Electrodynamics of Moving Bodies." He spends the first couple of pages on the problem of synchronizing clocks at different locations A and B. And he says there is no way that we can meaningfully compare the times they measure "unless we establish by *definition* that the 'time' required by light to travel from A to B equals the 'time' it requires to travel from B to A." He's essentially defining that the speed of light in opposite directions is the same. And he puts by definition in italics to remind us that this is only a convention. It's known as the Einstein synchronization convention.
> So, the idea that the speed of light is the same in opposite directions, as Einstein would later write, "is neither a supposition, nor a hypothesis about the physical nature of light, but a stipulation that I can make of my own free will to arrive at a definition of simultaneity."

Although simultaneity is not explicitly discussed further, we make the following observation a key point.

Key Point 2. Einstein apparently realized that his free-choice convention for the one-way speed of light as c would provide a *definition* of simultaneity. Ultimately this *convention* establishes simultaneity as relative.

Subsequently, the video reiterates that, “There’s no way to define the one-way speed of light, so the only thing we can really define is the two-way speed of light.” It is also shown that Einstein defined light using the two-way calculation shown for c earlier, $c = 2d/t$. A discussion of light clocks begins at 8:12.

> I don’t know if you saw [i]n your physics classes, but whenever there was a light clock, it would always bounce the light up and then back. You would never see a light clock just bounce light one-way. And this is why the only thing we can be certain is constant for all inertial observers is the two-way speed of light.

In the background, an animation is played of two trains where the train on top is at rest and the train on the bottom is moving to the right as shown in Figure 3.

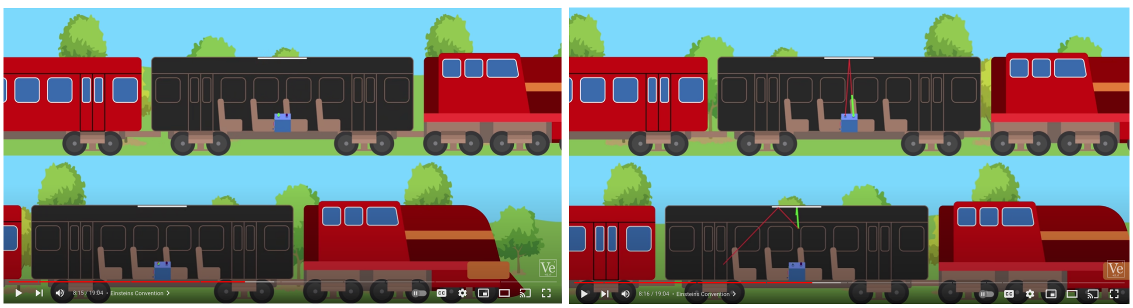

Figure 3: The top train is in the rest/ground frame and the bottom train moves to the right at a constant velocity. Light is emitted from a source and graphically appears to travel the same distance in each frame. Per the transcript of the text above, the animation indicates that each observer can only measure the two-way speed of light back to the source.

The video reviews failed attempts at measuring the one-way speed of light for over 100 years. The discussion includes some thought experiments, time dilation, clock synchronization, published papers, etc.

Although we will return to time dilation in the next section, the key point of the video (as discussed at 15:11) is that the one-way speed of light is unknowable.

> It is unknowable. That’s the whole point of the video… We’ve all agreed to just say it’s c in every direction, but the truth is that physics works the same whether it’s c or c over 2 [$i.e., c/2$] and instantaneous, or anything in between. As long as the roundtrip works out to be c, none of physics breaks - and that’s the crazy thing.

Although Einstein’s relativity used c as the *convention* for the one-way speed of light for all inertial observers, we make the following key point:

Key Point 3. If the one-way speed of light is truly unknowable, then we can say that the *speed of light changes according to the velocity of an observed frame* as long as the two-way speed of light is still c. This *alternative relativity* would produce the same results as the *many* experimental results of Einstein's relativity. If this is not true, then we would have a way to determine the one-way speed of light.

We will consider Key Point 3 further later. Here we make another observation as a key point.

Key Point 4. If two observers see a one-way speed of light (such as when they are separated by a fixed distance and a light bulb is turned on halfway between them), there is no way to tell when they both see it, since this is unknowable. However, they must have both seen the light within the timeframe that the light could be reflected back to the source.

