

# Newton's Theory of Universal Gravitation – A Critique Leading to A Revised Version

Bijon Kumar Sen\* and Subha Sen\*

Department of Chemistry, University of Calcutta, Kolkata, INDIA

## Abstract:

A close look into the Newton's theory of gravitational attraction reveals some controversial statements. The theory does not corroborate with the experimental observations on the properties of gravitational constant. An '*external push*' is suggested instead of the well accepted theory of '*attractive pull*' between two masses. This new concept is able to resolve the inconsistencies and drawbacks present in Newton's theory. An alternative approach is proposed for the theory of evolution of the Universe (Big Bang), that can account for the lacunas noticed in the existing theory.

*Keywords: Newton's theory of gravitation, Galileo's Experiments, Proposal of gravitational force as gravitational push, Terrestrial and cosmic gravitational forces, Super-heated and super-cooled metastable states, Alternative of Big Bang theory*

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\*Retired, Address for correspondence DD - 114, Street no. 269, Action Area I, Newtown, Kolkata 700156, INDIA, E-mail: [bk\\_sen@yahoo.com](mailto:bk_sen@yahoo.com)

## **Introduction:**

Few theories are universally accepted as infallible at the very moment of inception as that happened to the theory of universal gravitation propounded by Sir Isaac Newton in 1687 [1]. The theory resulted from a mathematical justification of the three famous laws of Kepler which qualitatively describe the motion of the planets. From the derivation, Newton concluded that “Every particle of matter in the universe attracts every other particle with a force proportional to the product of the masses of the particles and inversely proportional to the square of the distance between them”.

The theory was tested by physicists, astronomers, cosmologists for the last ~350 years and has been greatly modified by Einstein with the help of his famous General Theory of Relativity. Curiously enough neither Newton nor Einstein or for that matter any scientist has so far been able to answer the question of the nature of Gravitational Force in an unambiguous way. In analogy with nuclear force (involving  $\pi$ -mesons), Coulombic force (electrons), magnetic force (magnon), gravitational force has been thought to be derived from (graviton). This idea where gravitational field (in analogy with electrical or magnetic field for which line of forces could be experimentally proved) gives rise to hypothetical “graviton” could not be experimentally supported.

## **Forces:**

Newton identified gravitational force as an attractive interaction between two bodies depending on their masses and the distance separating them. Present day physics recognizes four types of forces operating in the universe: -

- i) The gravitational force which is independent of electrical charge and is the weakest of all forces. The force constant  $G$  is the Universal Constant. This acts mainly in planetary and stellar scale.

- ii) The mean nuclear force, which is important in proton-neutron interaction and is exhibited in beta neutrino emission.
- iii) The Coulombic or electromagnetic force which includes all atomic and molecular phenomena. It depends wholly on the charges of the bodies and may be attractive or repulsive depending on the similar or opposite value (+ve or -ve) of the charges.
- iv) Strong nuclear force involving nucleon-nucleon interaction does not depend on the charge of the nucleon. It is always active and operates within the dimension of the nucleus ( $10^{-13}$  cm). It does not persist outside the nucleus and considered not to affect molecule formation.

Relative magnitudes of these four forces are in the ratio of  $10^{-40}$  (gravitation)  $< 10^{-24}$  –  $10^{-13}$  (nucleon-nucleon force)  $< 10^{-2}$  (Coulombic)  $< 1$  (nuclear force).

***Attractive interaction:***

According to quantum mechanical principle, if there is possibility of an exchange of some common property between two particles, then an attractive force results with lowering of energy of the system. The common property for exchange may be either charge, or spin or position.

Heisenberg [2-4] proposed the exchange of electrons, protons and neutrons to explain the attractive force between nucleons but the idea was found to be inadequate to predict the high bonding force. It was later replaced by the exchange of pions ( $\pi^+$ ,  $\pi$  or  $\pi^-$ ) according to the prediction of Yukawa [5]. The  $\pi$ - mesons are massive particles with mass 273 times that of an electron and the doubt of such a mass transfer was overcome by Wick's application [6] of Heisenberg uncertainty principle. Bartlett [7] proposed exchange of spin of the particle with charge and position remaining unaltered to produce an attractive force between nucleons. Majorana [8] similarly employed exchange of position and Wigner [9-10] proposed that no property is exchanged.

