Emerging Gravity violates Energy Conservation

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Abstract

The justification of the idea of an emerging gravity by a comparison with conventional thermodynamics results from a small inaccuracy of the used equations. Two differing contexts of description coexist unnoticed and lead to a violation of energy conservation. Independent of the chosen description it is shown that no emerging gravity exists, if energy conservation is taken into account. This result is confirmed for one of the possible descriptions by an analytical solution of the problem within General Relativity.

Introduction

Since it's first publication in Jan. 2010 Verlindes article "On the Origin of Gravity and the Laws of Newton" (1) has found a widespread response in public media and scientific publications. Mostly heralded as an astonishing theory to explain the nature of gravitation, the criticism remains remarkable weak.
Even the publication of Kobakhidze "Gravity is not an entropic force" (2) could not dampen the enthusiasm despite the stated contradiction of the idea to experimental data.

To explain the nature of an entropic force in (1) an analogy of a polymer in a heat bath is introduced. This is a convenient and elegant method to describe the observed force by the entropy of the system. However it is undisputed, that a more or less complex model of the polymer, based on the involved molecular forces and statistics leads to the same result, without even mentioning entropy. In such a model force and entropy are considered as an output. Without this model and the knowledge about the nature of molecular forces, the underlying electromagnetic force could be singled out as "emerging" too.
Therefore it should be kept in mind, that the entropic force explains nothing on the microscopic level. Even an entropic force equal to a fundamental force doesn't prove it's entropic nature. Only by an independent criterion similar to the argumentation in (2) a distinction could be made between a fundamental or "emerging" force, if something like this exists.

The entropic force

In a thermodynamic system energy conservation for a constant volume is defined by

$$\Delta E = T\,\Delta S + \Delta M \;^{*} \qquad \text{(a1)}$$

with
E=energy, T=temperature, S=entropy and M=total mass
ΔM is used here in the same way as $\mu\Delta N$ for a conventional gas as a contributing form of energy in the system to describe.

* All constants used are set to $c, k_B, \hbar, G = 1$

This shows that the statement $\Delta E = T \Delta S$
only applies to a closed system with $\Delta M = 0$

A polymer, used as a model for an entropic force represents a closed thermodynamic system because no mass transfer into or from the system is allowed. The same is true for the osmosis, if the process is described by the total entropy of both sides of the membrane.
If this quality is important, it should be explicitly added to the used equations.

Keeping that in mind, a problem arises, first to see in eq. (2.3). of (1)

$$\frac{1}{T} = \frac{\delta S}{\delta E'} \quad , \quad \frac{F}{T} = \frac{\delta S}{\delta x'}$$

Citing the similarities with the polymer model, these equations should read correctly

$$\frac{1}{T} = \left(\frac{\delta S}{\delta E'}\right)_M \quad , \quad \frac{F}{T} = \left(\frac{\delta S}{\delta x'}\right)_M \qquad \text{(a2)}$$

Based on Black Hole thermodynamics the quantity $\Delta S = 2\pi k_B$
is introduced in (3.5) of (1).
It describes a minimum entropy change resulting from a particle by fusing it with the event horizon of a BH or a screen.
For a Schwarzschild BH the Bekenstein Hawking entropy is given by

$$S = 4\pi M^2 \qquad \text{(a3)}$$

and the temperature by

$$T = \frac{1}{8\pi M} \qquad \text{(a4)}$$

The condition $\Delta M = 0$ yields by (a3) $\Delta S = 0$ (a5)
and by (a1) $\Delta E = 0$.

That shows that the postulate $\Delta S = 2\pi k_B$ in (3.5) contradicts eq. (3.7) in (1) which is valid for a closed system only.
The entropy of a Schwarzschild BH only can be changed by changing it's mass. It has no internal degree of freedom, other than M. The same is true for the "screen" introduced in (1).

(a3) and (a4) lead for the open system (with E=M) to

$$\left(\frac{\delta S}{\delta E}\right) = \frac{1}{T} \quad , \quad \left(\frac{\delta S}{\delta x}\right) = \frac{F}{T} \qquad \text{(a6)}$$

These equations are identical to (2.3) , but have a totally different signification as for the closed system..

