Time Paradox, Zitterbewegung and Noncommutative Geometry

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The zitterbewgung of the electron is associated with an intrinsic time for an electron, or any of the fermionic particles with mass such as quarks and leptons. In this article it is shown that a massless particle coupled to noncommutative coordinate geometry is subjected to a gauge-like force. This force acts to trap the massless particle in an orbit within a region. This bottled massless particle then has an induced mass. This is then argued to be tied to fundamental aspects of physics, such as a dynamical Higgs model, as well as strings and p-brane theory.

Overview

Zitterbewegung is a possible internal clock for an elementary particle. This trembling motion of the electron reduces the mass of a particle to the frequency of this motion. This motion of a massless particle is due to a fundamental interaction between this particle and the structure of spacetime. An underlying noncommutative geometry of spacetime induces a gauge-like force on the massless particle. This force is similar to a micro-Einstein lens that curves the path of the null geodesic into a spiral path. The motion is similar a photon in a material with a spatially variable dielectric. This quantal dynamics likely has connections to physical foundations, such as string theory and conformal field theory. The onset of this frantic motion likely represents a transition in the renormalization group flow of conformal field theory at the symmetry breaking point.

Time and Zitterbewegung

De Broglie proposed a wave with a certain frequency and wavelength was associated with the motion of an electron [1]. This motion is something which intertwines the proper time of a particle in relativity with the coordinate time of a quantum particle. Special relativity defines the invariant interval

$$ds^2 \ = \ -(cdt)^2 \ + \ dx^2 \ + \ dy^2 \ + \ dz^2 \ = \ \eta_{ab}dx^a dx^b \tag{1a}$$

as the time measured on a worldline. A multiplication of proper time by mass-energy $E \ = \ mc^2$ of the particle on the worldline defines an action

$$dS \ = \ mc^2 ds, \tag{1b}$$

and the motion of a particle is determined by the variational calculus. For special relativity $\delta ds \ = \ ds^{-1}\eta_{ab}dx^a\delta dx^b$, and the four velocity $U^a \ = \ dx^a/ds$ gives the result

$$\delta ds \ = \ \eta_{ij}U^i\delta U^j ds \ + \ \eta_{ij}U^iU^j\delta(ds). \tag{2}$$

The variation in the four velocity is dU^a determines a four acceleration, the orthogonality condition between acceleration and velocity in four dimensions, and the equations of motion in special relativity. This is easily extended to a general g_{ab} metric, where the variational calculus will reproduce the geodesic equations of motion.

Particle dynamics is determined by the extremization of the proper time of a particle. Yet, what does it mean for a particle to have a clock? A clock is a device on a frame which oscillates masses on springs or jostles electrons between atomic levels. In that case there is some sort of internal dynamics to the particle to define the clock. It is then tempting to say

$$dS \;=\; mc^2 ds \;=\; pdq \;-\; Hdt, \tag{3}$$

for some Hamiltonian which defines the working of the clock. The energy involved with driving this clock contributes to the mc^2 of the particle mass. Further, this time "t" is a coordinate time, which is assigned by a gauge-like coordinate condition or choice of spatial surfaces of clock synchronization, which fixed the Hamiltonian. This coordinate time in quantum mechanics defines the evolution of quantum waves. A working clock model of time intertwines the proper time with coordinate time.

What about an elementary particle such as the electron? The electron has no apparent spatial extent to hold a clock. So what does it mean for an electron to have a clock? The answer to this question might be found by early work by deBroglie, Schrodinger and Dirac. Louis deBroglie determined that the wavelength of a particle times its momentum equals a unit of action. This is rewritten as $E \;=\; \hbar\omega$, which tells us energy is frequency. Combined with famous Einstein equation, mass is energy or $E \;=\; mc^2$, the mass of an electron is equivalent to its frequency

$$\omega \;=\; mc^2/\hbar, \tag{4}$$

which is about $1.6\times10^{-21} sec^{-1}$. This formula can be easily rewritten to give the Compton wavelength of the electron, $\lambda \;=\; \hbar/mc = \; 3.9\times10^{-11}$cm, which defines the region of motion. Some care is in order for $E \;=\; mc^2$ obtains on the rest frame of a particle. An oscillatory wave function is written as [2][3]

$$\psi(r,\; s) \;=\; \psi(r) exp(i\omega s), \tag{5}$$

for s the proper time of the particle. In the lab frame there is the observed frequency ω_L in lab time T_L, and clearly $\omega_s \;=\; \omega_L T_L$ which gives us

$$\omega_L \;=\; \omega(s/T_L) \;=\; \frac{\omega}{\gamma},\; \gamma \;=\; 1/\sqrt{1 \;-\; (v/c)^2}. \tag{6}$$

This suggests how an electron has a clock without internal "parts" that requires internal dynamics dependent on coordinate time.