These quantum mechanical exchange interactions are adequate in explaining the formation of nucleons, atoms, molecules and other interactions resulting from the polarizability effect (London and van der Waal's forces).

A logical question thus arises about how the force involving macroscopic particles (gravitational force) could be explained in this way as there is no perceptible exchange of masses and the theoretical impossibility of the application of Heisenberg's Uncertainty Principle. The suggestion of a gravitational quanta (designated as "Graviton") that is being exchanged does not seem conclusive even if one accepts the presence of 'matter waves' of de Broglie.

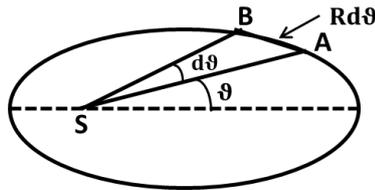
It is to be noted that Coulson [11] did not accept the exchange force as a phenomenon. If the formation of nucleons, atoms, molecules on one hand and plasma, solids, liquids and gases on the other be explained as arising from exchange of suitable particles, the force of gravitation cannot be explained by a similar concept as there is little possibility of mass exchange between macroscopic objects. Much work was done by Einstein and other scientists to work out a sophisticated mathematical theory which will unify all the known forces of nature. But in spite of all attempts of formulating a unified field theory, the gravitational field could not be accommodated there properly although other interactions like electrical and magnetic interactions could be combined together.

This leads to a query as to the similarity of nature of interactive forces which describes the nuclear, electric and magnetic interactions with that in gravitational force. It, therefore, seems necessary to examine the deduction of the gravitational force by Newton from the laws of Kepler which describes the elliptical revolution of planets and satellites around the Sun.

### **Kepler's Laws:**

Early in the 17<sup>th</sup> century, the German astronomer J. Kepler formulated the following three laws for orbiting of the planets and satellites around the Sun: -

- 1) Each planet revolves around the Sun in an elliptical orbit, with the Sun at one focus of the ellipse.
- 2) The speed of a planet in its orbit varies in such a way that the radius connecting the planet and the Sun sweeps over equal area in equal times.
- 3) The square of the periods of any two planets are in the same ratio as the cubes of their average distances from the Sun.



**Fig. I: Radial acceleration of a planet (A) around the Sun (S)**

### Newtonian deductions from Kepler's Laws:

Let 'A' be the position of the planet at a given instant of time 't', revolving in an elliptical path around the Sun, S, at one of the foci. After a small interval of time dt, 'A' moves to 'B' (Fig. I)

The area swept by the radius vector SA at this instant 'dt' is the area of the triangle SAB which is equal to  $\frac{1}{2} SA \cdot AB = \frac{1}{2} R \cdot R d\theta$  where SA = R and AB = R dθ

$$\text{So areal Velocity} = \frac{1}{2} R^2 \frac{d\theta}{dt}$$

According to Kepler's second law, this areal velocity must be a constant (= h/2)

$$\text{Therefore, areal velocity} = R^2 \frac{d\theta}{dt} = h = \text{constant.}$$

Since the planet moves in a curved path and constantly changes its direction, according to Newton, it must be under the action of a force and must consequently be possessing an acceleration in the direction of the force.

Resolving this acceleration into two rectangular components along and perpendicular to the radius vector, Newton obtained from simple dynamical consideration,

- 1) Component  $a_1$  along the radius vector i.e., radial acceleration of the planet is given by

$$a_1 = d^2R/dt^2 = R(d\theta/dt)^2$$

- 2) Component  $a_2$ , at right angle to the radius vector i.e., the transverse acceleration of the planet is given by

$$a_2 = 1/R \frac{d}{dt}[R^2(d\theta/dt)]$$

but since  $R^2(d\theta/dt) = h = \text{Constant}$

$$a_2 = 1/R \frac{d}{dt}(h) = 0$$

It follows, therefore, that the planet has no transverse acceleration and thus there is no force perpendicular to the radius vector. [Newton possibly did not notice the implication of this result. That  $m_1$  and  $m_2$  may be regarded as two masses which are attracted towards the Sun by the radial component of the force only. Due to the absence of a transverse force, the masses  $m_1$  and  $m_2$  will have no attractive or repulsive force between them in direct contradiction of Newton's theory of gravitation].

