

# Kolmogorov spaces which are injective

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## Abstract

Let  $K_0, K_1$  be Kolmogorov spaces, and there be an exact morphism  $K_0 \rightarrow K_1$ . Assuming  $K_0$  to be a “good moduli stack” gives us an exact injection into the category of hyperspace tilings. Here, we explore this link. We supply a healthy dose of background information to get the reader acquainted with the relevant topics in a lightning-round fashion. In the process we touch on automorphisms of spectral sequences.

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## Chapters

1  Compass homomorphisms	Pg. 1
2  Locally ringed spaces	Pg. 4
3  Degenerate displays	Pg. 6
4  SSets	Pg. 9
5  K-maps	Pg. 13

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## Chapter 1 | Compass homomorphisms

By a polynomial, we mean a *monic polynomial with constant term  $\pm 1$* .<sup>1</sup> Let  $f$  be “totally free,” up to a retract  $f(p^{-1})$ . Let  $f: j^* \rightarrow E$  be a rooted immersion at a stable point. Here, we will be interested in discussing *formal power series*, as an abstraction of polynomials. We start with a *complex space*; that is, an analytic stack over  $\mathbb{C}\mathbb{P}$  which is an  $n$ -fold covering of some building.

Let  $\text{fib}(\sigma) \rightarrow \text{cofib}(\delta)$  be a lifting of compasses. Then, there is some totally free root of unity which gives us a specific tension point along a suspended string with compass  $\Omega_\sigma^\delta \simeq \tilde{\Delta}$ . Following [Fib], pg. 4, when there is a relation  $\delta \prec \text{Stab}_{\mathbb{C}}$ , we will suppress the smallness of  $\delta$  by suppressing the phrase “stably almost” to refer to the somewhat awkward relationship between the tightly embedded subspace and the loose parent space.

**Proposition 1.0.1** Let  $\mathbf{A}$  and  $\mathbf{B}$  be two compasses. Then  $\text{fib}(\mathbf{A}) \simeq_p \mathbf{B}$  is a  $p$ -fold covering of the inertially weighted space of  $\mathbf{A}$ .

**Proof** See [Fib], proposition 2.3.

Let  $\mathbf{b}$  be the diagonal of a matrix over  $\mathbf{B}/p$ . Then, by the standard decomposition on objects, one obtains

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<sup>1</sup> See, [Aut].

$$\mathbf{b}(\mathbf{B}/\mathfrak{p})^{-1}=\mathbf{1},$$

which is a root of unity for a compass. We also have the chief stratum:

$$\mathbf{b}(\mathbf{B}/\mathfrak{p})^{-1} \circ \mathbf{b}(-1(\mathbf{B}/\mathfrak{p}))^n = \mathbf{B}^n/\mathfrak{p}_k$$

which is effectively a fusion rule (on objects) of a *braided* symmetric monoidal bi-category.

When we write a compass,  $\Omega_a^n$ , we call  $a$  the “inf-pole” and  $n$  the “sup-pole;”  $a$  corresponds to a positive root of unity, and  $n$  a complex root. The most direct path  $a \rightarrow n$  defines a spectral sequence (of flags),

$$a \subset \dots \subset n,$$

where each character in the chain is the successor  $a^{+k}$  on  $a$ . We single out this specific path as the “Countryman line,”  $\mathcal{W}$ , and it is the shortest possible walk from a once-compactified dipole to another.

Keep in mind, a once-compactified dipole has two poles,  $\{+,-\}$ , which double as identity actions, where every  $+$  is a section and every  $-$  is a retraction. We can write, somewhat dismissively,  $\Omega_{-}^{+}$  for the “barest bones” version of a compass, which maps signs to their retracts. Expressively, this gives us  $\mathbb{B}^n$ , the Boolean algebra on  $n$  characters. However, the interesting behavior of applying a compass arises when we *equip*  $\mathbb{B}^n$  with some *special structure*,

$$\mathbb{B}^n \oplus \mathbf{S},$$

which is essentially topological. We set the addition to be *pointwise*, and so there is an auto-equivalence  $\mathbf{S} \simeq \mathbb{B}^n \times \mathbf{k}\mathbb{B}^n$ ;  $\mathbf{k} > 0$ .

**Definition 1.0.2** A *partial flag variety*,  $\mathbf{V}$ , is a collection of types  $\alpha:\beta:\dots:\Omega$ , which are successively embedded into one another. A *flag variety*,  $\bar{\mathbf{V}}$ , is some partial flag variety which is closed.

**Proposition 1.0.3** Pointwise addition (of  $\mathbf{S}$ ) creates  $S^n$  partial flag varieties.

**Proof** Let  $\mathbb{B}^n \oplus_n \mathbf{S}$  be an  $n$ -small pointwise addition. Then, for every  $n$ , there is a unique diagonal  $\text{diag}(\mathbb{B}^n \oplus \mathbf{S})$  so that each form a unique partial flag variety.

**Definition 1.1.1** A *homomorphism* (of compasses),  $\Omega_x \rightarrow \Omega_y$ , is an injective immersion of schemes such that the ind-object of every point in  $x$  has the same rank as the corresponding point in  $y$ . We write this typically as a pair of injections,  $[X \rightrightarrows Y]$ .