The concluding section of the Veritasium video poses a question about simultaneity, which we will return to later. The passage refers to the convention for the one-way speed of light as c (starting at 16:03):

> …but I think it's important to point out that it is just a convention, not an empirically verified fact. Personally, I find it fascinating that this is something about the universe that is hidden from us. Sure, the round-trip speed of light is c, but does the one-way speed even have a well-defined value? And if it doesn't, what does that mean for the concept of simultaneity?
> …maybe, when physics takes the next paradigmatic leap, our inability to measure the one-way speed of light will be the obvious clue to the way General Relativity, Quantum Mechanics, space, and time are all connected. And we'll wonder why we didn't see it before.

Key Point 5. Although it is usually stated that, "The speed of light is the same for all inertial observers", it might be more accurate to state that, "The ***two-way*** speed of light is the same for all inertial observers" since no convention for the one-way speed of light needs to be assumed.

Part 3: Light clocks, the two-way speed of light, time dilation and length contraction.

In the previous section, we saw how important the two-way speed of light is for the determination of c. The one-way speed of light is conventionally taken to be c, but it is really unknowable.

In this section, we go back to the series, "The Mechanical Universe", Episode 42 and the discussion of light clocks in the determination of time dilation. The video implicitly assumes a one-way speed of light c.

As before, Albert stands stationary on the ground while Henry travels to the right on a train. Henry has a light clock that emits light through a fixed distance, as shown in Figure 4. Henry's frame uses a prime (*i.e.* a single quote) to distinguish it from Albert's frame.

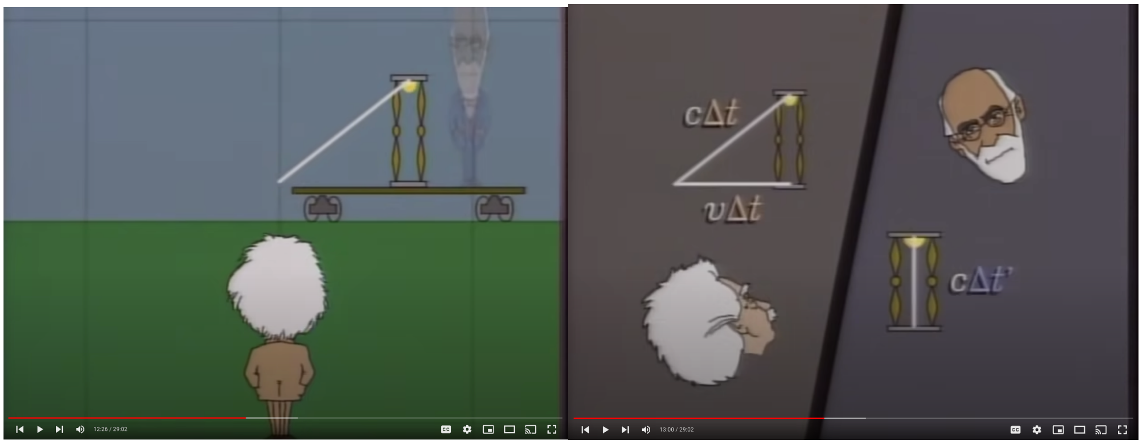

Figure 4: Henry travels to the right relative to Albert, who stands on the ground. Albert sees light travel a longer distance in Henry's light clock than Henry does. Note: A one-way speed of light is shown, but the argument actually holds for a two-way speed of light back to the light source. So, time dilation and length contraction do occur with the same mathematical result.

In Henry's frame, he sees light traverse a vertical up and down path. In Albert's frame, he sees the light in Henry's clock traverse a diagonal path upwards and to the right.

The video then uses the Pythagorean theorem and that the (implicit convention for the one-way) speed of light in all frames is c to show time dilation:
$\Delta t = \gamma \Delta t'$, where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ and $v < c$.

Due to symmetry, Henry would see the same time dilation. So, each frame observes the other to undergo time dilation.

Although the video is using the convention of c for a one-way speed of light, we must note that the video is correct for the calculated results for the two-way speed of light *c*. That is, when the light reflects back to the source, the two-way speed of light back to the source would provide for the same calculated results. (This is why it was suggested that Albert stand next to the light source earlier for calculations done in Henry's frame.)

Key Point 6. Time dilation from relativity is correct because the same results are obtained using only a two-way speed of light *c*, even though a one-way speed of light *c* is used in the video.

Key Point 7. It is important to note here, for later reference, that time dilation is always Δt, a time interval, due to light traveling through a distance. There is no time dilation effect at an instant in time.

The equations for time dilation and length contraction use $v < c$ and γ shown earlier. Here the direction of motion is in the x direction:

$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma(t - vx/c^2)$$

The following text is from the video (starting at 17:29):

> Together, these equations are the Lorentz transformation. They express the mathematical essence of the special theory of relativity. The Lorentz transformation slows time and contracts distances in a moving frame no matter which frame is taken to be moving. The observer in the moving frame thinks he's at rest, and that the other frame is really moving. But these equations do more than that. They actually join time and space together. When an event occurred has no meaning without seeing where it occurred.

