SUPERLUMINAL CONSEQUENCES OF QUANTUM DIFFUSION ON SPECIAL RELATIVITY

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Abstract:

With super-luminal neutrinos being observed in 2011 then refuted in 2012 we present an argument, based on Jensen's inequality, that they might still exist only with a smaller excess speed than initially thought. Specifically, the quantity measured by OPERA and ICARUS is the average of the length of the displacement over time which is greater than the length of the average velocities - which determines and does not break Lorenz invariance. We examine quantum diffusion to explain the physics behind the particle's variance resulting in an excess average velocity of $\hbar/mc^2(t+\tau)$, where t is the baseline and τ is the coherence time. We examine the experimental setup at OPERA to estimate the excess velocity and show it is within the error as observed in the tighter re-run. We also show consistency with Fermilab 1979 and supernova 1987A. In conclusion we comment on arguments refuting superluminal neutrinos and note a similar consequence of quantum mechanics that conservation of energy can be violated, if only for a short time.

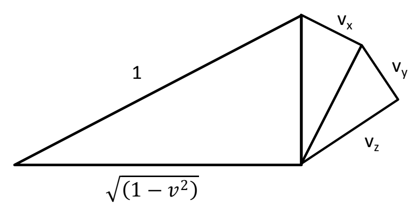
INTRODUCTION:

While the findings from OPERA [1] where neutrinos were clocked traveling faster than light (and seemingly break the laws of special relativity) seems to have been refuted by ICARUS [2], it behooves one to revisit those laws [3]. I argue that special relativity is secure in its postulates and conclusions if there is no variance in the particle's velocity. However we know from Quantum Mechanics and the Heisenberg Uncertainty Principle that this is not possible without the variance in position being infinite [4]. I argue that if a particle moves from location (0,0,0) to position (x,y,z) in time t, (where x, y, and z are random variables drawn from the squared magnitude of their corresponding wavefunction, then the quantity $v_0 = \sqrt{\bar{x}^2 + \bar{y}^2 + \bar{z}^2}/t$ is what special relativity postulates is always less than or equal to the speed of light [5], c. However on the contrary, the quantity measured by OPERA was $\bar{v} = \overline{\sqrt{x^2 + y^2 + z^2}}/t$ which can be greater than the speed of light, when v is close to c and the variance on x, y, or z is large enough. This difference between v_0 and $\bar{v}$ is well known in Information Theory as Jenson's Inequality [6]; namely that the average of a function is not the same as the function of the average.

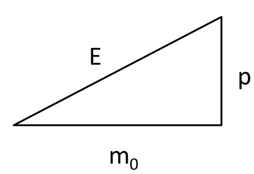
1.0 GEOMETERTY:

A way to visualize what is happening in special relativity is by using right triangles and Pythagoras's theorem.

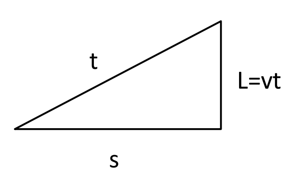
With $c = 1$, we begin by decomposing unity into four orthogonal dimensions with magnitude, v_x, v_y, v_z, and $\sqrt{(1-v^2)}$ where $v^2 = v_x^2 + v_y^2 + v_z^2$.



This decomposition remains valid for the four orthogonal dimensions with real magnitudes when $v^2 \in [0,1)$, or $-1 < v < 1$. We can now apply this decomposition into the energy domain by multiplying by the energy E. We can see that for a given particle with a rest mass m_0 and momentum, p_x, p_y, p_z we have the relationship, $E^2 = m_0^2 + p^2$ where $p^2 = p_x^2 + p_y^2 + p_z^2$, $p^2 = v^2 \cdot E^2$ and $m_0^2 = (1-v^2) \cdot E^2$



In the absence of a force and with $x(0) = 0$ we have $x = v_x t$. Re-writing the right triangle again we have $t^2 = s^2 + L^2$ where $L^2 = (vt)^2 = x^2 + y^2 + z^2$ and $s^2 = (1-v^2) \cdot t^2$



This decomposition of unity forms the basis for the Lorentz transformation and special relativity. We can see that if the non zero rest mass m_0 and momentum, p_x, p_y, p_z are all real (i.e. no imaginary component) the velocity obeys $-1 < v < 1$ as guaranteed by orthogonality [7].