**Example** Fix a point,  $d$ , on a manifold  $M$ . Deform the manifold via a power-set function  $f(d) \rightarrow M$ . Say that  $M$  is  $(2k-2)$  dimensional. Then, we have the maps

$$\{2,k\} \rightrightarrows (2)^{\mathfrak{p}}$$

for each of  $2,k$ . We let  $\mathfrak{p}=2\pi k i$ , and it is a distinct geometric invariant in the category of topological spaces with lenses.

**Example** Let  $\mathbf{V} \subseteq \bar{\mathbf{V}}$ , and  $\mathfrak{F}$  be an overring of  $\mathbf{V}$ . There is always a map

$$\text{proj}(\mathfrak{F}) \rightarrow \mathbf{V}' \subseteq \mathbf{V}$$

into a *subtypical chamber* (subspace) of a given type  $\mathbf{T}$ . This map is actually a Hurewicz isomorphism [Ap], and in addition is a homomorphism of compasses with inf-poles at most  $\mathbf{V}'$ .

**Example** Let  $\mathbb{C} = (\mathbb{R} + \mathfrak{F}) \simeq \mathbb{R}^2$  be a homomorphism of compasses. The supremum, which is a diagonalization of the *unit* differential  $\hat{\epsilon}$ , is  $\hat{\epsilon}^2$ . Thus, we have  $\Omega_{-\infty}^{\hat{\epsilon}^2}$  as the compass for our “good moduli space” of  $\mathbb{C}\mathbb{P}^1$ .

## Chapter 2 | Locally ringed spaces

Let  $\mathbf{R}_N$  be a ring. Let  $R_{N_{ext}}$  be its extension. Letting  $\mathbf{R}_{[p]}$  be a “good moduli space” for these settings; we develop the flag variety

$$\mathbf{R}_{[p]}: \mathbf{R}_N \subset R_{N_{ext}} = \{\phi_Y | Y = \{p, N\}\}$$

In a certain sense, this imitates the function of a *weakly chained space* by supplying co-cycles with “boundary conditions,” such as  $p$  and  $N$ , which are cohomological invariants. If, for every such  $\mathbf{R}_N$  there exists at least one  $R_{N_{ext}}' \subset R_{N_{ext}}$ , then we say that  $\mathbf{R}_N$  is *Prüffer-extensible*.

We have that  $\mathbf{R}_{[p]}$  and each Prüffer extensible ring is an Artin stack.  $\mathbf{R}_{[p]}$  with a simplicial realization are automatically a Deligne-mumford stack. We have:

$$\mathbf{LocRng} \subset \mathbf{LocSys}(\tilde{\Delta})$$

So in some sense, restricting ourselves to locally ringed spaces may give us more “control” over the modularity of the site in question. If  $Y$  consists of all locally coherent terms, then  $\mathbf{B}_Y$ , the “display block” of the ring space section being considered, is automatically coherent.

For two “good moduli spaces” with finite products, and all finite intersections,  $MM_2$ , they are left indecomposable. Thus, the metric torsor  $\mathbf{t}_k \in M \times M_i$  is totally free, but not in general guaranteed to be regular. For every *tensor equation*, we have a set of *fundamental facts*  $M \times M_i$  about which the spectrum of the theory is based. Of course, if we wish to work *topologically*, we cannot simply think of a “theory” as a purely modelable structure. Instead, we think of a “theory” as a certain sort of topos, with refinements being made rationally about certain generative loci. So, we transfer from  $\mathbf{t}_k$  to the *cardinal invariant*  $2\kappa+i$  by the *sharp* morphism  $Q$ .

$$Q = \mathbf{LocSys}(\tilde{\Delta})^{-1} = \mathbf{t}_k^\#$$

Here, we consider  $\tilde{\Delta}$  as a *sitewise deformation* of  $\mathbb{B}^\omega$ . So, if  $\mathbb{B}^\omega$  is locally presentable as a category of *commutative rings*, then  $\mathbb{B}^\omega$  is a *genuine isomorphism* of displays  $\phi_\omega \mathbb{B} \simeq \tilde{\Delta}_\phi$ .

### 2.1 Relative Functors

Let  $\mathbf{R}_N, \mathbf{R}_M$  be two distinct, locally perfect fields. If there is a *split epimorphism* onto a trivial co-character  $\xi$ , then we say that  $\mathbf{R}_N \xrightarrow{\xi} \xi \leftarrow \mathbf{R}_M$  is a *Chow-trivialization* of said fields. The Chow-trivialization preserves a certain distinct invariant, polarity,  $p$ , as well as the generic property of analyticity. If two functors,  $p \rightarrow, \leftarrow q$ , are related by a Chow-trivialization, then we say that their pullback onto a space of equal genus is *cohomologically Cauchy*.