With Key Point 6 in mind about the one-way and two-way speed of light, the following text from the video is a little ironic, because we can see aspects of Henry Lorentz and Albert Einstein's positions as both being correct (starting at 25:31).

> For Lorentz, the constant speed of light for all observers was a mere appearance. For Einstein, this constant speed was a principle from which all else should be derived.

Since the one-way speed of light is unknowable (and a convention), then Henry is correct that it is a "mere appearance". However, given the two-way speed of light c, Albert is correct about the calculated results.

Part 4: Revisiting a point.

The first section appeared to provide some clarity on why Henry and Albert would view that Henry saw both moving light detectors triggered at the same time. Albert followed the light as it was reflected back to Henry's eyes. By symmetry, Henry saw the same thing for Albert. We also saw that time dilation could be calculated with the two-way speed of light c emitted from a light clock and reflected back to the source.

However, there was an underlying assumption throughout the previous sections: The light that reached every observer, except possibly one who is at the source, is always a one-way speed of light. Consider that Albert only sees events based on light that reaches his eyes. Albert sees the one-way light travel from the initial flash. At all other times, he only sees the one-way light that travels from Henry to him at varying angles as Henry moves to the right. Possibly the only thing that Albert can know for sure is that the light that travels to his eyes is not changing speed in time, for otherwise a confusing order of events could unfold (Henry could appear to be jumping around in space).

Part 5: Simultaneity.

In this section, we begin with Einstein's convention of c for the one-way speed of light. We return to the "The Mechanical Universe" for the next episode, Episode 43, which also shows Figures 1 and 2. The following text attributes differences in simultaneity to time dilation. The text begins at 6:00:

> Time is relative, [Einstein] said, and a matter of simultaneous events. And in Einstein's relativity, which is reality no matter how it's looked at, simultaneity is a matter of opinion. Notice, for example [Figure 1], what happens when a light is switched on as Henry's train moves by [Figure 1, top left]. The light reaches the front and rear the train at the same time at least that's how Henry sees it [Figure 1, bottom left]. But consider an outsider's point of view [Figure 2]. To Albert, at rest on the ground, the rear wall comes up to meet the light [Figure 2, top right] and the front wall moves away from it [Figure 2, bottom right]. These two events don't look simultaneous at all. And that's the essence of the time problem. And in physics, the explanation of the problem starts with time dilation given the constancy of the speed of light.

The author of the present paper disagrees about the impact of time dilation on "why two events don't look simultaneous at all" for this example. There is, in fact, no "time problem". The explanation for the differences in what is observed here has to do with the travel of light through different distances to reach the eye of an observer.

Time dilation is valid for understanding why a clock in another inertial frame would seem to *run* slowly. Time intervals between two events A and B can pass differently for observers in different inertial frames. So, time dilation is a phenomenon of the *passage* of time, but not an instant of time for an event. This was noted earlier in Key Point 7, "There is no time dilation effect at an instant in time." An instantaneous event occurs at a specific place and time.

In Key Point 6, we saw how time dilation for relativity is due to the two-way speed of light c. However, we will see that simultaneity is not necessarily relative without the convention of c as the one-way speed of light.

Let us consider a scenario from a World Science Festival video, "Relativity of Simultaneity", by Bryan Greene.[5] The video implicitly assumes a one-way speed of light c. At one minute into the video he says:

> "... many people have asked me: Do I have an tuition for these ideas in relativity? And the answer - the most truthful answer is - not really. I can follow the chain of reason, I can do the mathematics, but do I have sort of a

> deep inner intuition in my bones for these ideas? I don't think that I really do, so I don't know if I can get you to that point, but at least I want to show you the chain of reasoning…"

He then goes through an example (stated as being used in his book, “The Elegant Universe”) where the presidents of two different countries are on a train while sitting equidistant to a light bulb in the center, as shown in Figure 5 from the video. The light bulb is turned on (Figure 5a), and the presidents see the light at the same time on the train (Figure 5b). The train is moving to the right relative to a track, and outside observers along the track view events starting from when they are equidistant between the two presidents (Figure 5c) as the train goes by - so the train must be really long. The outside observers view that the “Forwardland” president sees the light before the “Backwardland” president (Figure 5d). Supposedly this was arranged by the UN so that the presidents see the light and sign documents at the same time - and this is all based on the (one-way) speed of light being the same in all reference frames.