In (a6) $\frac{\delta E}{T}$ (or $\frac{\delta M}{T}$) must be added to the screen from outside (and not transferred to outside) to change the entropy by δS . This reflects the fact, that in this open system S is a purely extensive state function.
So eq. (3.13) together with all the other assumptions don't describe the Newton force but a force equal to the Newton force with opposite sign.

That however is in contradiction to the statement in chap.(2) “An entropic force is recognized by the facts that it points in the direction of increasing entropy, and, secondly, that it is proportional to the temperature”. That would only be true for the closed system.

To correct for this error eq.(3.7) in (1) must be adapted to the open system condition. ΔM in (a1) is the change of the total mass of the system and Δm is the mass of the external particle which eventually fuses with the screen and is no longer present in the outside region.

That leads to $\Delta M = -\Delta m$

and with (a1) to the correct equation replacing (3.7) $F \Delta x = T \Delta S - \Delta m$ (a7)

For the existence of any entropic force it must be shown, that $T \Delta S \neq \Delta m$ (a8)

No indications are given in (1), by what means (other than the Newton potential) that could be accomplished. So there is no other choice as to state

$$\Delta S = \frac{\Delta m}{T}$$

which shows that $F \Delta x = 0$

That leads to the corrected results for eq(3.9) $F \equiv 0$ (a9)

and for eq(3.13) $F \equiv 0$ (a10)

In Fig.1 and Fig.2 this situation is graphically depicted

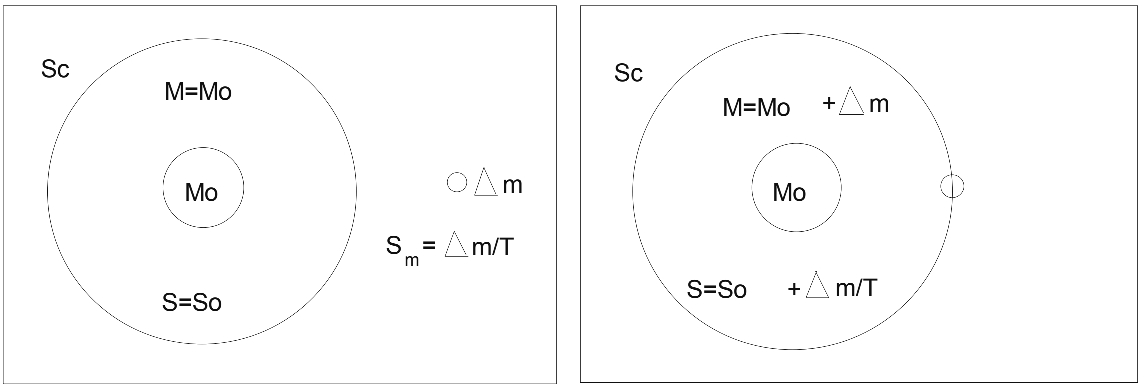

Fig. 1: Open system with external mass Fig. 2: Closed system without external mass

Fig. 1 shows an open system with the screen Sc and the enclosed mass M equal to the central mass M_0 in a heat bath with temperature T. The entropy of the screen Sc is equal to S_0 resulting from M_0. Outside the screen is a mass Δm with it's entropy S_m.

Fig. 2 shows the closed system resulting from the open system in Fig.1 by moving the mass Δm from the outside region to the screen S_C. The entropy S of the screen increases by this action by S_m. But the total entropy remains unchanged $S_{total} = S_0 + S_m$.

The results (a5), (a9) and (a10) are obtained by conventional thermodynamics valid in flat space, the same description as used in (1) . So the question arises, if these results still hold within GR.

Some answer to this question can be expected by looking on the influence of gravitational interaction of a BH with an external mass on the entropy of it's event horizon.

In GR there exists no analytical solution for the general 2 body problem. By such a solution it would be easy to see, how an external mass m_e in distance r changes the event horizon R and thereby the entropy S, used in (1) to justify the existence of an entropic force F.