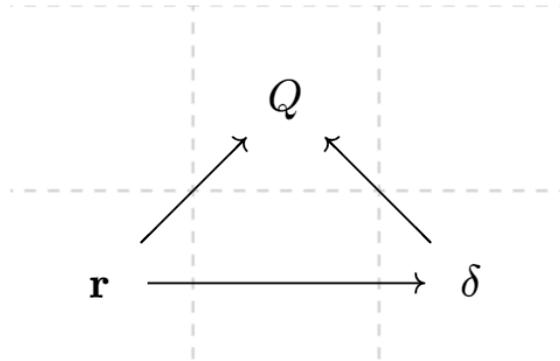
Cohomologically Cauchy pullbacks are also called “relative functors,” see e.g. [DK]. Let  $\mathbf{r}$  be a relative functor, and let  $\mathbf{K}_0, \mathbf{K}_1$  be Kolmogorov spaces. If and only if, for every  $n$ -cell in  $\mathbf{K}_0$  there is a bijection in  $\mathbf{K}_1$ , and the intersection of  $\mathbf{K}_0$  and  $\mathbf{K}_1$  is well inside  $\mathbf{K}_1$ , we say that  $\mathbf{K}_0$  is *relatively inside*

$\mathbf{K}_1$ . For two objects A and B, with A relatively inside B, then, from the “point of view” of the good moduli space, A is locally flat whence it is relatively stable. We denote a transition from the finer space to the larger space as  $\mathbf{r}: \mathbf{K}_m \rightarrow \mathbf{K}_n$ . This encloses the interior of a (rank n) space in a rank  $>n$  space of equal or lower codimension.

If two locally ringed spaces are connected by a relative functor, then they admit a homothetic scaling via  $\pi$ -weighted proportional re-representation. In other words, if two spaces look different on the outside, they may always be repackaged on the inside to look the same so long as there is the relative functor mediating the two! This is highly motivating for us, as it encourages us to explore new ring spectra which are *forced* by the inclusion of a smaller component within the larger.

**Warning 2.1.1** We encourage the reader to look for differences between the *sharp morphism*  $Q$  and the relative morphism  $\mathbf{r}$ . They are not to be confused. We note that  $Q$  is more special than  $\mathbf{r}$ , which is more generic.

We have the normal cone



and  $\delta$  is the stabilizer of  $\mathbf{r}$  at a point in  $Q$ .

**Remark 2.2.2** The restriction  $\delta \rightarrow Q$  is an opf-map from the stabilizer to its co-domain. We denote by  $\delta^{\text{opfib}}$  the relevant op-fibration in  $\mathbf{r}'$ . We have  $\delta^{\text{opfib}} \Vdash \mathbf{r}'_\infty$  as a coherent forcing notion, assuming the source and target of the forcing notion have equivariant display blocks.

### Chapter 3 | Degenerate displays

Let  $\delta^{\text{opfib}} \rightarrow \mathbf{A}$  be an *obstruction* to *lifting* the display block about a fixed algebraic object  $w$  to a transcendental ordinal  $\omega$ . What can we say about the existence of such an obstruction? Firstly, it is important to realize that not all of these obstructions will be of “one type.” Certainly, there are many varied causes for such a failure to lift outside of the locally ringed space of an action.

In any scenario, these cases comprise the majority of the objects in the category of locally ringed spaces. We will shy away from making explicit propositions regarding them, but will describe in some details their interesting properties. We see, by the formula

$$\mathbb{B}^{-n}(\delta) = \delta_{ij}^{-1}$$

that they are more prevalent in the coarser sites. Here, we abuse notation by letting  $\mathbb{B}$  be the radius of an open ball, with the Boolean metric, in a space. We call the “twist”

$$\delta_{ij}^{-1} \otimes \delta_{ij}^{1+\tau}$$

the “adic brake” for the analytic co-cycle of a ringed space with a dimension  $\tau$  for flow. The associated half twists act by an according action on the upper half-plane, which is confusingly labeled  $\mathbb{H}^2$ . So we have

$$\mathbb{I}=\{i,j\} \propto \tau \mathbb{H}^2$$

describing the cyclotomic action of the ideal miniscule co-weights over a perfect space. We have now enucleated one of the most visceral properties of degeneracy – *potency*. We call a matrix potent if it corresponds to either  $i$  or  $j$ .

So, for two sets of mutually orthogonal copies of  $\mathbb{P}^n$ , if we have a time evolution

$$\tau^n(\mathbb{P}^n) \rightarrow \mathbb{M}_{\text{ST}}$$

to a mapping stack, the density of characters in  $\mathbb{M}_{\text{ST}}$  is still determined by the density of co-characters in  $\mathbb{P}^n$ ! Assuming that this is a standard monomorphism, then the resulting co-characters, in the *mapping stack* cover of a space, are *totally free*, as there is a torsion-free inclusion at the level of  $\delta$ -objects.

**Proposition 3.0.1**  $\sum_{ij=i}^j \mathbb{I}=\Omega_N \rightarrow \Omega^M$

**Proof** If we have two potent half-twists, then we have a map into both halves of an interval. This corresponds from a walk from an inf-pole to a sup-pole, where  $M > N$ .

**Definition 3.0.2** We call two characters *co-degenerate* if  $\sum_{ij=i}^j \mathbb{I} = \sum_{ij=i}^j \mathbb{I}'$

**Remark** The time-evolution of co-degenerate kernels is identical, assuming a real locus is preserved.

If we have two display blocks,  $\mathbf{B}_i, \mathbf{B}_i^!$ , and a homomorphism  $\mathbf{B}_i \rightarrow \mathbf{B}_i^!$ , then we have at least a *single* fixed display  $\phi_n$ , which is a relative functor. So, if

$$\mathbf{B}!SO(p-1) \times_n \phi_n \times \mathbf{B}!SO(p)$$

is co-degenerate with  $\mathbf{B}_!$ , then there is a partial flag variety whose diagonal consists of scalar multiples of  $n$ .