Figure 5:
(a) *the presidents' frame where the bulb is turned on at the same time [4:25 in the video],* (b) *the presidents' frame where light reaches the presidents simultaneously [4:29 in the video],* (c) *the outside observers' frame just as the bulb is turned on inside the moving train [5:20 in the video],* (d) *the outside observers' frame where the light reaches "Forwardland" president first [5:41 in the video]*

This scenario is different from the one with Albert and Henry. There is no common observer to follow the light back to, as there was with Henry. What are we to make of this?

Let us digress here. Key Point 4 suggested that, since the one-way speed of light is unknowable, that we cannot even be sure that the presidents would see the light at the same time in their frame. This would be a one-way speed of light that is dependent on direction. And we might expect that in a similar situation of the train going in the opposite direction, the same presidents (maintaining their names in spite of the change of direction) would see the light in exactly the opposite order. That is, result are directionally dependent. However, there is a convention for the one-way speed of light in which *everyone*, including the presidents and outside observers, would agree that events occurred.

Let us refer back to Key Point 3. Just like Einstein, we are free to make a choice for the one-way speed of light. However, instead of making a choice for the one-way speed of light based on direction, we will make a convention in terms of the *velocity of inertial frames*. That is, we will make a choice whereby the one-way speed of light always moves with the velocity of an observed moving frame – such that the two-way speed of light is still c.

Key Point *8*. We are free to take the one-way speed of light, itself, as relative, where the corresponding two-way speed of light is still c. We will consider that the one-way speed of light includes the value of the velocity v of the moving frame. This will be called *alternative* relativity to distinguish it from Einstein's relativity which used a fixed one-way speed of light convention.

In the example with the train moving to the right of the track, observers on the platform will consider that the one-way speed of light moves *with* the train at velocity v. So, the one-way speed of light toward the “Backwardland” president’s eye is $c + v$, and the one-way speed of light towards “Forwardland” president’s eye is $c - v$. Platform observers, and the presidents themselves, would all view that the presidents saw the light at the same time and place - simultaneously. (Note that for the presidents in the moving frame, the velocity of the frame is 0, so their one-way speed of light is just c).

The outside observers would view that the light *reflected* off the “Backwardland” and “Forwardland” presidents faces would have speeds of $c - v$ and $c + v$ respectively. When the observers back-calculate the time it took light to reach them, they will determine that the light hit both presidents faces at the same time (like the example of Albert and Henry here). So, we must do some simple algebra for the frame of the outside observers. We will consider that the observers are between the presidents throughout the observation, they stand next to the track, and that $v < c$. Observers will start measuring time (time = 0) at the event of the light bulb flash.

v is the speed of the train to the right
l is half the train’s length
t_1 is the time for the light from the bult to reach “ForwardLand” president, *F*
t_2 is the time for reflected light from *F* to reach the observers, *O*
t_3 is the time for the light from the bulb to reach “BackwardLand” president, *B*
t_4 is the time for reflected light from *B* to reach *O*

The following calculations are based on the one-way speed of light that moves with the observed frame (so v either adds to or subtracts from c).

The time t_1, from the start until light traveling at the one-way speed $(c - v)$ reaches *F*, can be found from the distance formula:

$$(c - v)t_1 = l - vt_1, \quad \text{so } t_1 = \frac{l}{c}$$

The time t_2, for reflected light traveling at the one-way speed $(c + v)$ from *F* to reach *O*, can be found from the distance formula (considering where *F* is after time t_1)::

$$(c + v)t_2 = l - vt_1, \quad \text{so } t_2 = \frac{l - vt_1}{c+v} = \frac{l(c-v)}{c(c+v)}$$

The time t_3, from the start until light traveling at the one-way speed $(c + v)$ reaches *B*, can be found from the distance formula:

$$(c + v)t_3 = l + vt_3, \quad \text{so } t_3 = \frac{l}{c}$$

The time t_4, for reflected light traveling at the one-way speed $(c - v)$ from *B* to reach *O*, can be found from the distance formula (considering where *B* is after time t_3):

$$(c - v)t_4 = l + vt_3, \quad \text{so } t_4 = \frac{l + vt_3}{c-v} = \frac{l(c+v)}{c(c-v)}$$

It is already clear that the observers *O* would determine that the light hit *F* and *B* at the same time, since $\boldsymbol{t_1 = t_3 = \frac{l}{c}}$. However, when the observers see the light is different, although this can be accounted for by following the light.