However, maintaining spherical symmetry, a metric similar to the Schwarzschild metric can be derived from Einstein's field equations.
This is done by Zhang and Liu (3), where the metric is calculated for a thin spherical shell of mass m_e around a BH from the Schwarzschild type.

$$ds_I^2 = \left(1-\frac{r_H}{r}\right) dt_I^2 - \left(1-\frac{r_H}{r}\right)^{-1} dr^2 - r^2\left(d\Theta^2 + \sin^2(\Theta)\, d\Phi^2\right) \qquad \text{(a11)}$$

(a11) is valid for region I which extends from $0 < r < R_1$ with

R_1 = radius of the spherical shell of mass around the BH and

$R_1 > r_H$ with the Schwarzschild radius $r_H = 2\text{M}$

This shows, that the shell of mass around M has no effect on the event horizon of the BH enclosed, since it is not different from the Schwarzschild metric without the shell of mass. The only difference of (a11) compared to the Schwarzschild metric is $dt_I = F(R_1, m_e)\, dt$ when dt denotes the time in the Schwarzschild solution. This however has no influence on the parameter of interest , the entropy

$$\text{S} = \pi r_H^2 = 4\pi M^2$$

It is a constant and not influenced by any gravitational interaction of the mass m_e outside the

event horizon. ($\frac{\delta S}{\delta m_e} = 0$)

This leads to $\Delta S = 0$ in contradiction to eq.(3.5) of (1) but in line with (a5) for the closed system.

The confirmation within GR of (a5) for the flat space comes not unexpected due to the Birkhoff theorem and since no reversible process like gravitational interaction can be described by BH entropy due to the second Hawking law. No clues are given in (1) why this should not apply to the "screen" in the same way.

Conclusion

Violating energy conservation, ΔS in (3.5) is introduced for free whereas it costs $T \Delta S$ in energy which is nowhere accounted for in (1). By this violation the Newton potential is no longer needed to stand for the gravitational interaction and thus allows to speculate about an entropic gravity.

That shouldn't have been unnoticed, since Bekenstein is cited in chap(2) with : “Therefore, it (the particle) increases the mass and horizon area by a small amount, which he identified with one bit of information.”

Safeguarding energy conservation leads to the description within an open thermodynamic system. In the absence of a Newton potential the energy is brought by the mass of the particle from outside. By merging with the screen it creates new entropy there but vanishes itself totally or in part, depending on the size of the stated entropy change.

Thus the emerging gravity becomes a zero sum idea in the open system.

In the closed system it doesn't exist anyway since ΔS=0.

The Newton potential in flat space yields $\Delta m = 0$ and $T \Delta S = F_N \Delta x$.

To demonstrate the existence of an entropic gravity it must be shown that (a8) applies without using the Newton potential . Entropy alone is unlikely to give an answer, since it only describes how the energy in a system flows, is distributed or transformed. It never creates (or absorbs) energy as falsely stated in (1). But exactly that would be required to replace the Newton potential.

Even for a credible deduction of an entropic gravity nothing would be gained. Verlinde showed why:

”Equation (3.8) should be read as a formula for the temperature T that is required to cause an acceleration equal to a. And not as usual, as temperature caused by acceleration.”

He should have added :

Equation (3.5) (if true) should be read as a formula for the entropy that is required to cause a force equal to the Newton force. And not as usual, as entropy caused by the Newton force.

A difficult choice for the reader, given Verlindes other recommendations.

1) Erik Verlinde, “On the Origin of Gravity and the Laws of Newton”
arXiv:1001.0785v1 [hep-th] 6 Jan 2010

2) Archil Kobakhidze, “Gravity is not an entropic force”
arXiv:1009.5414v2 [hep-th] 30 Dec 2010

3) Shuang Nan Zhang (zhangsn@tsinghua.edu.cn) and Yuan Liu
“Observe matter falling into a black hole”
arXiv:0710.2443v1 [astro- ph] 12 Oct 2007