There are measurable consequences of zitterbewegung. The distance traversed by the the particle in the lab time T_L is $d = vT_L$ which in a clock period is equal to $d = hp/(mc)^2$. An electron moving through a crystalline lattice with atomic spacing d exhibits a resonance between the zitterfrequency and the occurrence of an atom in the lattice. For an silicon crystal with $d = 3.84A$ the resonance energy pc is easily computed as $80.87MeV$. A channeling experiment was performed with the linear accelerator at Saclay. There was an observed drop in the transmission resonance of electrons at $81.8MeV$. The publication received scant attention, in part because of the venue, *Annales de la Foundation Louis de Broglie*, and a subsequent account [4] was largely ignored. Zitterbewegung has historical connections to subquantal interpretations and Louis de Broglie's "French school" of quantum mechanics. However the data is sufficient to be taken seriously.

The Dirac equation $(\gamma^\mu\partial_\mu + m)\psi = 0$ may be written as a Schrödinger equation

$$H\psi(\mathbf{q},\ t) = i\hbar\frac{\partial\psi}{\partial t}(\mathbf{q},\ t), \tag{7}$$

with the spatial gamma matrices $\alpha_i = \gamma_0\gamma_k$ [5]. The free field Dirac Hamiltonian $H = \alpha_0 m + \alpha_i\hat{p}_i$ determines the evolution of any observable $\mathcal{O}(t)$ by $i\partial_t\mathcal{O}(t) = [\mathcal{O},\ H]$. The position of the electron $\mathcal{O}(t) = q_i(t)$ is governed by the equation of motion

$$\hbar\partial_t q_i(t) = i[H,\ q_i] = i\alpha_j[p_j,\ q_i] = \hbar\alpha_i. \tag{8}$$

The acceleration $\ddot{q}_i$ of the particle is determined again by the Schrödinger equation

$$\hbar\partial_t\alpha_i = i[H,\ \alpha_i] = i([\alpha_0,\ \alpha_k]m + [\alpha_i,\ \alpha_j]\hat{p}_j = 2i\gamma_i m + i[\gamma_i,\ \gamma_j]\hat{p}_j, \tag{9}$$

where the last commutator defines the spin tensor $\sigma_{ij} = (i/2)[\gamma_i, \gamma_j]$. Hence the equation of motion clearly involves a rotational motion. The motion is in a more compact form $\dot\alpha_i = 2i(p_i - \alpha_i H)$. Hence the equation of motion is $\alpha_i(t) = \alpha_i(0)exp(2i(p_j q_j - Ht))$, and particle in a frame with $p_i = 0$ the motion is

$$q_i(t) = \frac{i\hbar}{2H}\alpha_i(0)(e^{-2iHt} - 1). \tag{10}$$

This motion occurs in all three directions q_x, q_y, q_z and is a Lissajous type of motion in the stationary frame of the electron. For a massless fermion the null worldline exhibits this trembling motion confined in a region with an induced mass $m = \langle H\rangle/c^2$.

This motion appears paradoxical. It is clear that $\dot p_i = 0$ and H is time independent, which contradicts the fact that the zitterbewegung motion is an oscillatory motion. The Newton-Wigner representation with $H_0 = \beta m$ removes zitterbewegung motion [6][7]. However, this requires a bare mass for the particle. For a massless particle this motion still exists, but with a changing momentum which contradicts $\dot p = 0$. So something appears problematic. Zitterbewegung has been suggested as a source of the magnetic moment of the electron, and the angular momentum or spin of the electron, in addition to its mass. So this motion may point to deeper foundations, and this problem with momentum might suggest some interaction on a deeper level.

Gravity, Gauge Potentials and Noncommutative Geometry

A potential method for removing this apparent contradiction is to consider a gauge potential ω so that $p_i \rightarrow p_i + \omega_i$. The nature of this gauge potential may then define a geodesic-like equation $P_i = 0$. Further, the gauge potential must act as a confining interaction around one particle, which is suggestive of a soliton-like object or a geon. The equation of motion for α_i suggests this motion is identified with $g^{\mu\nu}{}_{;\nu} = 0$, with $\gamma^\mu{}_{;\nu} =$

$\gamma^\mu{}_{,\nu} + \Gamma^\mu{}_{\nu\sigma}\gamma^\sigma = 0$.[8] Since covariant momentum is constant, the two conditions $\dot{\alpha}_i = 0$ $p_i = 0$ implies a type of geodesic motion. If the Hamiltonian transforms according to the unitary operator $U = exp(i\alpha\cdot\int_{\omega_0}^{\omega}\mathbf{A})$ as $H \rightarrow UHU^{-1}$ into the form

$$H' = \alpha\cdot p\, exp(2i\alpha\cdot\int_{\omega_0}^{\omega}\mathbf{A}) = \alpha\cdot\omega(cos(2\theta) - isin(2\theta)) \qquad (11)$$

for $\theta = 1 - |\omega/\omega_0|$ and small. This factors out the Hamiltonian as $H' = \alpha\cdot(p + \omega)$ where the angle θ is absorbed into the gauge connection. This produces the zitterbewegung motion for the gauge transformed Hamiltonian in equation 10, and where the covariant momentum $P = p + \omega$ is time independent.

