NON CONSERVATIVE GRAVITY MODEL

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ABSTRACT. I will explore a non-conservative gravity field model with base idea rooted in trying to re-think free-falling observers.

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1. Motivation

Motivation behind this idea is that there could be alternative way of describing gravity field. Idea behind it goes back to definition of inertial frame of reference. If gravity field causes movement is observer in gravity field inertial, answer is yes from General Relativity. Reasoning behind it is that observer moves in shortest possible path in spacetime. But this observer is accelerating so let for a sake of argument assume that this observer frame of reference is true, what it observers is how things happen.

Observer , when not taking into account tidal forces will state that it is motionless and it is the gravity source that is in motion. On the other side gravity field source observes itself motionless and will observe that observer is the one who is in motion. Those two perspectives seem to contradict each other. My reasoning is that in truth they are not in contradiction to each other. Both gravity source and object that is under it's action will observe that they are stationary and the other one is motion. So what if they are really both stationary?

It would be a new view of gravity field, where each observer is stationary with respect to himself in any gravity field. It means that motion in gravity field never happens from point of view of any given reference frame, it's always the other frames that are in motion. Only change is that in truth all observers are not in motion. It means that there is something that makes any observer stationary from his point of view and for each observer so reference frame it holds true. And im not talking about transforming reference frames so each observer is stationary from it's point of view, rather im talking about physical facts about motion of this objects.

In this case being inertial means that object always stays stationary with relative to itself is not a reference frame point of view but it's a universal view in gravity field. It means that gravity field should behave always way that it's effect on any reference frame does not change it's motion. What does change is how spacetime behaves around it. But to build a model that agrees with this idea, there is need to change fundamental ideas about gravity field. Gravity field will be no longer a contracting force but rather expanding as it needs to make objects that move in all directions counteract that expansion not by contraction but by making spacetime behave exactly same way as object motion does. It means that gravity field will always behave exactly as motion of object does by so making objects form it's point of view always stationary.

2. Free-fall revisited

Free-fall is basic of gravity , object in free-fall not only does observers itself as stationary if we ignore tidal forces so make object point like, there is no force acting on that object. From falling object point of view it's gravity source that is expanding in all directions. I will assume that it's perspective as free-falling observer is correct. What does it mean to spacetime model?

First space is expanding with earth movement, second earth does move in all directions with close to constant speed in all directions and third closer to earth observer is faster it's expanding. This at seems may seem as matter of perspective not a true picture of spacetime, but I will assume the opposite as what observer "sees" is light signals send from gravity source it's point of view about how spacetime behaves is true. How then earth can be expanding in all directions and stay at constant radius at same time?

Space has to expand exactly same as earth does so one does counteract the another and radius does not change from someone measuring it on earth. it's opposite view of gravity field than one that comes from General Relativity. Here motion and spacetime are behaving exactly same allowing all observers to be stationary from their point's of view.

So when observer free-falls in gravity field it's space around it that it's expanding with earth expanding with exactly same rate. It means that observer is stationary with relative to earth and earth stationary with relative to that observer as from earth perspective it's object that is expanding and earth is stationary. So both perspective will say same fact about space around it, it's expanding in all directions so they observe same physical reality from two perspectives.

But why object does fall onto earth eventually? If space is expanding around earth with earth more distance there is between earth on falling object slower earth is expanding as it depends on earth's mass and energy. But from fact that space is expanding and closer object gets to earth faster it's expanding it means that space further away from earth is expanding slower and closer to it faster so eventually that faster expanding space with earth will reach falling object and expand it with itself.

