Null-sum physics I

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Abstract

We show that the free propagation and interactions of all fundamental particle fields can be derived from arithmetic summations over a suitably parametrized null-cone. Our approach represents a considerable simplification of the path integral formalism and leads to essentially identical results as those obtained from conventional procedures, but without using i) differential operators, ii) the principle of least action or iii) explicit gauge symmetry.

1 Introduction

In the Lagrangian formulation of particle physics, one constructs a sum of various Lorentz scalar, charge-neutral products of fermion and boson fields and their derivatives. The interactions between these fundamental fields are then deduced by invoking the principle of (broken) gauge symmetry. There exists a transparent one-to-one correspondence between terms in the Lagrangian and irreducible single vertex Feymann graphs.

This approach to physics has enjoyed an unbroken run of successes since its introduction early last century, culminating in the so-called Standard Model [1] based on the $SU(3) \times SU(2)_L \times U(1)$ symmetry group. The predictions of the Standard Model are probably consistent with all experimentally determined properties and interactions of the known elementary particles. Such is the dominance of this paradigm that attempts (e.g. SUSY) to extend the Standard Model so to encompass gravitational phenomena also invariably start from some more intricate, supersymmetric, Lagrangian.

The Lagrangian approach to physics appears to lead to renormalizable theories of extraordinary precision. It may therefore safely be regarded as "true" in some sense and very few people would dare to suggest that it needs to be replaced by something better or even that such a replacement is possible. Nevertheless, a small number of mathematicians have in the course of the past decades made brave attempts to formulate elementary physics in terms other than the least action principle [2, 3, 4].

For all its power and undoubted success, the Standard Model is not without its weaknesses. For one thing, it requires a relatively large number (twenty-one) of independent constants associated with particle masses, mixing angles and gauge coupling constants. Another unsatisfactory aspect is that it discloses more bosonic degrees of freedom than actually exist in the real world, and the unphysical modes must be eliminated by choice of gauge, which is aesthetically unpleasing. A more fundamental problem with the Lagrangian approach is that it presupposes a smooth space-time manifold over which the fields are supposed to be differentiable, right down to the smallest Planckian dimensions ($\approx 10^{-33}$ cm). For these and other reasons, no one today seriously believes that the lagrangian density represents the ultimate physical reality.

In the present paper we present a novel way of deriving the equations of motion for fundamental particle fields without recourse to the principle of least action or that of explicit gauge symmetry. Our *ansatz* does not have any obvious connection with the ultimate goal of grand unification and is hence not as ambitious as earlier schemes based upon pre-geometrical concepts[2, 3, 4]. However, insofar as the causal structure is partially embodied in the coordinate system itself, our null-cone approach does provide some hints as to how space-time may arise as an emergent property of quantum events constituting a network of charge-exchanging operations over Clifford algebras [5]. It is explicitly supersymmetric insofar as that fermions and bosons appear together in a homogeneous structure that somewhat resembles a single multiplet.

2 Definitions

Consider the following spinorial parametrization (c.f. Eqn. (4.4) of [4] and [6]), of the past ($t > 0$) null cone in Minkowski spacetime with vertex at the origin:

$$M_P x_\mu = \bar{\mathbf{N}} \sigma_\mu \mathbf{N} \tag{1}$$

where M_P represents some large (Planckian) mass and the dimensionless spinor comprises 4 real quantities: n_A, n_B, n_C, n_D:

$$\mathbf{N} = \begin{pmatrix} n_A + i n_B \\ n_C + i n_D \end{pmatrix} \tag{2}$$

It is easy to see that $x_\mu x^\mu = 0$ and it can be verified that elements of volume in N-space are Lorentz scalar invariant:

$$d^4 n \equiv dn_A \cdot dn_B \cdot dn_C \cdot dn_D = M_P^2 t^{-1} \cdot dx \cdot dy \cdot dz$$

where $t = \sqrt{x^2 + y^2 + z^2}$
Unbounded N-space has the following relation to momentum space:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{ik_\nu x^\nu} d^4 n \equiv \int e^{ik_\nu x^\nu} d^4 n = \frac{M_P^2}{k_\mu k^\mu} e^{ik_\nu x^\nu} \tag{3}$$

$$\int x_\lambda e^{ik_\nu x^\nu} d^4 n = \frac{i k_\lambda M_P^2}{(k_\mu k^\mu)^2} e^{ik_\nu x^\nu} \tag{4}$$

$$\int x_\lambda x_\mu e^{ik_\nu x^\nu} d^4 n = \frac{k_\lambda k_\mu M_P^2}{(k_\mu k^\mu)^3} e^{ik_\nu x^\nu} \tag{5}$$