Let the connection one-form be $\underline{\omega}^i = \Gamma^i{}_{00}dt$, which defines a geodesic equation of motion for the covariant momentum $dP^i/ds = dp^i/ds + \omega^i(U^t)^2$ which vanishes. This to the prospect the momentum may be constant where position variable may change. There is a slight problem however, for geodesic motion is determined by the proper time, while the Dirac equation is governed by a coordinate time. It is tempting to say that $U^t \simeq 1$, yet the frequency of motion is $\omega = 10^{21}sec^{-1}$ and a Newtonian acceleration in a region $\sim 10^{-11}$cm is $a \simeq 10^{30}cm/s^2$. This extreme condition calls into question this assumption. Further, if we assume a gravitating mass with $\Gamma^i{}_{00} \sim a = GMq_i/r^3$ the mass would have to be $M \simeq 10^{15}g$. Clearly there is no mini-black hole with this mass, and radius $\sim 10^{-13}cm$, in an electron.

Let the spacetime be tessellated by Planck units of volume, similar to a crystalline lattice. The quantum wave is a Bloch wave with a symmetry according to the Voronoi structure of the lattice, with solid state physics parallels [9]. A Brillioun reciprocal $\mathbf{k}$ space is determined by the polytope dual to the fundamental unit of tessellation. There is a translational symmetry to the wave with $T(r)\psi(q) = \psi(q + r)$, which equals $\psi(q)$ for $r = D$ a lattice spacing. Assume the Voronoi cell is the root lattice for the gauge group $\mathcal{G}$. The translation of the wave function is $UT(r)U^\dagger\psi(q) = U\nabla_r U^\dagger\psi(q)$, $U \in \mathcal{G}$. Define gauge-like fields $A_z = \hbar U\nabla_z U^\dagger$, for a symplectic $z = \{q,\ p\}$, which define a system of general translations in the lattice and reciprocal lattice. This is a noncommutative coordinate system:

$$[q,\ p] = i\hbar + i\hbar^2\nabla_{[q}A_{p]}$$

$$[q,\ q'] = i\hbar^2\nabla_{[p}A_{p']},\ [p,\ p'] = i\hbar^2\nabla_{[q}A_{q']}, \qquad (12a)$$

where these commutators are spacetime covariant, and the first of these is

$$[q,\ p'] = i\hbar + i\hbar^2 e_\mu((\partial_{[q}A_{p]})^\mu - iR^\mu{}_{\nu\alpha\beta}A^\nu q^\alpha p^\beta). \qquad (12b)$$

The first of these is employed in equation 8, so that $\alpha_i \rightarrow \alpha_i + i\hbar^2\epsilon_{ijk}\nabla_{[q^j}A_{p^k]}$, which adds a magnetic-like field $B_i = \epsilon_{i\mu\nu}\nabla_{[q^\mu}A_{p^\nu]}$, to the Dirac matrix element. In a manifestly covariant form the dynamical equation for α_i will now be extended to

$$\dot{\alpha}_i = 2i(p_i - \alpha_i H) + \alpha_l\partial_{[l}B_{i]}. \qquad (13)$$

By analogy with the Maxwell-Faraday equation this is a field equation with a current term. The identification of a displacement current $\dot{\alpha}_i = \alpha_0\partial_t B_i$ leads to a general wave equation of the form $\Box A^\mu = 4\pi j^\mu$, for $j^\mu = 2i(p^i - \alpha^i H)\delta_i{}^\mu$.

An obvious question is how near Planck scale physics, $10^{33}cm$ has this effect upon nature on a scale much larger scale $10^{-11}cm$. This issue is addressed at the end of this section. The physics is scale invariant on a quantum critical point. The Planck scale effects then scales to the Fermi length, as indicated below.

This gravity-like field and its wave equation binds the electron in a bottle, similar to a geon [10]. This is particularly if the bare mass of the electron is zero and the mass of the electron is due to energy trapped in this "bottle."