3. First guess of field equation

Next step is to create a mathematical model that supports this idea. Simplest guess I can arrive at is that Ricci tensor is equal to negative energy momentum tensor. But there are problems with this equation. First off both side of equation contradict each other as one is conserved and another one is not:

$$R_{\mu\nu} = -\kappa T_{\mu\nu} \tag{3.1}$$

$$\nabla^\mu R_{\mu\nu} = \frac{1}{2} g_{\mu\nu} \nabla^\mu R \tag{3.2}$$

$$-\kappa \nabla^\mu T_{\mu\nu} = 0 \tag{3.3}$$

So I need to create a energy momentum tensor that is not conserved in general sense. So both side of field equation will be not conserved. But that creates a general problem if matter field is not preserved, how does conservation of energy works? Let's go back to free-fall example, if there is earth expanding in all directions and it's caused by matter field expanding in all directions that is caused by earths movement (earth pressure), what kind of matter field causes more far away space to expand? And here is an answer, matter field has to be in all points of space that explains why matter field is not conserved in general sense. I will denote this kind of non conservative energy momentum tensor as $\tilde{T}_{\mu\nu}$ to not confuse it with normal energy momentum tensor. It's easy to check that for a dust that tensor does not vanish at any point of space as it's density never gets to zero. Same with it's conservation, it does not vanish when I take partial derivative as it's derivative is non zero for all space. It's only one example of such tensor but general rule follows it has to have non vanishing matter field. And from it follows that it's covariant derivative is non zero. So first guess of field equation makes more sense now if I use this kind of energy momentum tensor:

$$R_{\mu\nu} = -\kappa \tilde{T}_{\mu\nu} \tag{3.4}$$

But it still lacks need that for empty spacetime I will get Minkowski space. Before I arrive at second guess for how field equation could encode this idea, let's write formally that non vanishing energy momentum tensor has this properties:

$$\nabla^\mu \tilde{T}_{\mu\nu} = -\frac{1}{2} g_{\mu\nu} \nabla^\mu R = \frac{1}{2} g_{\mu\nu} \nabla^\mu \tilde{T} \tag{3.5}$$

So to sum it up non vanishing energy momentum tensor has value everywhere non equal to zero, that represents matter field that in general sense is not conserved as it's energy will be defined not as a point vanishing source but rather a field of matter that is in whole space. It comes from need that spacetime movement and matter movement are exactly same.

4. Second guess of field equation

To make field equation work I need to not use Ricci tensor but instead Riemann tensor. But problem is that non vanishing energy momentum tensor is second order tensor and Riemann tensor is four order tensor. To solve this problem I need to create a new tensor from non vanishing energy momentum tensor that is order four tensor. This tensor has to be have same contractions as Riemann tensor, and has to vanish when Riemann tensor vanishes. I can denote this tensor as $\tilde{T}_{\alpha\mu\beta\nu}$, so I will arrive at field equations that state:

$$R_{\alpha\mu\beta\nu} = -\kappa \tilde{T}_{\alpha\mu\beta\nu} \tag{4.1}$$

Both side of equation are not conserved. And equation states clearly that if there is no matter field I will arrive at flat spacetime:

$$R_{\alpha\mu\beta\nu} = 0 \tag{4.2}$$

It's still very complicated equation and what is left is to figure out right side of equation so generalized non conservative energy momentum tensor. Where last assumption is true only if there is matter present, so if energy momentum tensor is non zero. Now I will write all Riemann tensor contractions that are needed to create non vanishing energy momentum tensor contractions, from fact that both side of equation have to be equal:

$$g^{\alpha\mu} R_{\alpha\mu\beta\nu} = 0 \tag{4.3}$$

$$g^{\alpha\beta} R_{\alpha\mu\beta\nu} = R_{\mu\nu} \tag{4.4}$$

$$g^{\alpha\nu} R_{\alpha\mu\beta\nu} = -R_{\mu\beta} \tag{4.5}$$

$$g^{\mu\beta} R_{\alpha\mu\beta\nu} = -R_{\alpha\nu} \tag{4.6}$$

$$g^{\mu\nu} R_{\alpha\mu\beta\nu} = R_{\alpha\beta} \tag{4.7}$$

$$g^{\beta\nu} R_{\alpha\mu\beta\nu} = 0 \tag{4.8}$$

From it I can start construction of non vanishing generalized energy momentum tensor. Where it will contract same way but it will give non vanishing energy momentum tensor instead of Ricci tensor.