We assum the reader is familiar with the OPERA and ICARUS results relating to super luminal neutrinos [1,2]. Since the particle has a wavefunction and x y and z are no longer eigenstates, but rather random variables with a variance, the ensemble average no longer obeys this decomposition. We still have $\bar{x} = \overline{v_x} t$ where the average is over the ensemble, but $\bar{L} = (\bar{v}t) = \overline{\sqrt{x^2 + y^2 + z^2}}$ is no longer equal to $L_0 = v_0 t = \sqrt{\bar{x}^2 + \bar{y}^2 + \bar{z}^2}$. By Jensen's inequality we have $\bar{L} \geq L_0$. In this respect special relativity no longer applies because the length function in an orthogonal basis is convex and thus taking the ensemble average ruins the equality by Jensen's inequality.

2.0 DIFFUSION:

First consider the diffusion of a free particle in its rest frame. We know that the variance of a free particle's wavefunction will diffuse due to quantum effects. With no force, $x(t + t_0) = x(t_0) + p(t_0)t/m$ and we have,

$$\Delta x^2(t) = \langle x^2(t_0)\rangle + \langle [x(t_0), p(t_0)]_+ \rangle\, t/m + \langle p^2(t_0)\rangle t^2/m^2$$

Balancing the terms and relating them through the Heisenberg uncertainty principle we have $\langle x^2(t_0)\rangle = \hbar t/2m$ and $\langle p^2(t_0)\rangle/m^2 = \hbar/2tm$. We also have $\langle [x(t_0), p(t_0)]_+\rangle = 0$, leading to $\Delta x^2(t) = \hbar t/m$ [8].

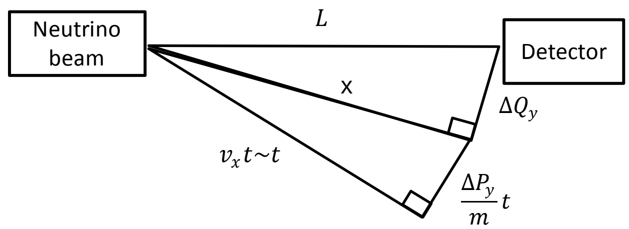
The Langevin equation confirms this when the time is greater than the coherence time $\tau = \hbar/2k_B T$. If $m\ddot{x} = -m\dot{x}/\tau + F_x$ where F_x is the noisy driving force (uncorrelated with x), the variance of x as a function of time is [9]

$$\Delta x^2(t) = 2\mathrm{D}\left(\mathrm{t} - \tau\left(1 - e^{-t/\tau}\right)\right)$$

where $D = k_B T\tau/m$ from Einstein's relation [9]. Beyond the coherence time $\Delta x^2(t \gg \tau) \cong 2\mathrm{Dt} = \hbar t/m$. Used later we also have $\Delta x^2(t \ll \tau) \cong (k_B T/m) \cdot t^2$, or the thermal velocity, $\Delta v_T{}^2 = (k_B T/m)$, which is also derivable from the equipartition theorem [4].

The particle at rest has width $\Delta x = \Delta y = \Delta z$, however in a reference frame moving with velocity $v_x = v$ the width of the x dimension will shrink by the factor $1/\gamma = \sqrt{1 - v^2}$, such that $\Delta x'^2 = (1 - v^2)\Delta x^2$, which we consider as zero when v_x is close to one (the speed of light). Yet in the other two dimensions transverse to the motion, there is no shrinkage [3].

If the particle starts at (0,0,0) and moves for t seconds in the x direction we will have two effects. First the displacement of the y and z dimensions will be according diffusion of the position. Second the effective velocity along the line of sight will also be slightly greater due to the momentum diffusing. When the average velocity in the x direction is just under the speed of light the following equation holds, as visualized in the figure for one of the two transverse dimensions.



$$L = \sqrt{t^2\left(1 + \frac{\Delta P_y{}^2}{m^2} + \frac{\Delta P_z{}^2}{m^2}\right) + \Delta Q_y{}^2 + \Delta Q_z{}^2}$$

Thus the velocity $v = L/t = \sqrt{x^2 + y^2 + z^2}/t$ is a random variable with probability

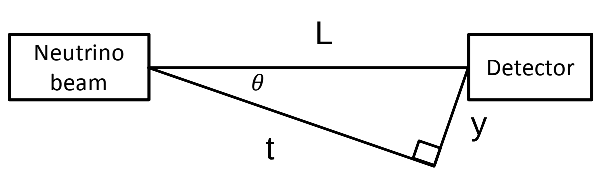
$$P(v)dv = \frac{tvdv}{\sqrt{2\pi(\Delta v)^2(L^2 - t^2)}} e^{-(L^2 - t^2)/2t^2(\Delta v)^2}$$

Where $\Delta v^2(t) = \Delta Q_y{}^2/t^2 + \Delta P_y{}^2/m^2 + \Delta Q_z{}^2/t^2 + \Delta P_z{}^2/m^2$. Taking the first moment and assuming $\Delta v \ll 1$ we have, $\bar{v} \cong 1 + (\Delta v)^2/2$

However before we further reduce this answer, we must consider that the width of the neutrino beam is finite and non-zero.