By way of comparison we consider elements of geon physics. Consider the noncommutative aspects of the spacetime as a small $O(\hbar)$ correction to a flat background. The unitary operators on a flat spacetime, large enough so the noncommutative physics is small, the $(U^\dagger U)_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, which are transition functions identified with a metric. The order parameter for this expansion is the wavelength of the field λ induced by noncommutivity and the background scale for either spacetime or a background field with a scale variation L. Consider the noncommutative aspects of the spacetime as a small $O(\hbar)$ correction to a flat background. The unitary operators on a flat spacetime, on a large scale so the noncommutative physics is small, then $(U^\dagger U)_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. The order parameter for this expansion is the wavelength of the field λ induced by noncommutivity and the background scale for either spacetime or a background field with a scale variation L, with order parameter $\epsilon = \lambda/L$. Let the variation in the spacetime metric due to noncommutativity be small so that the average $\langle g_{\mu\nu}\rangle = \langle \eta_{\mu\nu}\rangle$ on the scale L. This zero average applies as well with higher order derivative of $h_{\mu\nu}$. Within the scale of the spacetime field the deviations have the order

$$O(h_{\mu\nu,\alpha}) = O(1),\ O(h_{\mu\nu,\alpha\beta}) = O(\epsilon^{-1}). \tag{14}$$

A form of the Ricci curvature is calculated easily as

$$R_{\mu\nu} = \frac{1}{2}\eta^{\alpha\beta}(h_{\alpha\beta;\mu\nu} + h_{\mu\nu;\alpha\beta} - h_{\alpha\mu;\nu\beta} - h_{\alpha\nu;\mu\beta}). \tag{15}$$

The overall derivative on the metric subtracts the $i\hbar^2\nabla_{[q}A_{q']}$, whichdoes not depend upon the gravitational constant G and is larger. The field effect due to noncommutative coordinates expressed according to a special derivative notion is $h_{\mu\nu;\hat{\alpha\beta}} = U_\mu\nabla_\beta U^\dagger_\nu$. So while the gravitational field component may be small the field effect due to noncommutative coordinates may be appreciable. For this reason the zitterbewegung motion can be tied to spacetime without a mini-black hole source for the field.

There is a series in the Ricci tensor according to powers in the perturbing metric term $h_{\mu\nu}$[11]. This series is similar to the Ricci series expansion of the Einstein field equation in string theory for some Ricci-like tensor $R_{\mu\nu} = \lambda g_{\mu\nu}$, and an eigenvalued system for derivatives of the metric corrections. Possible connections between string theory and geons are an open question to be examined. Yet the geon is similar to a soliton, which might have connections to the dynamics of an open string and a D-brane.

The gauge-like field is similar to a gravi-electromagnetic geon [12]. The wave equation for the gauge-like field

$$\Box A^{\mu} = 8\pi i(p^{i} - \alpha^{i} H)\delta_{i}{}^{\mu}, \tag{16}$$

is examined for a Hamiltonian that contains the $H = \frac{1}{2}B^2$ for this magnetic-like, or gauge-like field, with $p_i = 0$. For the sake of simplicity we reduce this differential equation to $1 + 1$ dimensions and consider the gauge-like potential as a scalar field ϕ. The magnitude of the field strength is $|B|^2 \sim k^2\phi^2 - g^2\phi^4$, and we replace the α^i with a conjugate momentum operator $\alpha^i \rightarrow -i\partial/\partial\phi$ which results in a differentiation of H by ϕ. In this approximation the curl term is replaced by ϕ_x. The approximate simplified wave equation is then,

$$\phi_{xx} - \ddot{\phi} + 8\pi(\kappa\phi\psi - 2k^2\phi + 3g^2\phi^3) = 0, \tag{17a}$$

for ψ a massless one-dimensional field for the fermion and κ a coupling constant between the massless fermion and the gauge-like field. This wave equation is a cubic Klein-Gordon equation, with some similarities to the $1 + 1$ spacetime differential equation for a gravi-electromagnetic geon. The coupling includes an equation for the motion of the massless fermion, which is

$$\dot{\psi} = -i\phi\psi, \tag{17b}$$

which in a numerical treatment is considered to be a point-like particle.

The evolution of the particle is a helix in two dimensional spacetime, and the gauge-like field is in a soliton-like configuration. The solution is approximately $\phi \sim \phi_0 sech(kx - \omega t)$, for $\phi << 1$ and weak coupling. If the initial configuration of the field is described by some other configuration, such as a periodic function with a gaussian envelope, it numerically evolves into a soliton-like configuration. The wave function of the massless particle oscillates back and forth as if in a potential well. The differential effective index of refraction from the particle coupling to noncommutative spacetime defines a path which slows the particle down the further it moves from $\phi_x = 0$ of the potential field ϕ.

