Non-existence of looping in Collatz conjecture integers series

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Abstract

If there is looping in the series of Collatz conjecture integers, the conjecture cannot be realized. Therefore, non-existence of looping should be required for the correctness of the conjecture. Here it is tried to prove that there is no looping in it.

1. Introduction

'All positive odd integers are on any series of NOO from integer 1 if there is no looping or divergence.' is mentioned. (*1, 7. page6.)

About the looping,'There is a looping which is only one member looping or self-looping when n=1.' is mentioned. (*1, 6. page6.)

But the proof should be insufficient about other case than n=1, and additional proof is required.

2. Characteristics of NOO series and nonexistence of looping

NOO series from integer 1 (*1, 2. Page1) can continue to expand NOOs infinitely because there are multiple paths from a NOI to NOOs avoiding NOO which is multiple of 3. (1)

This means NOO series starting from integer 1 can reach to every positive odd integer because it reaches to infinite number of positive odd integers and it has no looping (described as below about (2)).

If there is a NOO which is not on NOO series from integer 1, it can also continue to expand NOOs infinitely same as integer 1.

This means there are at least two infinite number of positive odd integers sets. One is on NOO series from integer 1, another is not on it.

It is contradiction that there are multiple sets which has infinite number of positive odd integers, and no fact support this yet.

NOO series from integer 1 has no looping except integer 1 itself. (2)

The reason of this is;

A series of NOO (3) is considered.

$$n_1 \rightarrow \cdots \rightarrow n_{m-2} \rightarrow n_{m-1} \rightarrow n_m \quad (3)$$

Starting integer n_1 of the series is integer 1.

If (4) is satisfied, it is looping from n_l to n_{m-1}.

$$n_m = n_l \quad 1 \leq l < m \tag{4}$$

Relation $n_{m-1} \to n_m$ and $n_{l-1} \to n_l$ is unique and same. Therefore $n_{m-1} = n_{l-1}$.
This is repeated in turn to $n_{m-(l-1)} = n_{l-(l-1)}$.
Here $n_{l-(l-1)} = n_1$ is integer 1. Therefore, n_{m-l} which is before of n_1 does not exist.
This means that (4) is not satisfied then there is no looping.

3. Conclusion

About looping, we have additional proof here. This should support the conclusion of the report (*1).

4. Consideration

Collatz conjecture operation is doing following formula. (*1, 6. (4) page6.)

$$\frac{3^i n}{2^{m_1+m_2+\cdots+m_i}} + \frac{3^{i-1}}{2^{m_1+m_2+\cdots+m_i}} + \frac{3^{i-2}}{2^{m_2+\cdots+m_i}} + \cdots + \frac{3^1}{2^{m_{i-1}+m_i}} + \frac{3^0}{2^{m_i}} = 1$$

This can be modified.

$$3^i n + 3^{i-1} + 2^{m_1} \cdot 3^{i-2} + \cdots + 2^{m_1+m_2+\cdots+m_{i-2}} \cdot 3^1 + 2^{m_1+m_2+\cdots+m_{i-1}} \cdot 3^0 = 2^{m_1+m_2+\cdots+m_i}.$$

This means
'Regarding to binary representation of integer, mantissa 1 integers (right side) can be expanded as the formula of left side using arbitrary odd integer n.
Therefore, Collatz conjecture operation is doing this expansion process.'

*1 https://vixra.org/abs/2302.0015