However, it is needless to repeat the calculation of Newton. From condition  $a_1$  above i.e., radial component of acceleration towards the Sun, from which following three results could be derived successfully.

- 1) The only force acting on the planet is towards the Sun.
- 2) The force acting on the planet is inversely proportional to the square of the distance from the Sun.

$$\text{i.e., } a_1 \propto -1/R^2$$

Newton interpreted the negative sign as an indication that the force is attractive. [But it is imperative that the sign only indicates the direction of the vectorial force and has

nothing to do with either attractive pull or external push. It only indicates the effect, not the cause].

- 3) The force of 'attraction' between the planet and the Sun is directly proportional to the product of their masses.

### **G, universal gravitation constant:**

From these results, Newton presented his theory of universal gravitation which led to the equation

$F = G \times M \times M' / R^2$  which is very similar to the formula for attraction between electric charges and magnetic poles. The gravitational constant G has a value of  $6.67 \times 10^{-11} \text{ Nm}^2\text{Kg}^2$ .

G, is the universal gravitational force constant. It is known that determination of the value of G under different experimental conditions exhibits a few characteristic properties which distinguish gravitation from analogous electrical and magnetic counterparts.

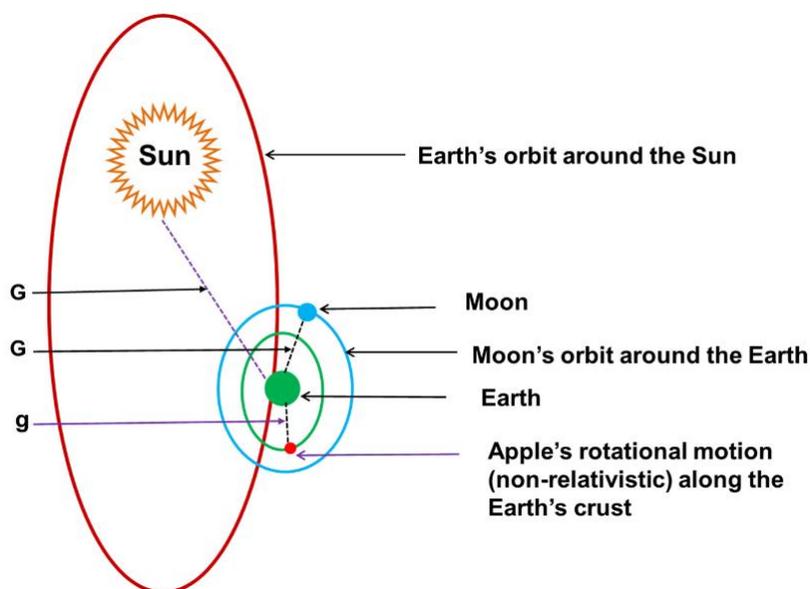
1. The value of G is independent of the intervening medium separating the two masses. Be it vacuum in extra-terrestrial space or air in terrestrial space or placing any substance as barriers with diverse dielectric properties in between, the interacting masses do not affect the value of G. This permeability therefore, leads to the conclusion that no effect is produced in gravitational 'attraction' between the masses by the nature of the medium interposed in between them. This leads one to doubt if there is any *attractive pull* between masses at all.
2. Gravitational forces are by no means a selective phenomenon and depend only on the magnitude of the masses and not on their composition. The nature of the masses, their chemical composition or even radioactivity do not have any effect on G.
3. Gravitational forces, unlike anisotropic properties like refractive index, heat and electrical conductivity do not depend on the orientation of crystallographic axes and, therefore, free from any directivity.

4.  $G$  does not show any appreciable dependence on difference of temperature.

**g, acceleration due to gravity:**

All the above properties of  $G$  are more or less applicable to 'g'. We find that no attempt has been so far made to explain their peculiar behaviours.

The motion of falling bodies is the subject of study from ancient times. Aristotle ascertained that heavy objects fall faster than light objects in proportion to their weights. Galileo, however showed that the motion of a falling body is nearly independent of its weight. He made his legendary experiment from the leaning tower of Pisa [12]. He dropped cannon balls and bullets which reached the earth at the same time. It has been found that, in absence of air resistance, all bodies whatever be its weight fall with the same acceleration at the same point of the earth irrespective of its distance from the earth which is ordinarily negligible in comparison to earth's radius. This is called the acceleration due to gravity, denoted by 'g'. The value of 'g' near earth surface is  $980 \text{ cm/sec}^2$ , on moon the value is  $1.67 \text{ m/sec}^2$  and on sun its value is  $274 \text{ m/sec}^2$  (Fig. II).