The gauge-like field induces the fermionic spin to the massless field, so the electron observed is a quasi-particle, similar to spin density fields. The Hamiltonian for quasiparticles near the Fermi point is $H = E^i_a \sigma^a (p_i - p^0_k)$, for p^0_i on the momentum representation of the Fermi surface, σ^a Pauli matrices. Thus near the Fermi surface the quasi-fermions behave as Weyl 2-component spinors, and E^i_a a tetrad which acts on the field. The tetrad may define the gauge-like field, "lens" or an effective gravity field. Such quasi-fermions will acts as Landau fluids which "break down" at a quantum critical point as $p_i \rightarrow p^0_i$. At the quantum critical point the physics becomes scale invariant, and the Planck scale physics scales to much larger values. In this way elementary particles (quarks and leptons) acquire their masses at $E \simeq 10^{-16} L_p$. The vacuum configuration is close to a quantum critical point, which corresponds to a phase transition point on the abscissa, where $T \rightarrow 0$. In certain condensed matter systems the effective mass of a Landau heavy electron fluid diverges near the quantum critical point. The ratio m/m^*, for $m^* \rightarrow \infty$ near this point is a measure of the overlap between electron states and heavy spin states (heavy electrons) vanishes: or $Z = |\langle e^- | p^- \rangle|^2$, which is a scale invariant quantum state. The imaginary time $t_i = \hbar/kT$ for a quantum process is large as the temperature $T \rightarrow 0$, which induces a scaling invariance.

The Higgs Field, M-theory and Beyond

This section considers how zitterbewegung may have connection with deeper aspects of physics. Zitterbewegung hints at a deeper structure to quantum mechanics, and the above illustrates connections with the foundations of spacetime physics. These connections lead to quantum gravity and string theory.

Higgs field, mass and zitterbewegung

The zig-zag of the two component Weyl field theory with

$$\nabla_{AA'}\psi^{A} ~=~ \frac{m}{\sqrt{2}}\psi_{A'},~ \nabla_{AA'}\psi^{A'} ~=~ \frac{m}{\sqrt{2}}\psi_{A}, \tag{18}$$

describes the oscillatory motion of a particle between its two helicity states through a scattered by its mass. In the standard model this mass is determined by a Yukawa Lagrangian with the Higgs field. The underlying gauge-like structure induced by noncommutative geometry underlies a dynamics for the Higgs field. This gauge particle has the characteristic of a bound state of fermions, such the above scalar as a dynamical Higgs particle composed of the top and anti-top quark [13]. The gauge-like particle is associated with the pair production of a fermion and anti-fermion, which is an underlying quaternionic structure to the theory. To bring this in line with the standard model and QCD these fermions are the top and anti-top quarks.

The Higgs field with gauge fields determines lepton or quark masses in a complicated manner. The mass is induced by the term $V ~=\bar{u}Hu$, for $u ~=~ q,~\ell$ for quarks or leptons respectively. We consider the fermion as transformed by a gauge transformation $u ~\rightarrow~ uexp(i\int A\cdot dx)$, which we consider for a small gauge shift with a variation. Then the expectation of the potential variation is $\langle\delta V\rangle ~\sim~ \langle~|A^{\dagger}\bar{u}HuA|~\rangle$. This is a version of the "zig-zag" which is seen with the two component Weyl equations. The Higgs field induces the zig-zag, or this zitterbewegung, and with the inclusion of the gauge field the effect is enhanced. Further, the Higgs field and the gauge field may in principle be mixed into each other. The Higgs field may be exchanged for a pair $(A,~A^\dagger)$ or $(t,~\bar t)$. In the asymptotic freedom case the gauge field becomes weak and so the diagram is reduced to the quark interaction only with the Higgs with some small QCD interaction. In this case the quark mass is the bare Higgs induced mass. When the interaction is at low energy the QCD interaction strength becomes very large and contributes to this mass.

For a gauge field the momentum-energy tensor is.

$$T^{ab}~=~-F^{ac}F_c{}^b~-~\frac{1}{4}g^{ab}F_{cd}F^{cd}$$

which quantum mechanically is evaluated as $\langle T^{ab}\rangle$. Let us consider the uncertainty in this quantity ΔT^{ab}

$$\Delta T^{ab}~=~\delta x^c\partial_cT^{ab}~+~\frac{1}{2}\delta x^c\delta x^d\partial_c\partial_dT^{ab}$$

Under the expectation $\langle\delta x^c\rangle~=~0$ and the second order term remains. The second order term $\delta x^c\delta x^d$ has symmetric and antisymmetric portions. The antisymmetric portion is clearly zero since $\partial_{[c}\partial_{d]}~=~0$ and so

$$\Delta T^{ab}~=~\frac{1}{6}\delta x^2\partial_c\partial^cT^{ab}$$

The term δx^c for a massive particle evolves by the generalized Lorentz equation $mD^2\delta x^c/ds^2~=~U_aF^{ac}$, and for an oscillator motion $\delta x^c~=~\delta x(0)^c exp(i\nu t)$ then

$$\nu^2\delta x^c~=~U_aF^{ac}.$$

The frequency ν is then related to the mass of the particle through $E~=~mc^2$ and the deBroglie relation $E~=~h\nu$, and so $\nu~=~mc^2/h$, or

$$\delta x^c~=~\frac{2p\hbar}{mc^2}U_aF^{ac},~\langle\delta x^2\rangle~=~\frac{2p\hbar}{mc^2}\langle~|F^{ab}F_{ab}|~\rangle,$$

which is a result related to zitterbewegung. The expectation of the uncertainty in the momentum energy tensor leads to the Lamb shift for the box normalization condition on the electric field $(E_0)^2 \ = \ \hbar ck/2\epsilon V$. The requirement of a mass term in the δx^c motion is somewhat artificial. To remove this the noncommutative term being set to zero condition is removed. This leads to noncommutative variables which are responsible for the zitterbewgung, which then feeds into the mass m in the dynamics of the variation in δx^c.