It follows from (3) and (4) that solutions of the Dirac equation for a fermion of mass m in the presence of a gauge field A_μ:

$$\left[i\gamma^\mu(\partial_\mu + eA_\mu) - m\right]\psi(x) = 0 \tag{6}$$

- also necessarily satisfy the integral equation:

$$\int \left[\mathbf{I}_4 + i x_\mu \gamma^\mu [m + eA^\nu \gamma_\nu]\psi(n)\right] d^4 n = 0 \tag{7}$$

This equivalence can be verified by differentiating three times from the left.
In the chiral representation (7) has the explicit form:

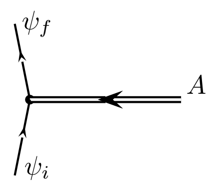
$$\int \left[\begin{pmatrix} \mathbf{I_2} & 0 \\ 0 & \mathbf{I_2} \end{pmatrix} + \frac{i}{M_P} \begin{pmatrix} \sigma_\mu \bar{\mathbf{N}} \sigma_\mu \mathbf{N} & 0 \\ 0 & \sigma_\mu \bar{\mathbf{N}} \sigma^\mu \mathbf{N} \end{pmatrix} \begin{pmatrix} eA_\nu \sigma_\nu & m \\ m & eA_\nu \sigma^\nu \end{pmatrix} \right] \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} d^4n = 0$$

By separating out the contribution to the zero-sum from the immediate vicinity of the origin, this identity assumes the character of a real space propagator. Moreover, provided M_P is sufficiently large compared to m, we can replace the 4-dimensional integral, with negligible loss of accuracy, by a summation over all discrete integer values :- $-\infty < n_A, n_B, n_C, n_D < +\infty$:

$$\begin{pmatrix} \psi_L(0) \\ \psi_R(0) \end{pmatrix} \approx - \sum_{n \neq 0}^{\infty} \left[\begin{pmatrix} \mathbf{I_2} & 0 \\ 0 & \mathbf{I_2} \end{pmatrix} + \frac{i}{M_P} \begin{pmatrix} \sigma_\mu \bar{\mathbf{N}} \sigma_\mu \mathbf{N} & 0 \\ 0 & \sigma_\mu \bar{\mathbf{N}} \sigma^\mu \mathbf{N} \end{pmatrix} \begin{pmatrix} eA_\nu \sigma_\nu & m \\ m & eA_\nu \sigma^\nu \end{pmatrix} \right] \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} + \mathcal{O}(m/M_P) \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \quad (8)$$

The above expression tells us that, because single fermions obey the Dirac equation, the value of each chiral component is given by the unweighted sum of its amplitude at all past null-cone "sampling" points *plus* a t-weighted sum of the opposite chirality amplitude multiplied by the (bare unrenormalized) mass m *plus* a t-weighted sum over the product of all gauge fields with said chiral component.

The first-order gauge interaction corresponding to Feymann graphs like:



is according to (8):

$$\psi_f(0) = \frac{ie}{M_P} \sum_{n=-\infty}^{\infty} \begin{pmatrix} \sigma_\mu \bar{\mathbf{N}} \sigma_\mu \mathbf{N} \cdot A_\nu \sigma_\nu & 0 \\ 0 & \sigma_\mu \bar{\mathbf{N}} \sigma^\mu \mathbf{N} \cdot A_\nu \sigma^\nu \end{pmatrix} \psi_i(n) \quad (9)$$

It follows from (3) that current source equations of the form $\partial_\nu \partial^\nu A_\mu = J_\mu$ can also be expressed as a sum over the null-cone:

$$A_{if,\mu} = \frac{e}{M_P^2} \int J_{\mu,if} d^4n = \frac{e}{M_P^2} \int \bar{\psi_f} \gamma_\mu \psi_i d^4n = \frac{e}{M_P^2} \sum_{n=-\infty}^{\infty} \bar{\psi_f} \gamma_\mu \psi_i \quad (10)$$