**Fig. II: Relativistic motion of Sun, Earth, Moon and non-relativistic motion of an Apple (not to scale)**

This means that in case of terrestrial motion where (Example: earth and apple) there is no revolution of a mass with respect to one another, Newton's law is directly contradicted. This is due to the fact that Newton derived the law from Kepler's planetary (Cosmic) motion. But most serious is the fact that this raises doubt about the existence of attractive pull between masses. An attractive pull must depend on the distance between the masses and on the nature of the intervening medium. It is only possible if the attractive pull can be replaced by an externally imposed force for conglomeration.

Cavendish's method of determination of 'G' has been taken as the proof for the "attraction" between two masses. But it was not considered that the same effect of relative displacement of balls in Cavendish's experiment is equally possible by a push imposed from outside.

**G and g:** G is a universal constant while 'g' is a conditional constant. The value of 'g' depends on the constancy of latitude and longitude on the planet (earth) and significant values of mass and distance in comparison to the mass and radius of the earth. G and 'g' both represent conglomerative forces tending to coalescence of masses. While G is weak and does not result in the process of condensation, 'g' is stronger and leads to a condensation.

### **Celestial interaction vs Terrestrial interaction:**

Celestial interaction involves relative revolutionary motions of the planets resulting in a centripetal force which does not lead to conglomeration and the virtual centrifugal force keeps the masses away.  $F = G M M' / R^2$ .

In terrestrial interaction there is no relative revolutionary motion. The force 'g' does not depend on distance appreciably but depends on mass (contrary to Galileo's experiment). As there is negligible centrifugal force from the planets, the mass 'm' is directed towards the centre of the earth.  $W = F = m g$

## Gravitation:

Gravitation is the conglomerating force caused by the revolution of masses.

- a) If the masses are comparable and R is also high, celestial motion results (example: Sun and Earth).
- b) If the masses are not comparable, the lower mass tends to move towards the latter one (example: Apple to Earth).
- c) If the masses are comparable but 'r' is small, gravitation force may not be appreciable (example: in interstellar space known as Lagrange's point, where the gravitational force of the Sun and the Earth cancel each other out, the mass is 'weightless' as 'g' is equal to zero, the masses do not conglomerate as is seen by the movement of the astronauts in the space. There is no visible attractive or repulsive force between them). This is true for any free object in space contrary to Newton's law.

Newton considered the rotation of moon around the earth and calculated its rotational (centripetal) acceleration towards the earth as  $2.74 \times 10^{-3} \text{ m/s}^2$ . An apple on earth which is subjected to a gravitational force 'g' will be accelerated to  $981 \times (60.2)^{-2} = 2.7 \times 10^{-3} \text{ m/s}^2$  in proportion to the ratio of the distance of the apple from moon and earth [13]. Thus, Newton found a similarity between the two accelerations. On the basis of the assumption that 'g' on earth is very much similar to G of moon inferred that the same type of interaction is operative in both the cases. Newton assumed that 'g' of a static (not in relative motion) system may be identical with G of a dynamic (in relative revolutionary motion) system. The apple is in same rotational motion as the earth with same velocity and same direction whereas the moon rotates with its own characteristic angular velocity around the earth.

It is imperative that Force on a dynamic system  $F_{dy} = G M. m /R^2$  cannot be compared with the force on a static system  $F_{st} = g m$  (where m is = mass of the apple). Both  $F_{dy}$  and  $F_{st}$

are conglomeration forces but they cannot be equated as such, but on combination yield a relation  $g/G = M / R^2$  which is tentative with limited applicability.

That forces G and 'g' are not 'attractive' in nature as described in the following section.

