Some problems in Hungarian mathematical competition. IV.

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Abstract

In this work, we continue to present some interesting problems in the Transylvanian Hungarian Mathematical Competition held in 2012.

D1st Problem. Solve in $\mathbb{Z}$ the following equation: $\frac{3}{\sqrt{x}}+\frac{2}{\sqrt{y}}=\frac{1}{\sqrt{2}}$.

Ferenc Kacsó

D2nd Problem. Let's consider set $M=\left\{a^2-2ab+2b^2|a,b\in\mathbb{Z}\right\}$. Show that $2012\notin M$. Prove that M is a closed subset of $\mathbb{N}$ in respect of the multiplication of integers.

Béla Bíró

D3rd Problem. Solve in $\mathbb{R}$ equation $5x^3-18x^2+43x-6=3\cdot 2^{x+2}$.

Béla Kovács

D4th Problem. In the not isosceles triangle ABC we have $m\left(\widehat{BAC}\right)=90^\circ$, AD, AE, AO are altitude, angle-bisector and median, respectively $(D,E,O\in(BC))$. Prove that if $OE=2DE$, then $AB^2+AC^2=4AB\cdot AC$.

Lajos Longáver

D5th Problem. Uncle John has taken blood pressure drops for a long time according to the following rule: 1 drop for one day, 2 drops daily for two days, ..., 10 drops daily for ten days, 9 drops daily for nine days, ..., 2 drops daily for two days, 1 drop for one day, 2 drops daily for two days, ... One day he forgot how many drops he should take, finally he took 5 drops. What is the probability that he guessed right the daily dose? Later he remembered taking 5 drops previous day, so he calmed down that he guessed the dose correctly with high probability. What is this newer probability?

Ágnes Mikó

D6th Problem. a) At least how many elements must be selected from the group $(\mathbb{Z}_{2k},+)$ such that among the selected elements surely there exist three (not necessarily distinct) with sum $\hat{0}$?
b) The same question for $(\mathbb{Z}_{15},+)$.

Szilárd András