5. Form of generalized energy momentum tensor

Going back to contractions, I need to have same contractions as for Riemann tensor. That makes figuring out generalized non-vanishing energy momentum tensor alot easier. First two indexes and last two will give zero, second and third , second and fourth will give non-vanishing energy momentum tensor and rest two will give minus it. I have six contractions that pair up to three results:- zero, minus and plus. My strategy is to use metric tensor and normal non-vanishing energy momentum tensor combination with trace of energy momentum term. I will first write form of that tensor then explain and check does it work as it should:

$$\tilde{T}_{\alpha\mu\beta\nu} = \frac{1}{2}\left(\tilde{T}_{\alpha\beta}g_{\mu\nu} + \tilde{T}_{\mu\nu}g_{\alpha\beta}\right) - \frac{1}{2}\left(\tilde{T}_{\mu\beta}g_{\alpha\nu} + \tilde{T}_{\alpha\nu}g_{\mu\beta}\right) - \frac{1}{6}\tilde{T}\left(g_{\alpha\beta}g_{\mu\nu} - g_{\beta\mu}g_{\alpha\nu}\right) \tag{5.1}$$

Let me start with vanishing components of Riemann tensor, if first two indexes are equal to last two it does vanish so does this tensor, where I still assume that it's symmetric tensor:

$$\tilde{T}_{\alpha\alpha\beta\nu} = \frac{1}{2}\left(\tilde{T}_{\alpha\beta}g_{\alpha\nu} + \tilde{T}_{\alpha\nu}g_{\alpha\beta}\right) - \frac{1}{2}\left(\tilde{T}_{\alpha\beta}g_{\alpha\nu} + \tilde{T}_{\alpha\nu}g_{\alpha\beta}\right) - \frac{1}{6}\tilde{T}\left(g_{\alpha\beta}g_{\alpha\nu} - g_{\beta\alpha}g_{\alpha\nu}\right) = 0 \tag{5.2}$$

$$\tilde{T}_{\alpha\mu\beta\beta} = \frac{1}{2}\left(\tilde{T}_{\alpha\beta}g_{\mu\beta} + \tilde{T}_{\mu\beta}g_{\alpha\beta}\right) - \frac{1}{2}\left(\tilde{T}_{\mu\beta}g_{\alpha\beta} + \tilde{T}_{\alpha\beta}g_{\mu\beta}\right) - \frac{1}{6}\tilde{T}\left(g_{\alpha\beta}g_{\mu\beta} - g_{\beta\mu}g_{\alpha\beta}\right) = 0 \tag{5.3}$$

Now I can check all the contractions that add up exactly how they should. So they are equal to:

$$g^{\alpha\mu}\tilde{T}_{\alpha\mu\beta\nu} = 0 \tag{5.4}$$

$$g^{\alpha\beta}\tilde{T}_{\alpha\mu\beta\nu} = \tilde{T}_{\mu\nu} \tag{5.5}$$

$$g^{\alpha\nu}\tilde{T}_{\alpha\mu\beta\nu} = -\tilde{T}_{\mu\beta} \tag{5.6}$$

$$g^{\mu\beta}\tilde{T}_{\alpha\mu\beta\nu} = -\tilde{T}_{\alpha\nu} \tag{5.7}$$

$$g^{\mu\nu}\tilde{T}_{\alpha\mu\beta\nu} = \tilde{T}_{\alpha\beta} \tag{5.8}$$

$$g^{\beta\nu}\tilde{T}_{\alpha\mu\beta\nu} = 0 \tag{5.9}$$

Strategy was to pair non-vanishing energy momentum tensor with metric tensor this way that each contraction will lead to zero for first or last two indexes by using all other indexes and making terms add up to zero. For positive value of contractions I used same trick but term with trace of energy momentum tensor cancels out remaining trace that comes from contractions.

References

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