The time for the initial flash of light to hit F, and then the time for the light to be reflected back to *O* is:

$$t_1 + t_2 = \frac{l}{c} + \frac{l(c-v)}{c(c+v)}$$

The time for the initial flash of light to hit B, and then the time for the light to be reflected back to *O* is:

$$t_3 + t_4 = \frac{l}{c} + \frac{l(c+v)}{c(c-v)}$$

Since $t_1 + t_2 < t_3 + t_4$, then the observers see that the light reflected off *F* before they see that light reflected off *B*. But they also know that light had to travel further to reach them from *B*.

Of course, the back-calculation accounts for the discrepancy of when they see the light for the same event, which happened simultaneously at time $t_1 = t_3 = \frac{l}{c}$. [See the appendix for this calculation, which obviously must account for the different distances that light must travel to account for the time differences – that is, the same amounts of time are added and subtracted.] So, observers can determine that the light reflected off the presidents at the same time using *alternative relativity* and *following the light*.

There is no experiment that could disprove this relative use of the one-way speed of light, for otherwise, we would be able to measure the one-way speed of light, which is impossible.

Conclusion

In this paper, we consider an *alternative relativity* where the one-way speed of light includes the value of the velocity v of the moving frame (although the two-way speed of light is still c). The choice of the one-way speed of light impacts simultaneity, as summarized next.

Einstein's relativity with a constant one-way speed of light c:

- Simultaneity is relative for inertial observers

Alternate relativity with a relative one-way speed of light:

- Simultaneity is absolute for inertial observers

This alternative relativity cannot be disproved, without impacting the one-way speed of light, which is unknowable. Yet, there are other things that we could consider with this alternative relativity. But that can wait for another time (pun intended).

Appendix:

The following is the back-calculation that accounts for the discrepancy of when observers see the light from each president. This shows how observers would determine that the light actually reflected off the presidents simultaneously – by following the light. As previously mentioned, this is due to adding and subtracting the same amounts of time.

Reflected light from *F* to *O* only had to travel a length of: $l - vt_1$, or $\frac{l(c-v)}{c}$
Reflected light from *B* to *O* had to travel a length of: $l + vt_3$, or $\frac{l(c+v)}{c}$

Since the light from *F* to *O* traveled at speed $(c + v)$ for distance $\frac{l(c-v)}{c}$, then the reflection off *F* took place (using *time* equals *distance* divided by *velocity*) a while ago:

$$\frac{l(c-v)}{c(c+v)}$$

So, observers would calculate that the light they see reflected off of *F* after time $t_1 + t_2$, really reflected off him at:

$$t_1 + t_2 - \frac{l(c-v)}{c(c+v)} = \frac{l}{c} + \frac{l(c-v)}{c(c+v)} - \frac{l(c-v)}{c(c+v)} = \frac{l}{c}$$

Since the light from *B* to *O* traveled at speed $(c - v)$ for distance $\frac{l(c+v)}{c}$, then the reflection off *F* took place (using *time* equals *distance* divided by *velocity*) a while ago:

$$\frac{l(c+v)}{c(c-v)}$$

So, observers would calculate that the light they see reflected off of *B* after time $t_3 + t_4$, really reflected off him at:

$$t_3 + t_4 - \frac{l(c+v)}{c(c-v)} = \frac{l}{c} + \frac{l(c+v)}{c(c-v)} - \frac{l(c+v)}{c(c-v)} = \frac{l}{c}$$

The observers calculate that the reflection occurred off of both presidents as the same time, $\frac{l}{c}$.

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[1] Caltech "Episode 42: The Lorentz Transformation - the Mechanical Universe." YouTube, YouTube, 19 Dec. 2016, https://www.youtube.com/watch?v=feBT0Anpg4A&list=PL8_xPU5epJddRABXqJ5h5G0dk-XGtA5cZ&index=42. Figures used with permission of David Goodstein (via private email on March 1, 2023).

[2] "About the Japan Prize the Grand Prix Winners." *Japan Prize International Contest For Educational Media*, http://www.nhk.or.jp/jp-prize/english/about/grandprix_winners.html.

[3] Caltech "Episode 43: Velocity and Time - the Mechanical Universe." *YouTube*, YouTube, 19 Dec. 2016, https://www.youtube.com/watch?v=BFLUa0ciMjw&list=PPSV.
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[4] Veritasium. "Why No One Has Measured the Speed of Light." *YouTube*, YouTube, 31 Oct. 2020, https://www.youtube.com/watch?v=pTn6Ewhb27k.

[5] World Science Festival. "Your Daily Equation #4: Relativity of Simultaneity." *YouTube*, YouTube, 31 Mar. 2020, https://www.youtube.com/watch?v=558zAduRYMk.