The relativistic quantum field theoretic perspective on zitterbewegung suggests that the motion is connected with the virtual production of particles in the vacuum state. The current source in equation 13 may be interpreted as a stochastic source. The delta function δ^μ_j in a quantum stochastic setting is augmented by a distribution determined by $\int dV\bar\psi\psi$ or that the current is determined by $\langle\ j^\mu\rangle$. Within this interpretation the current is a quantum stochastic source that confers a mass to the fermion. This is a coupling of the massless field to the vacuum state, so that the current expectation has the form $j\ \sim\ \langle\ |\{\bar q,\ \bar\ell\}H|\{q,\ \ell\}\rangle$, where $\{q,\ \ell\}$ represent quarks or leptons. It is possible to consider the direct interaction to be due to QED, but ultimately the mass term is conferred through the Higgs interaction.

Strings and M-theory

There are possible connections with M-theory. The soliton-like field induced by noncommutative coordinates is similar to braney solitons induced through Chan-Patton factors for open strings. Spacetime is then induced by oscillations on the p-brane by a target map. Further, M-theory predicts on small scales the fabric of spacetime exhibits noncommutative coordinate conditions. The gravitational contribution from noncommutative geometry is the Ricci curvature term in equation 12b. This becomes only apparent at scales approaching the Planck length. However, there is in addition a gauge-like field effect which may exist on much larger scales comparable to the Compton wavelength of the electron.

How does this physics really scale to 20 orders of magnitude? We consider the geon-like physics as tied to a soliton field on a p-brane. The superspace Lagrangian contains $dX_n \wedge dX_{10-n}$ 10-form [14] for $n = 3$ the states of the vacuum are $|0\rangle$ and have other states by $|1_k\rangle = b_k^\dagger|0\rangle$, where the b_k, $b_k^\dagger$ operators are the quantization of the modes on the D3-brane. We consider the electron as an open Type II with fermionic modes, which are confered to the brane. The states on the dual D7-brane are spinon fields, which are dual to the states on the 3-brane. The three brane can't be identified with the spatial manifolds of evolution in LQG until some constraints are imposed. This happens for a vanishing overlap between these states. Hence the 3 brane will have states that correspond to states $\langle p|$ that have minimal overlap with states $|q\rangle \in$ D7-brane $|\langle p|q\rangle| \simeq 0$. The states $|q\rangle$ have their dual then with 3-brane states $|*q\rangle$ that similarly have

$$|\langle p|*q\rangle|^2 = m/m^* \rightarrow 0, \tag{19}$$

where m^* is the ZPE of the vacuum state for other D3-branes in a foliation with the D3-brane corresponding to states $\langle p|$, and a dual D7-brane with ZPE $\sim 1/m^*$. This is analogous to the heavy mass of Landau electron fluids when the "die" near the quantum critical point [14]. The states $|*q\rangle$ may then be arrived at from a magnetization M_q from the vacuum $|0\rangle$ and states $|p\rangle$ by

$$|*q_k\rangle = M_q b_{k-q}^\dagger|0\rangle \tag{20}$$

and the minimal overlap is then a statement of how the vacuum in our universe is "light," or has a small cosmological constant. This also means that the net field content on the spatial manifold satisfies a constraint

There is an induced target map to a spatial. The noncommutative coordinates define a magnetic-like field $B = dA$, which is by the map defines the magnetization M_q. The vector potential under this map defines a LQG connection term $p_{ij} \rightarrow p_{ij} + i\sigma^a A_{ij} b_a$. This field obeys a Gauss' law with a net charge $Q^a = \nabla^i A_i$, for A_i the 3-dim Hodge dual to A_{ij}. The minimal overlap then defines a constraint $B_a Q^a = 0$. These spin fields, similarly those associated with twistors, exist on a Fermi surface. This surface also has some topological features. A space of evolution may be a spin-net in the LQG sense, or a D-brane in the string theory. For the space of evolution, which defines a world volume $V = \Sigma \times R$, where V is the evolute of the surface Σ, which has a target map to the spacetime or super-spacetime M^n. The compactified winding of a D-brane on this world volume is given by a unitary group $U(n)$, where n is the winding number or coincidence number of these branes. These winding numbers define the brane charges on the volume, which define charges in K-theory groups on the manifold M^n [14], which are closely related to the cohomology $H^p(M^n, R)$. Within twistor theory for $n = 4$ this is a sheaf cohomology, where these charges are helicity states or frequencies for the $PT^{\pm}$ subspaces of twistor geometry.