Recall, [Fat], that “any acyclic simplicial sets [sic] is a filtered colimit of finitely presentable acyclic simplicial sets.” Write  $\mathcal{X}^\square$  if a simplicial set  $\mathcal{X}$  has the “right lifting property”<sup>2</sup> and  $\square \mathcal{X}$  if it has the left-lifting property. For any homomorphism of degenerate displays, we have a lifting  $CH_0^\square \mathcal{X}$  or  $CH_0 \mathcal{X}^\square$  which has the other as an adjoint. If  $\mathbf{D}_m$  and  $\mathbf{D}_n$  are co-degenerate displays, then there is a *presented finite cell complex*<sup>3</sup> from the inner hom-set  $\text{Hom}(\mathcal{X}, \mathcal{X}^\heartsuit)$  to the left (resp. *right*) lifting of a display  $\mathbf{D}_n^\pm$ .

**Proposition 3.1.0** Finite cell complexes correspond to partial flag varieties.

**Proof** We see clearly that there is a lifting  $\text{Hom}(\mathcal{X}, \mathcal{X}^\heartsuit) \vdash \mathbf{D}_n^\pm$ . We model this as an inclusion of a compass,  $\Omega_{\mathcal{X}}^D$ , and a map  $\mathcal{X} \rightarrow \mathbf{D}$  from the inf-pole of a fat object to the sup-pole of its parent category. That this is a partial flag variety follows by replacing the relational symbol “ $\rightarrow$ ” with the notation  $:\dots:$ , so that it reads  $\mathcal{X}:\dots:\mathbf{D}$ .

In this case, the flag variety is a tower of scalars that act via the type-inclusion relation to induce homothety. Here, we are interested in working with chains of *types*, or perhaps *categories*; we have

$$\mathbf{Cat}_1 \subset \dots \subset \mathbf{S}\mathbf{Sets},$$

so that  $\mathbf{S}\mathbf{Sets}$  is *co-fibered over* by the globalization of objects in  $\mathbf{Cat}_1$ . We let the truncated portion be a *transitive inner model* whose op-fibrations are injections to locally ringed spaces. To this extent, we obtain *charts*, which are *transition fibers* with pullbacks onto concrete objects. Each of these objects have the *left* (resp. *right*) lifting property whence there is a right (resp. *left*) earthquake  $\tilde{\mathcal{E}}$  from a chart which is locally a copy of  $\mathbb{P}^1$  to a chart which is locally a smash product of two copies  $\mathbb{P}^1 \times \mathbb{P}^1$ .

We render this information, by writing, as a *block of displays*

$$(\mathbb{P}^1 \times \mathbb{P}^1 \rightarrow \mathbb{B}) \simeq \mathbf{B}_{\mathbb{P}}$$

which “promotes” a finite cell complex to the level of an *affine*, locally ringed space. This is essentially an “extension of a single object” along a *bundle gerbe*.<sup>4</sup> We let the cokernel of  $\mathbf{B}_{\mathbb{P}}$  be cohomologically affine, and such that its weak minimal extension is an object in the inner hom-space of a neighborhood locally resembling itself.

We have

$$\text{Hom}(\mathbf{B}_{\mathbb{P}}, \mathbf{B}_{\mathbb{P}}) = \mathbf{Map}(\mathbb{P}^1 \times \mathbb{P}^1, \mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{B}) \simeq \mathbf{LocRng}(\mathbb{A});$$

The following commutative diagram represents the shtuka:

<sup>2</sup> Loc cit. Pg. 2

<sup>3</sup> Ibid, pg. 4

<sup>4</sup> See, [ap.], sec. 1.2

$$\begin{array}{ccc}
 \mathbb{P}^1 \times \mathbb{P}^1 & \longrightarrow & \mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{B} \\
 \downarrow & \nearrow f & \downarrow \\
 \mathbb{P}^1 \times \mathbb{P}^1 & \longrightarrow & \mathbf{LocRng}(\mathbb{A})
 \end{array}$$

where  $f$  is a sharp lift from the total space  $\mathbb{P}^1 \times \mathbb{P}^1$ .

**Warning** The space  $\mathbb{P}^1$  is *not* an analytic space!

## Chapter 4 | S Sets

One of the most important categories for working with a discrete, or more loosely, any quasi-separated continuum, is S Sets, the category of simplicial sets whose objects are vertices and whose morphisms are edges. An object in S Sets with a correspondence to a transcendental ordinal has the property that it is (locally) the sup-pole of some compass on the category Sets. We have used such correspondences previously in our account of the “adic brake” of a ring.<sup>5</sup>

Say we have two topological objects,  $P_1$  and  $P_2$ , and  $P_2$  is a suspension of the first. Then, for our purposes, we can treat the strata of  $P_2$  as the *codimension* for our model. On some level, this makes sense for us, because we are accustomed to working with spaces that are partitioned into flat representations of  $\mathbb{R}^n$ . We can actually treat two local copies of  $\mathbb{R}^n$  as being the *Dehn twist* of one another. If there are two copies of an object,  $P$ , which are mutually orthogonal, we can always adjoin a third object  $P_3$  at the inf-pole to create a new mutually orthogonal vector.