## **Origin of gravitational force**

### **Evolution of the universe**

What happened when the universe was born some billions of years ago is impossible to be known to anybody. Astronomers, cosmologists, physicists and nuclear astrophysicists all converge on the Big Bang theory [14-15] which proposes that at the time of spatio-temporal singularity, a super dense mass ( $10^{63}$  gm/cc) exploded to form masses of different weights and dimensions which occupied the Universe. From the analogies of actual occurrence, we find that when an incendiary explodes, its splinters are ejected in all directions at straight lines. If Big Bang theory is to be accepted, the components of the Universe will drive through space in straight lines and would have been lost in vast expanse of space and the existence of the present Universe can never be explained. It is absolutely necessary that the ejected particles must have revolutionary or rotational motion so as to check their outgoing tendency. This makes possible the existence of stars and planets rotating around each other making the Universe stable. Big Bang explosion is thermodynamically allowed as the entropy increases accompanying the explosion.

### **Alternative approach**

Before the start of the Universe, the vast expanse of the space was filled with energy (although one does not know wherefrom the energy originated). This energy was in the form of heated plasma (fourth state of matter different from solid, liquid or gas). Drastic condition of temperature, pressure etc., can only be conjectured. The equilibrium condition of the plasma was disturbed by quantum mechanical tunnel effect whereby a highly energetic metastable energy state was produced accompanied by violent rotational and revolutionary motion.

(Analogy: when a cyclonic storm develops into a hurricane, the cloud particles rotate and revolve around the “eye” of the storm and proceed with a revolutionary motion in whatsoever direction possible governed by the existing meteorological condition). At the point of spatio-temporal singularity, a thermodynamically irreversible implosion of energy started with decrease of entropy.

(Analogy: at the time of cloudburst, the water particles of the cloud become heavily charged and are in metastable condition. Suddenly, accompanied by lowering of temperature the clouds begin to condense into ice particles of different sizes in the form of hailstorm and comes down on earth. The process is thermodynamically irreversible with decrease of entropy, but ultimately the ice particles become liquid and then to a vapour with gain of entropy).

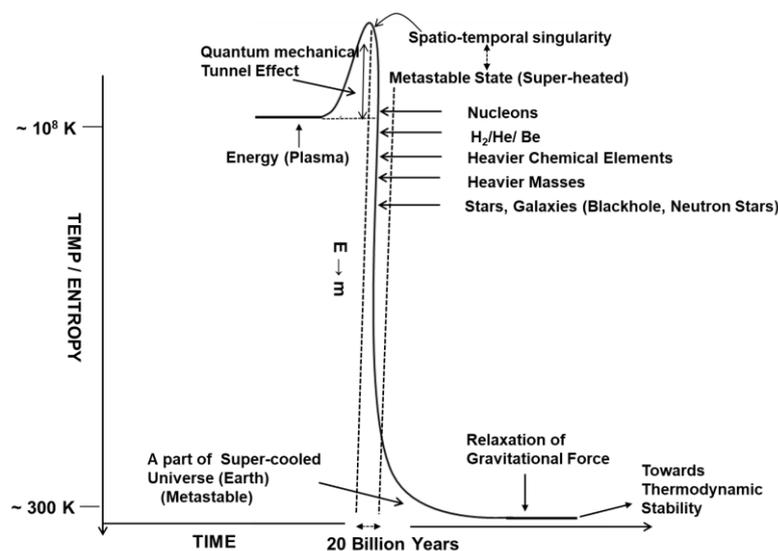
The implosion of energy results in the production of mass according to the famous mass – energy conversion formula of Einstein ( $E = mc^2$ ) but the formation of mass is accompanied by a revolutionary and rotational motional of the plasma. The condensation process forms masses of all types starting from quarks, nucleons, atoms, molecules, masses, stars, galaxies etc., depending on time and the prevailing conditions. The first and foremost entity to be produced in the Universe was the nucleons of hydrogen, proton, deuteron, Paulion [16] and triton. Paulions are the correct species for production of an alpha particle and are also responsible for the formation of nuclei of elements through Bose-Einstein condensation. These nuclei are bonded strongly by  $\pi$  - mesons exchange which arises from the interchange of the protons and neutrons. These exchange forces are the strongest possible force in the Universe.

The nuclei of all elements combine with electrons to produce different atoms. The electrons revolve round the nuclei which spin along their own axes thereby giving rise to spin – orbit coupling forces.

Atoms then combine whenever possible to form molecules; the molecules chemically combine to form compounds of masses of different structures.

The revolution and spinning of electrons produce electrical and magnetic forces. Thus, the binding energy of molecules results from Coulombic or electromagnetic interactions.