4.0 ANGULAR LIMIT:

One complication we find is that the width of the neutrino beam in the OPERA and ICARUS experiments are finite as the beam is not radially distributed. Thus $\langle y^2(t_0)\rangle$ and $\langle z^2(t_0)\rangle$ as seen at the detector are reduced. While the neutrinos still diffuse, the initial spread of the beam is such that the ones that diffuse greater than $t\Delta\theta$ end up past the detector. Considering only the spatial effects (and not the momentum effects) we have



If t is the baseline and y is a Gaussian random variable with zero mean and variance $\langle y^2(t_0)\rangle$ and if $Prob(\theta)$ is the probability the neutrino starts out at an angle θ away from the x direction with variance $(\Delta\theta)^2_\nu$, we have with $\sin(\theta) \sim \tan(\theta) \sim \theta$

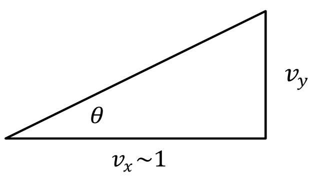
$$Prob(y = \mathrm{t}\sin(\theta)) = \mathrm{Norm}(0, \Delta Q_y{}^2)$$

Where $\Delta Q_y{}^2 = 1/(1/t^2(\Delta\theta)^2_\nu + 1/\langle y^2(t_0)\rangle)$.

For the width occurring from the momentum space we find the same result but for a different reason. Here the width is limited because the initial beam width is not zero as it has a finite variance in velocity and thus momentum. $\langle y^2(0)\rangle = \hbar^2/4\Delta p_T{}^2 = \hbar^2/4m^2\Delta v_T{}^2$. From the Heisenberg uncertainty principle we have

$$\Delta P_y{}^2 = \frac{\hbar^2/4}{\langle y^2(0)\rangle + \langle y^2(t_0)\rangle}$$

Since we can re-write $(\Delta\theta)^2_\nu$ as $\Delta v_T{}^2$ (given the geometry of velocity right after the neutrino is generated as below), one can show that $\Delta P_y{}^2/m^2 = \Delta Q_y{}^2/t^2$



Lastly if we associate the temperature along the transverse direction equal to the variance in the velocity times the mass, $k_B T = m\Delta v_T{}^2$ [10], we end up with our primary result, where $\tau = \hbar/2k_B T$.

$$\bar{\boldsymbol{v}} \cong \mathbf{1} + \hbar/\boldsymbol{m}(\boldsymbol{t} + \boldsymbol{\tau})$$

4.0 BEAM WIDTH:

Relating this to the measurements of the CERN neutrino beam we have the following:

Assuming the angular variance of the muon neutrino beam is equal to the angular variance of muons we have $t^2(\Delta\theta)^2_{\nu\mu} = (800\ \mathrm{m})^2$, where the baseline is $t = 730km$. This is calculated from the FWHM of $2.8km$ [11] and distribution similar to the triangle distribution.

However we should also expect the subsequent diffusion of the neutrinos to spread the beam. If ΔD is is the width of the neutrino beam at the detector we have $\Delta D^2 = (\Delta\theta)^2_{\mu\nu} \cdot t^2 + \langle y^2(t_0)\rangle$. Note that the variance in momentum does not impact the spread of the neutrinos since it only impacts the velocity.

We can find ΔD by looking at the expected number of neutrinos that pass through the detector. We have

$$Events = \frac{Events}{Neturino\ detector} \cdot \frac{Neutrino\ detector}{Neutrino} \cdot \frac{Neutrino}{pot} \cdot pot$$

From [1] we have CC events = 15223, and from [12] pot=2.8e19 and 1 neutrino for 2.2 pots. Averaging the cross section $6.7X10^{-43}\ m^2/GeV$ [13] over the flux as a distribution of energy [14], we have probability of an event given the neutrino passed through the detector is, $P = \sigma N/A$, where $\sigma = 1.23X10^{-41}\ \mathrm{m}^2$, $A = (6.7\mathrm{m})^2$ is the area of the detector and N is the mass of the detector over the mass of a proton; $N = 7.8X10^{32}$.