$$\{P_n \wedge \mathbb{A}^m | m < n\}$$

We restrict ourselves to Cauchy injections  $P_n \rightarrow P_m$ , with uniformizer  $m_{ij}$ . In the category of local rings, one defines the system

$$\mathbf{LocRng}|_{\mathbf{LocSys}} = \tilde{\Delta} \in \mathbf{SSets}$$

We have

$$\mathbf{Pull}(\tilde{\Delta}) = \Omega_{m_{ij}} \sum_{ij=i}^j \mathbb{I} =^{\mathbf{HUR}} \mathcal{T}_{\mathbf{Nis}}(\mathbf{LocSys}(\mathbf{Quiv}), \mathbb{G}^{\#})$$

so that any inf-pole at a locally based section of  $\tilde{\Delta}$  is “glued to” an associated apartment in S Sets. We let the generative factor of  $P_n$  be such that any Hurewicz isomorphism  $\frac{1}{\xi} \tilde{\Delta} \rightarrow \mathbf{g}P_n$ , there is a unique uniformizer  $n_x \in \mathbf{N}$ . Lastly, the Hurewicz isomorphism to  $\mathbb{G}^{\#}$  is given by considering *a certain portion of Quiv* which looks locally like the category of simplicial sets. This gives us a special kind of **g**-small “portable” Countryman line which forms a diagonal in each display over the geometric realization of  $\frac{1}{\xi} \tilde{\Delta}$ .

It should be remarked that there are a fair number of ways of treating **SSets** like a combinatorial category; for instance, by letting it “locally imitate” the Giriy monad or the Kleisli category by opfibered inclusion. The vertices of S Sets may also be thought of as indices (Roman letters)  $i, j$ , etc., and the edges may be thought of as capital Greek letters,  $\Pi, P, \Sigma$ , etc. In this respect, the *relational* aspects are privileged over the object-level instances of the graph. For instance, one has  $\Pi_{ij}$  corresponding to a certain location in the incidence matrix of the graph. In a hypergraph, there are

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<sup>5</sup> Ibid, pg. 5

$$2^{ord(\tilde{\Delta})}$$

ways of combining the infinitesimal slices of a smooth curve to form second-order branches. Here,  $ord(\tilde{\Delta})$  denotes the *filtered* number of vertices in the first-level order on  $\tilde{\Delta}$ . For a third level branch, one has:

$$2^{2^{ord(\tilde{\Delta})}} \geq 3^{ord(\tilde{\Delta})},$$

and follows the typical rules for Boolean exponentiation to achieve higher and higher power cycles.

For a Kolmogorov space, one assigns a coordinate to each outcome of the power cycle in order to determine the number of unique singleton neighborhoods, and by Hurewicz we map these neighborhoods to edges on the boundary of a space. Increasing this number acts via the Weierstrass function to increase local smoothness, but also “spikiness,” as in Hausdorff dimension.

**Proposition 4.1.0** Increasing the Hausdorff dimension of a space sharply regularizes its adic brake.

**Proof** As we see, the fractal dimension is directly related to the number of closed neighborhoods of a space. The “spikier” it becomes, the “sharper” the jump from the adic brake to the surrounding separated set becomes.

We model the jump  $\frac{1}{\mathbb{B}^n} \rightarrow m > n$  to  $\Omega_- \rightarrow \Omega^+$  as the *packing number* of edges at the boundary, which corresponds to the number of curves in a complexified field. This is essentially a “tilt” of perfectoid fields

$$\left(\frac{1}{\mathbb{B}^n}\right)^b \mapsto \Omega_-^+,$$

with the right hand side isotonicly directed upwards. This is a map

$$\mathbf{SSets}^b \subset \mathbf{SSets} \rightarrow \mathbf{SSets}$$

which takes the étale component of a geometry and maps it via projection to its “less bounded”<sup>6</sup> counterpart. In the image, we frequently end up at the Zariski site. This is why we are so often interested in the “Zariski density” of a set, but little do we hear of an “étale density.”

## 4.2 The Main Diagonal

In a weighted combinatorial set, or a hyper-regular graph, we write the main diagonal as:

$$2^{\dots 2^{ord(\tilde{\Delta})}} ;$$

and, we are interested in the singular value,  $k$ , which is the count of twos from the bottom-left to the top-right of the equation. If a space with a diagonal of  $k$  has a property  $P$ , then it shall be called  $k$ - $P$ -able. For instance, if the property of left division is  $P$ , then the space has the property of  $k$ -left-divisibility. If it is descent, then we say that it is  $k$ -descendable. Etc.

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<sup>6</sup> Or, more loosely restricted; laced

When we are working with the fractal dimension of a ruled space, we are interested in the *adic* decomposition of the right-hand side of the irrational dimension number. For the number of digits considered, it will be called k-digital.

The main drawback to this approach is a matter of computational constraint; for instance, we must satisfy ourselves with a very small sample size of digits. Another drawback is that this may be used to produce k-adic analysis, while the k-adic case is simply inferior to the p-adic case. In spite of this, we can still take k-adic samples, and to some degree of success, use them to compare two types of spaces. We see that for the “zero spaces”, for example, there is no difference between a space “0” and a space “000”, so the k-value of a zero-space is irrelevant. They both belong to the first class,  $\text{ord}(\tilde{\Delta}) = \mathbf{id}_{\text{ord}}$  under their reduction to the absorption kernel, zero.