This process of coalescence is continuing even today which is the driving force of gravitation (Fig: III).



**Fig. III: Variation of Entropy with Time during evolution of the Universe showing Origin of Gravitational Force (Not to scale)**

The time elapsed is some 20 billion years for this irreversible process to occur and we are at present in a metastable super-cooled state of the Universe (earth). The process of condensation (gravitation) will proceed for billions of years onwards till a stable state is attained or the heliocentric Universe will end with the burning out of the Sun. This super-cooled state assisted the formation of virus, bacteria, enzymes, life process and vegetation on earth. Exobiologists have been searching for evidence of extra-terrestrial life for years. They have not yet found such evidence to claim with certainty that undisputed biological activity exists on any planet other than earth [17].

Transformation from super-heated metastable state to a super-cooled metastable state took place on account of the heat liberated dissipating to a limitless space which acts as a sink. The kinetics of the unidirectional process, spanning a time period of ~20 billion years and

involving innumerable condensation processes producing all types of matter from nucleons to galaxies is beyond the comprehension of human being.

Thus, we find in gravity, free particle which are not in relative rotational motion tends to condense with earth (apple falling from trees). The revolutionary motion of particles in an orbit (planets) are prevented from condensation/coalescence by the Gravitational centrifugal force. Gravitation is, therefore, an external centripetal force of implosion of masses.

With all due respect to the genius, Sir Isacc Newton, the revised theory of universal gravitation is to be considered as follows:

- a) *All masses of the universe are being pushed by forces from outside tending to their coalescence.*
- b) *In case of terrestrial objects where there is no relative revolutionary motion (no centrifugal force), the force of condensation is dependent on masses but independent of the distance from the centre of the earth ( $W = g m$ ).*
- c) *In case of cosmic objects, where masses are in relative revolutionary motion, the external force of condensation (gravitation) manifests itself as centripetal force that depends directly on the product of their masses and inversely as the square of distances between them (Newton's law:  $F = GMM'/R^2$ ). This is the binding force of the universal planetary system. If this force was not present, the universe could not have stability and would have been lost in the limitless space.*
- d) *Objects of masses which are not in relative revolutionary motion may be interacted by a planet through centripetal force but they will exert no interaction among themselves due to the absence of any transverse interaction (Example: motion of astronauts in space).*

### **Concluding Remarks:**

In conclusion, we feel that the notion of gravitational force should be replaced by an external push in lieu of an inter-mass pull. This will explain all the characteristic properties of

G and g. Newton did not distinguish between Celestial Centripetal force  $F = GMM'/R^2$  from Terrestrial gravitational force (conglomeration)  $F = g m$ . He tacitly assumed that the forces are attractive and originates from same source and found that the magnitude of attraction is comparable by considering only the distance but omitting the consideration of their masses.

The former (celestial force) originates from relative rotational motion while the latter (terrestrial force) is a result of non-relative linear motion. Vectorially both the forces are directed towards the heavier mass { earth towards sun; apple towards earth }. But any two bodies which are free from any relative revolutionary motion do not affect one another either by attraction or by repulsion.

Thus, Newton's formula will not be applicable to the falling bodies (apple) and the downward acceleration of a free-falling body (gravity, g) but will depend on the distance from the centre of the earth. This will not be a constant considering the non-spherical nature of the earth with polar and equatorial distance being different.

In 1915 Einstein proposed his General Theory of Relativity with the idea that gravitation is an effect of curvature of space-time. Results of Einstein's theory differ slightly from those of Newton's theory and predicted exact motion of Mercury [18] which could not be derived from Newton's theory.

Predictions from Einstein's theory of General Theory of Relativity that rotating masses emit Gravitational waves was indirectly confirmed by rapidly rotating neutron stars (binary pulsar). The observation of shift in wave length of light from distant stars (indicative of an expanding Universe) are taken to be the justification of general theory of relativity. But this theory is actually silent on the nature and origin of gravitational forces.

It is to be noted that attempts are made to 'modify' Newton's 'Law' of gravitation with the help of the effects of generalized uncertainty principle (GUP). Some approaches are made

through quantum gravity such as string theory, black hole physics and doubly special relativity theories (DSR) on the area law of entropy [19].

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