The scalar value given by the null-cone sum of the product of a fermion, a conjugate fermion and one $\bar{\mathbf{N}} \gamma \mathbf{N}$ factor expresses the amplitude that a state ψ_i remains in a state ψ_f, which is the orthonormality condition:

$$\delta_{if} = \frac{e}{M_P^2} \sum_{n=-\infty}^{\infty} \bar{\mathbf{N}} \gamma^\mu \mathbf{N} \cdot \bar{\psi_f} \gamma_\mu \psi_i \quad (11)$$

The scalar amplitude that a gauge boson is absorbed in a volume defined by the normalization factor on the fermion field requires two $\bar{\mathbf{N}}\gamma\mathbf{N}$ factors as in (5) in order to get the correct dimension and tensorial rank of zero:

$$c_{if} = \frac{e}{M_P^4} \sum_{n=-\infty}^{\infty} \bar{\mathbf{N}}\gamma^{\mu}\mathbf{N} \cdot \bar{\mathbf{N}}\gamma^{\nu}\mathbf{N} \cdot A_{\nu}\bar{\psi_f}\gamma_{\mu}\psi_i \tag{12}$$

Combination of (10) and (12) yields an amplitude for two photon processes like Compton scattering and Bremsstrahlung that agrees with traditional treatments.

For the case of self-interacting massive bosons like the W and Z there are additional contributions to the null-cone summation corresponding to the Higgs interaction, 3-boson and 4-boson interactions. Neglecting tensor indices and group structure constants, the propagated boson amplitudes will be of the general form:

$$W = \frac{1}{M_P^2} \sum_{n=-\infty}^{\infty} \left[g_1 J_W + M_W^2 W + g_2 M_W \bar{\mathbf{N}}\gamma\mathbf{N} W_a Z_b + g_3 W_a W_b W_c\right] \tag{13}$$

At this point we must acknowledge a discrepancy from the Standard Model regarding the 3-boson vertex term. According to Eqns. (6) and (7) of [1], we would expect a contribution to the boson amplitude like

$$\frac{1}{M_P^2} \sum_{n=-\infty}^{\infty} \left[g_2 \partial [W_a Z_b]\right] \tag{14}$$

- instead of the g_2 term in (13). There is no way of getting a derivative operator into our theory and there is no term one could introduce into a Lagrangian that would yield a term like our g_2 term in the ensuing equations of motion, so this discrepancy can not be resolved. Experimental observations of the momentum form factors for WWZ and $WW\gamma$ interactions could therefore provide a test of the validity of the ideas presented in this paper.

Subject to the above previso, it looks as though nearly every term in the Standard Model Lagrangian [1] corresponds to a sum over the null-cone of the tensor product of the participating fields and that the amplitude of any leg of any irreducible interaction vertex can be derived from the product of all the other legs.

3 Conclusions

The above findings embolden us to put forward the following hypothesis, namely that the amplitude of any fundamental fermion or boson at a point is given by the sum over the past cone of all products of masses, gauge, fermion fields and $\bar{\mathbf{N}}\sigma_{\mu}\mathbf{N}$ that yield the charge,spin and metric dimension appropriate to that fundamental particle. Each term is associated with a dimensionless coupling factor, but the number of independent factors is limited by group symmetry considerations. Equations (8) and (13) above exemplify the fermion and gauge boson instances of this principle, but one would also expect corresponding propagators for Higgs bosons and other possible types of particle. In the case of fermions, the contribution from the origin is omitted from the sum on the RHS, but the sums have otherwise essentially the same sum-of-products form.

This, we believe, is basically how the principle of least action in 4-dimensional spacetime is manifest on the null-cone, and by extension the whole causal network.

It remains to be seen whether and how gravity can be incorporated into this picture. Further work will be directed to determining whether the very rigid constraints imposed by the null-cone summing procedure impose dependencies between the twenty-one independent constants of the Standard Model. As to the question of what operators constitute the absolutely conserved charges, we allow ourselves to be guided by the ideas of [5], and can already see a way to naturally incorporate three spinor families into our scheme by means of a trivial departure from the approach presented in that paper.

References

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