**Theorem 4.2.0.** The equivalence class of any number of zeros is k-equivalent to 0(1).

**Proof** As we say, for non-unary number fields, every sequence of k zeros is equivalent to any other sequence of zeros in a sample of k digits. That is to say, if a sample is taken, and all its digits consist in zeros, then the number of such does not impinge upon the realization of the sequence.

This gives us

$$(0(1) \Leftarrow 0(n) + \{\emptyset\}) = 0 + \{\emptyset\}$$

**Proposition 4.2.1** Let A be a k-adic sequence of zeroes, and A+1 be a k+1-adic sequence of zeros and a one. Regardless of the place of the digit 1, the sequence is larger in cardinality than  $0 + \{\emptyset\}$ .

**Proof** The above proposition gives us the vocabulary  $0, \{\emptyset\}, 1$ , which is larger than  $0, \{\emptyset\}$ .

**Proposition 4.2.2** A k+1-digital sequence is not necessarily smaller than a k+2-digital sequence.

**Proof** This depends upon where we insert the k+2nd digit. If it is to the left (resp. right), the sequence will be larger (resp. smaller) in the non-*adic* case.

This is significant, because the main diagonal truncates potentially *lots* of data! Because of the shortcut used for observing the first and the last digits of a k-series, one may lose lots of information about the digits in the middle of the series. So, we have  $A < A+1 \leq A+2 \leq \dots \leq A+n$ ; all we know is that  $A < A+n$ . Here, k is equal to n+1, so that the entire sequence is a k-sequence.

There is a *lot of potential* information included in the sequence A+1 through A+n, but if none is known, it will be defaulted to a forced homotopy with the zero-set. If the number of non-zero elements is known, then a random distribution of non-zero elements will be injected into the forgetful space. Thus,

**Proposition 4.3.0** the map

$$\mathbf{SSets}^b \subset \mathbf{SSets} \rightarrow \mathbf{SSets}$$

is forgetful.

**Proof** Trivially follows. The map is an *injection* but is not *surjective* on **SSets**. Thus, it is not a categorical isomorphism.

**Proposition 4.3.1** There is some set  $\mathbf{SSETS}^b$  in the target of the above morphism, which locally resembles  $\mathbf{SSETS}^b$ .

**Proof** If a set of morphisms are injective on objects, then one can always construct the disjoint union of those objects as a separate set which the map is one-to-one and onto. This locally resembles the first at the loop level, assuming the source is quasi-separated.

**Proposition 4.3.2**  $*$   $\rightarrow \mathbf{SSETS}^b$  is  $*$ -absorbing.

**Proof** Assume that  $\mathbf{SSETS}^b = \inf(\mathbf{SSETS})$ . Then, assuming we have a complete lattice,  $\inf(\mathbf{SSETS})=0$  is a surjective source for every object  $*$ , it is unital, and absorbing.

## Chapter 5 | $\mathbf{K}$ -maps

A  $K$ -map  $[\mathbf{K} \rightrightarrows \mathbf{K}]$  is a continuous pair of morphisms  $\alpha, \beta$ , from a  $\mathbf{T}_{\geq 0}$ -space to a  $\mathbf{T}_0$ -space. For our purposes, these are both considered *sober spaces*. Without loss of generality, let each morphism be injective (resp. surjective); then, there is an op-fibration  $\mathbf{K}^{-1} \rightarrow \mathbf{K}$  which is surjective (resp. injective). When  $\mathbf{K}$  is equipped with a *compass* structure (and is hence, *compassionate*), we say that it is tensored in the category  $\mathbf{KComp}$  of Kolmogorov spaces with compasses.

We have the identity

$$(\mathbf{K} \rightrightarrows \mathbf{K} \times \mathbf{K}) \simeq \text{Hom}(\mathbf{KComp}, \Omega) \simeq \mathbb{M}_{\text{ST}}(\mathbf{K}, \mathbf{K}^{-1})$$

on fibered objects of Kolmogorov spaces, so that  $\mathbf{K}$  is both the *source* of a Hurewicz fibration and is also a *good moduli* space for its opfibration.

**Warning 5.0.0**  $\mathbf{K}$ , oftentimes, is not Zariski. However, the product  $\mathbf{K} \times \mathbf{K}$  induces a Zariski density on objects of the mapping stack. This is because

$$\text{QCoh}(\hat{\epsilon}) \twoheadrightarrow \hat{\epsilon}^n$$

is fibered over inductive colimits. So, for an  $\epsilon$ -chain stratifying the diagonal of a space, its  $n$ -dimensional representation as a convolved<sup>7</sup> product is precisely the structure sheaf of the main diagonal of  $\mathbf{K} \times \mathbf{K}$ .

**Reminder 5.0.1**  $\mathbb{M}_{\text{ST}}(\mathbf{K}, \mathbf{K}^{-1})$  is a differentiable stack. This means that we can associate a vector bundle  $\mathcal{V}$  at the level of the site to induce a metric. Kähler and Weil-Petersson metrics<sup>8</sup>, in specific, are interesting to work with for producing good, locally trivial spaces. We can tensor  $\mathbb{M}_{\text{ST}}(\pm \mathcal{V})_{\mu}$  to obtain a supersymmetric orbifold, which is homotopic to the quaternions. We get our transition maps for free by matching each object in  $\mathbf{K}^{-1}$  to an object in  $\mathbf{K}$ , and lifting the correspondence to the level of sets.