The fraction of neutrinos that pass through the detector is area integral over the squared radial distribution of the neutrino beam. Analysis on the published radial distribution [11] shows, the fraction of neutrinos that pass through the detector is $\mathrm{A}/4(\Delta\mathrm{D})^2$.

Putting this all together we have $(\Delta\mathrm{D})^2 = (1400\ \mathrm{m})^2$

5.0 NEUTRINO MASS:

We should be able to calculate the mass of the neutrino with a measurement $\bar{v}$ assuming there is not a complication of flavor eigenstates and mass eigenstates which would make the effective mass different than the mass eigentstate. Putting this possible complication aside we have the following:

With a measurement of $(\Delta\mathrm{D})^2$ and $t^2(\Delta\theta)^2_{\nu\mu}$ we can determine $\langle y^2(t_0)\rangle = (1150\ \mathrm{m})^2$. From here we can estimate $\Delta Q^2 = 1/\left(1/t^2(\Delta\theta)^2_{\nu\mu} + 1/\langle y^2(t_0)\rangle\right) = (1000\ \mathrm{m})^2$. Since we found that $\bar{v} \cong 1 + 2(\Delta Q^2)/t^2$ we have $\bar{v} - 1 = 1.5X10^{-6}$ as the excess average velocity that should have been measured by OPERA. Resulting in $\delta t = 4\ nsec$, less than the main refuted results from OPERA[1] and ICARUS [2].

From here we should be able to calculate the 1st mass eigenstate which would dominate the other mass eigenstates in determining the width of the wavepacket since it relates to the inverse of the mass. We have $\langle y^2(t_0)\rangle = \hbar t/2m_1$, or $m_1 = 5.5X10^{-43}\mathrm{kg}$; a very small mass indeed.

How if we were to consider an effective mass we would look at the transition matrix between flavor and mass eigenstates, but for now that effect is less than an order of magnitude.

6.0 FERMILAB 1979:

A review of another experiment from Fermilab 1979 gives us a second measurement of the average excess speed of muon neutrinos with a baseline of 550m[15].

Assuming the physics of the beam from Femilab and OPERA are similar we can use the same $(\Delta\theta)^2_{\nu\mu}$ we just calculated. Also using m_1 we just calculated we see that, $\Delta Q^2 \sim (\Delta\theta)^2_{\nu\mu} \cdot t^2$ and thus the excess average velocity is $< 5X10^{-5}$ as the paper finds.

6.1 SN1987A:

In the case of SN1987A (where neutrinos were found to arrive 3 hours before the photons over a baseline of 168,000 light years [16]), we see the opposite thing happen to limit ΔQ^2. Since the supernova produced a circularly uniform distribution of neutrinos we have $\Delta Q^2 = \langle y^2(t_0)\rangle$. Using the first mass eignestate as calculated above we have $\bar{v} - 1 \sim 10^{-20}$. From this it appears likely that the original explanation of early arrival time of the neutrinos remains as the delay of the visual light coming from the dense core.

7.0 CONCLUSION:

We find that OPERA should have measured super luminal neutrinos, but only at $\bar{v} - 1 \sim 1.5X10^{-6}$. There are a number of rough estimates in this number like the width of the beam but this analysis should give us the correct mass of the neutrino within an order of magnitude.

It would make sense to redo an experiment with a wider beam to rule out the beam limiting the effect of the diffusion. Of course a short baseline experiment would give better fidelity in calculating the first mass eigenstate and would allow a direct comparison to a photon beam for timing.

Other refuting arguments of super luminal neutrinos include the requirement that a super luminal neutrino will radiate energy and thus the neutrinos would have a different energy distribution at collection than at creation [17,18,19]. However I claim the quantity which determines the validity of Lorenz invariance and any subsequent radiation is $\sqrt{\overline{v_x}^2 + \overline{v_y}^2 + \overline{v_z}^2} = \sqrt{\overline{v_x}^2} = \overline{v_x}$ which is always less than the speed of light. Thus the assumption in these refuting arguments is invalid.

One might think of a similar consequence of quantum mechanics where a violation of the conservation of energy is permitted if only for a short amount of time (giving rise to virtual particles for example) [4]. In this case we have a situation where information could travel faster than the speed of light, but only by one part in hbar divided the action of the path.

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