The entire structure suggests a format by which gauge theories and gravitation exist in a single system. Noncommutative geometry can put gauge fields and gravitation on the same footing. The two fields exist on the same frame bundle. The commutator of coordinate or momentum variables is a type of operator which produces a vector field, or a differential of a connection term, with both gauge or gauge-like content and gravitation. The perturbation series for geon Ricci curvature is remarkably similar to the Ricci curvature expansion in string theory in orders of α_g.

The framing of these gauge or gauge-like fields differ by the important gravitational constant, and the invariance under $R \leftrightarrow \alpha/R$ in T-duality, exchanges bundle information by imposing the constant $G \sim \alpha_g^2$. This is an intertwining on the frame, which in noncommutative geometric context exchanges the two fields. This connects with the AdS/CFT duality, where in ten dimensions conformal fields on S^5 are dual to the five dimensional space of AdS. The Anti-de Sitter spacetime is $SO(3,2)$ which embeds in $SU(4)$. In a toy model system we may decompose $SU(4)$ into $SU(3) \times U(1)$. The additional $U(1)$ is an abelian B-L transformation and the $SU(3)$ is a dual field theory to the strong nuclear force. This is a reflection of QCD, and a duality between the chromodynamics of type IIB strings which bind p-branes in a foliation, and gluon flux lines which bind quarks in a hadron.

Conformal fields and renormalization group flows

The zitterbewegung may represent the endpoint of conformal renormalization group (RG) flow. The conformal boundary of the Anti de Sitter (AdS) spacetime is equivalent to a Conformal Field Theory (CFT) at high energy [16], which exhibits a simple RG flow according to the geodesic arcs on the hyperbolic AdS, similar to the arc on the Poincaré disk. The low energy theory, where the curve returns to the boundary, appears the same as the high-energy theory. This is because there is no intrinsic scale — the AdS is conformally invariant. But if you perturb the theory by adding mass terms to certain fields, the RG flow is nontrivial and one obtains a different theory at low energies. The zitterbewegung is a manifestation of how the RG flows continue into the low energy, or sub-electroweak scale. The conformal fields are equivalent to the AdS, and at very low energy addition boundary conditions enter into the physics. The conformal infinity of AdS is a timelike Minkoswki space which requires additional BC information on top of spatial surfaces for their specification. So this appears to be the energy cut off of RG flow, but in fact the apparent masses are simply manifestation of RG flows as energy approaches zero. The zitterbewegung may encode this additional information.

Suppose that the endpoint of the RG flows is a point where masses and gauge field coupling constant are such that the total horizon area of black holes in the average cosmological horizon bubble of radius $r = \sqrt{3/\Lambda}$ is some extremum. In other words if the gauge fields were too strong or two weak or did not have their observed proportions the number of black holes in the universe would be smaller or larger. Further all of the physical field data outside of black holes in this region are holographically projected by both the cosmological event horizon and these black hole horizons. Now transfer to the equivalent AdS spacetime from the physical de Sitter spacetime. These black holes are "lumped" into a single BTZ black hole. The fields at the boundary of the AdS spacetime are CFT, and leave the boundary at $E \sim \infty$, and then approach it again at $E \sim 0$. However, with the BTZ black hole fields that leave the black hole by quantum tunneling with energy $E \sim \hbar/8\pi M$ approach the CFT with $\rightarrow 0$, and those which leave the $CFT \sim \partial AdS$ (∂ = conformal boundary) at $E \sim \infty$ approach the BTZ horizon with $K_t \cdot E = const$, and the Killing vector $K_t = 1/\sqrt{1 - 2M/r}$ means $E \sim 0$ as $r \rightarrow 2M$. So there is a difference in the energy scales for the two cases. This of course involves the mass of the BTZ black hole, which corresponds to the total black hole horizon area in the corresponding physical dS cosmology. This information provides the boundary conditions on the conformal boundary of AdS, which is encoded in the zitterbewegung of the massive elementary particles.

Back to time

Returning to the issue of time, this picture suggests that proper time is an emergent form of time. An electron, or any other fundamental particle is a massless particle trapped in a self-confining region. The particle then really has no proper time, but emerges from this interaction with noncommutative geometry. Coordinate time in general relativity is a group element that under covariant differentiation defines connection terms $de^t = \omega^t{}_\mu e^\mu$.

The differential of the time basis element defines a choice for a "flow of time," which due to its definition according to a connection term is not physically "real," since it does not transform homogeneously. The Dirac motion of the electron emerges from a time averaging over the zitterbewegung, which is where the quantum wave equation is defined by a coordinate time. From the perspective of relativity this averaging is what gives rise to the proper time of the particle, though here the world line is the most expected path which cooresponds to a classical solution.