We can always reverse engineer  $\hat{\epsilon}^n$  so that it is the convolution of a set of torsors  $\epsilon_{\mathcal{X}}^1, \epsilon_{\mathcal{X}}^2, \dots, \epsilon_{\mathcal{X}}^{\xi}$ , for example, by the formula:

$$\epsilon_{\mathcal{X}}^1 \partial_0 \otimes \epsilon_{\mathcal{X}}^2 \partial_1, \dots, \otimes \epsilon_{\mathcal{X}}^{\xi} \partial_{\mathcal{X}}^i = \text{LocSys}(\mathbf{t}),$$

and we couple to truth values over time

$$\text{LocSys}(\mathbf{t}) \leftrightarrow \tau_n \tau_{n-1} = d \mapsto \frac{d}{dt} = A_{\text{Eff}}$$

### 5.1 Effective action

For a Manifold  $M$  with a flat metric  $p$ , the effective action  $A_{\text{Eff}}$  is the average Hamiltonian of an ensemble of bi-products

$$d_i \theta_{\mu} = d \times d/dt \rightarrow d \star p$$

---

<sup>7</sup> Twisted

<sup>8</sup> See [WP]

In the above equation,  $\theta$  is a K-map of normal strength, and  $\mu$  is a measure of the time-decaying potential of  $A_{\text{Eff}}$ .

$$\begin{aligned} A_{\text{Eff}} \mathbf{Mod}_{\mathbf{R}}(\tilde{\theta}) &= \mu^{-1}(\text{d}/\text{dt}) \\ &= \mathbf{M}_{\mathbf{M}} \xrightarrow{k} \mathbf{M}^{\text{N}} \lambda_{\text{cur}} \Omega_{-}^{+} \end{aligned}$$

The monomorphisms are *k-split* over the product of the current flow and the induced sign metric of the compass. We rewrite this by saying:

$$\mathbf{M}^k(\mathbb{B}) = \{\{0\} + \emptyset\} \rightarrow \mathbf{1}$$

So that there are  $k$ -many ways of decomposing a manifold  $\mathbf{M}$  with a **Boolean** base to obtain the probability measure  $\mu=1$ .

For two manifolds,  $\mathbf{M}, \mathbf{M}'$ , with one Hermitian, the other Kähler, we obtain the Mochizuki link  $\theta: \mathbf{M}_{\pm} \xleftrightarrow{\pm} \mathbf{M}'$  from the étale picture to the Frobenius display. If, for a fibration  $f: \mathbf{M} \rightarrow \mathbf{M}'$ , there is an opf-map  $f \rightarrow f'$ , and if this map is injective, then the *twisted action* on  $\Sigma \mathbf{M}$  is  $\mathcal{L}$ -stable. If the map is  $\mathcal{L}$ -stable, and  $\mathbf{M}$  and  $\mathbf{M}'$  are perfectly flat and with a common metric, we call the action *perfectly trivial*. For two twisted, perfectly trivial maps with time-varying potential, it is possible to obtain (assuming both maps to be monotonic), a point in  $\mathbf{M}$  which is in the image of  $\mathbf{M}$  at a time  $t_k \mathbf{M} = \mathbf{M}'$ . The nonstandard Mochizuki link acts as a shortcut between these two points; it is a time-forgetful functor (t.f.f.). This is a canonical extension of the *universe-forgetful functors*  $\mathbf{U}(2^i) \rightarrow \mathbf{U}(-)$ , which surrenders information about the large quiver of a system of diagrams, at the upshot of accessibility to miniscule universe-dependent parameters. Technically, t.f.f.s are also *sign-forgetful functors*, as the absolute value is adopted to create portable metrics for scaling a model between positive and negatively oriented reference frames.

**Proposition 5.1.1** Hurewicz isomorphisms “lift”  $\theta_{\mu}$  from the class of “universe-forgetful functors” to the set of genuine isomorphisms.

**Proof** Say we have two universes  $\mathbf{U}(-)$ , and  $\mathbf{U}(\Omega_{\text{inf}(X+Y)}^{\text{sup}(X+Y)})$ , and there is a  $k$ -rational map between them.

We denote, by equivalence class,

$$S_1 \geq \frac{1}{2} \text{sup}(X+Y); S_0 < \frac{1}{2} \text{sup}(X+Y)$$

and we have the fibrations  $0 \rightarrow 1, 0 \rightarrow 0, 1 \rightarrow 1$ , and the op-fibration  $1 \rightarrow 0$ . We denote by  $\pm \mathbf{S}$  the set of *fibrations* (+), and *opfibration* (-). We have

$$\mu^{\pm} = \text{opfib}(\theta_{\mu})^{\pm};$$

if  $\mu^{\pm} \leftrightarrow (-)$  is a valid pairing, and if it is Serre, then it is bijective on opens. Thus, there is *at least one* bijection  $\mu^{\pm} \leftrightarrow \bullet \in \Omega_{\text{inf}(X+Y)}^{\text{sup}(X+Y)}$  from a compact unit to a compass of dilations. So,

$$\text{fib}(\theta_{\mu}) \rightarrow \text{opfib}(\theta_{\mu}) \simeq 1/k \rightarrow \mathbf{1}$$

Let  $\mathcal{K}$  be a  $\mathbf{K}$ -map. Let  $\varphi$  be the fibration of a topological space  $T$ . Then, assuming  $T$  to be sober, there is a unique indecomposable term  $\chi$  which bounds every  $\theta_\mu(\epsilon^{\wedge n})$ , where  $n$  is the dimension of the space. This forces an equivalence

$$\mathcal{K} \simeq \xi \epsilon^{\wedge n} \rightarrow \Phi_\xi$$

where  $\xi$  is equal to a sum of fibrations in  $\varphi$ . So, we have

$$\begin{array}{c} \mathcal{K} \simeq \sum_{\varphi=0}^n I_\varphi \epsilon^{\wedge n} \rightarrow \Phi_{\sum_{\varphi=0}^n I_\varphi} \\ \downarrow \text{HUR} \\ \mathbb{Z}/p \cup \{\infty\} \end{array}$$

yielding the correct isogeny of displays induced by  $\mathcal{K}$ . We let  $\mathbb{Z}/p \cup \{\infty\}$  denote the attaching space of a  $k$ -rational map which splits at  $\mathcal{K}$ . So, for a mapping stack  $\mathbb{M}_{\mathbf{ST}}$ , we let

$$\mathbb{M}_{\mathbf{ST}} \cup \mathbb{Z}/p \cup \{\infty\}$$

be an  $\infty$ -groupoid and we record the trace of a kernel of  $\mathbb{Z}/p$  as the Gromov-Witten invariant.

## 5.2 Yetter-Drinfeld Category

Let  $\mathcal{YD}$  be the Yetter-Drinfeld category. Let  $\mathbf{K}_1, \mathbf{K}_2$  be complete Kolmogorov spaces. Let<sup>9</sup>

$$(\mathbf{K}_1 \otimes \mathbf{K}_2^{-1} \otimes \mathbf{K}_1^{-1} \otimes \mathbf{K}_2) \mapsto_{\mathbf{H}} {}^{\mathbf{H}}\mathcal{YD} \simeq \mathbf{K}_{\text{cent}}$$

be the fusion rule on  $\mathcal{YD}$  assigning to each space  $\mathbf{K}_{\text{cent}}$  a nucleus. This is a more modern reformulation of the classical Hopf fibration<sup>10</sup>

$$\begin{array}{c} \mathbf{S}^7 \leftrightarrow \mathbf{S}^3 \\ \downarrow \\ \mathbf{S}^4 \end{array}$$

By assignment,  $\mathbf{K}^\pm$ , we have two end twists,  $\mathbf{K}_1, \mathbf{K}_2$ , and two half-twists at the center,  $\mathbf{K}_2^{-1}$  and  $\mathbf{K}_1^{-1}$ . Operationally, the whole ensemble yields a non-trivial two-fold covering of an *effectively lensed* Kolmogorov space, and – assuming that space to be sober, a restriction is made to  $\mathbf{S}^4$  so that it is indecomposable into tori. This preserves the genus of the cokernel maps from  ${}^{\mathbf{H}}\mathcal{YD}$ . The spinorial component of the Yetter-Drinfeld category is compactified from a genus  $2g+2$  space to a space with two closed points with a Baxter bundle over them.

This is achieved by performing the Dehn twist on  $\mathbf{S}^3$  in the projective preimage. More or less, this is what Weinstein means with his “Shiab operator.” We take the inequality

$$\mathbf{S}^n \neq \mathbf{S}^{n+1}/2$$

<sup>9</sup> See, [YD]

<sup>10</sup> [YM], pg. 9

as our starting point for coupling the pseudo-Riemannian geometry of a twistor to its mass term. We call this the Yukawa coupling if it is localized to a scale with ineffective or only weakly effective symmetry breaking.

Let  $H$  be a Hopf algebroid in some vect-enriched category. Let  $r$  be a weighted bi-module. By the formula

$$H(r)^{-1} \mapsto \mathfrak{h},$$

we obtain a genuine eigenalgebra  $\mathfrak{h}$  with commutative ends, making  $\mathfrak{h}$  monoidal and symmetrically closed. For our purposes, we want to describe  $\text{rep}(\mathfrak{h})$  as a *tiled space*, which gives it an isometry to the Anabelian Teichmuller space. This is a very free and general starting space, but we restrict to a set of mutually orthogonal vectors along a hyper-bundle so we obtain a “nicer” *simplicial space*,  $\mathcal{C}(S)$ .

What does a “twist” correspond to, simplicially?

Well, we can imagine that edges between vertices are labeled  $\Pi_{ij}$  where the Greek letter indicates a relationship and the subscripts denote two objects under said relationship. Interestingly for our (toy) purposes, we can let  $ij$  be a superposition, or perhaps even an entanglement. Anyways, on the *graph*  $G(\Pi_{ij})$ , we can write a series of half-twists, such as  $i^{-1} \otimes j^{-1}$  to represent swapping the poles in a partial flag variety, and we have a perfect correspondence between operations on these co-characters and maps between graphs. We write the composition of two functions to a negative term as a *twist*  $\mathbf{T} \in \mathbb{R}\mathbb{P}^1$ . Two path-connected spaces will still be path-connected after the twist, although their orientation may become reversed, which is the case with the spinorial particle’s behavior.

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