The zitterbewegung problem has been used to argue for the existence of subquantal physics. The helical motion of the massless particle is the source for the spin of the fermion. While there is connection here, it is preferable to see the spin of the particle induced by the noncommutative structure of spacetime, which couples to the particle. The connection coefficient in general relativity is more generally

$$\Gamma^a{}_{bc} ~=~ \left\{ {a \atop bc} \right\} ~+~ \hbar\gamma^a{}_{bc},$$

where the classical connection term is supplanted by a quantum torsional spin connection. For a classical system with action S, the contribution of the torsional term is $O(\epsilon^2)$, for ϵ the affine distance of transport, times $\hbar/S$. Thus in order to measure this torsional connection the geodesic motion of a rotating nucleus in a cavity would be required.

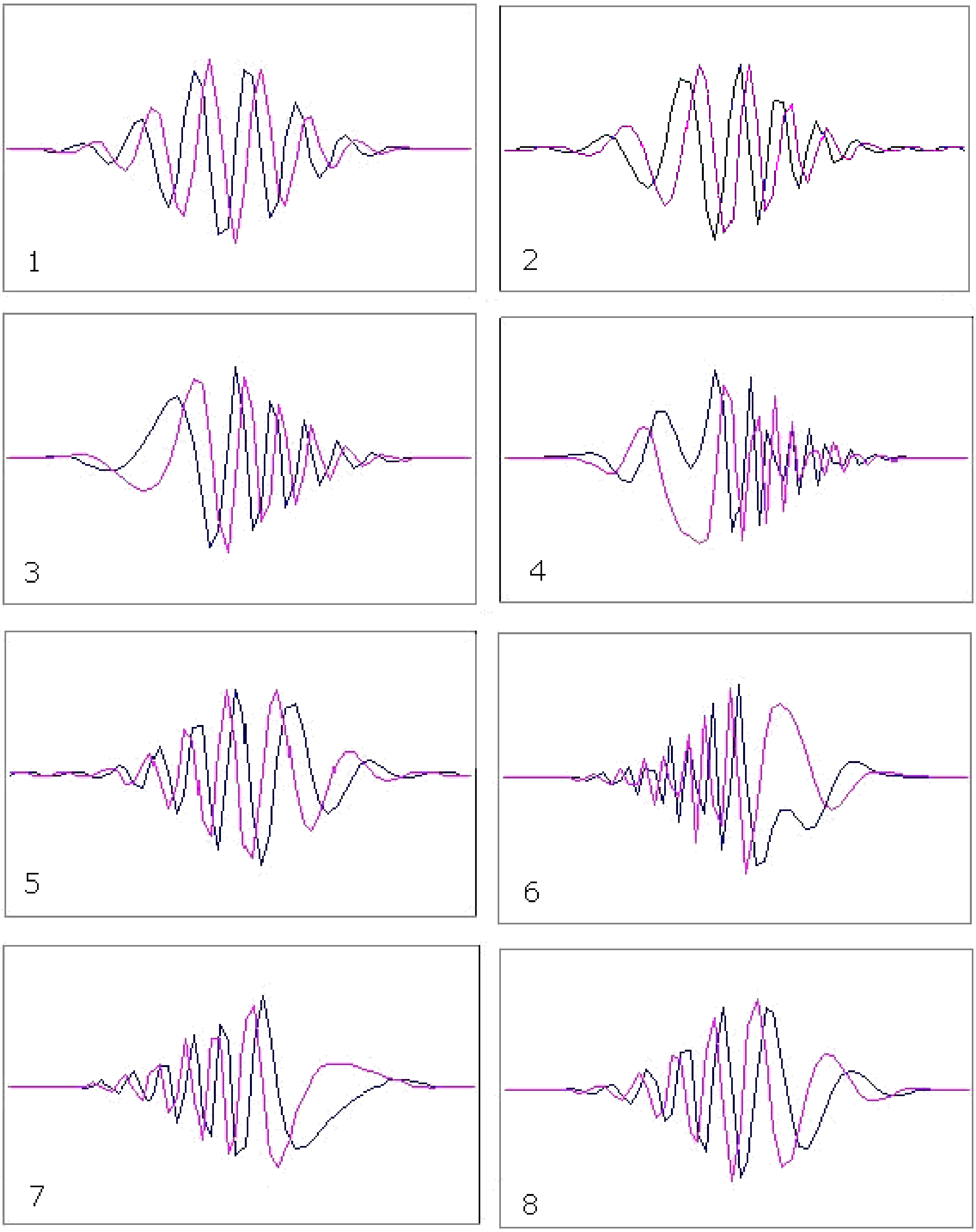

Figure 1: The evolution of the wave function due to noncommutative coordinate induced Zitterbewegung.

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