Theory of Gravitation and Quantum Uncertainty

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Abstract

We argue that invariance under continuous 4 dimensional space-time transformations is in conflict with determinism in physics due to the existence of families of smooth structures. We speculate that quantum uncertainty emerges as a result, suggesting a new way to unify gravity and quantum mechanics.

100 years ago, Einstein, using the equivalence principle, predicted the bending of light by gravity. Five years later, he published the general theory of relativity when he also corrected the predicted amount of light bending. Quantum theory, on the other hand, embodies the uncertainty principle that is very nonintuitive to the classical world view. Ever since the birth of the general theory and quantum mechanics, the unification of general relativity and quantum theory has been one of the most important goals of fundamental physics. In this essay, we will discuss how extending Einstein's pattern of argument in relativity may lead to a new direction in research to fulfill that goal.

As is well known, Einstein himself never felt that quantum theory, due to the critical role of uncertainty plays in the theory, is a fundamental theory of nature. In contrast, relativity theories rest on a simple proposition. In the popular misconception, relativity advocates that “everything is relative.” But the main proposition in relativity is about the absoluteness of physical laws. Thus, whenever there is a conflict between the absoluteness of individual perception and the absoluteness of physical law, we give up the absoluteness of the individual perception. In the case of special relativity, in order for the Maxwell equations for electromagnetism to be "absolute" (in different inertial frames), we make space and time relative. As a consequence, electric field and magnetic fields are relative as well, depending on the frame of reference. The transformations to relate the different inertial frames are the Lorentz transforms, which are Minkowski metric preserving linear transformations. General relativity goes one step further in demanding that physical laws be invariant under smooth transformations. Under this invariance, gravitational force becomes relative in that it is not experienced in a free fall frame, or it is experienced in an accelerating frame even in absence of a gravitational source. The gravitational force becomes equivalent to the inertial force. In this essay, we propose the possibility of quantum uncertainty emergent from an even more general class of invariance. The more general class of invariance consists of continuous transformations of the space-time continuum that may not necessarily be smooth. We regard the generalization as a natural next step in the progression of relativity. We will argue for the possibility that quantum uncertainty may actually be a consequence of relativistic principles.

As invariance becomes more general, the structure of space-time becomes less structured. A less structured space-time can accommodate richer varieties of phenomena. In special relativity, space-time has the rigid structure of a linear space. In general relativity, the space-time continuum is a smooth manifold and we demand invariance of physical laws under diffeomorphisms. Let us now imagine that the space-time continuum is only endowed with a structure of continuity, *i.e.*, it is the topological manifold R^4. In classical physics, physical laws are expressed as equations of motion for particles or field equation in the form of differential equations. Differential equations must be defined over differentiable structures. It has been several decades since mathematicians discovered[1] that on R^4, unlike in any other dimensions, differentiable (or smooth) structures are not unique. In fact, there are continuous families of differentiable structures in a R^4 space-time. If space-time has no *a priori* differentiable structure, then all the possible structures are equally valid. But the equations of motions in all the differentiable structures are then all equally valid. We then face the situation that physical motion are non-unique, which means we have broken determinism. In appealing to the relativistic principle, we must be willing to give up determinism in favor of the invariance of the physical laws. An orbit may be smooth relative to a particular smooth structure but non-smooth relative to a different smooth structures. Different smooth structures imply different equations of motions. But all are valid. The possible motions are therefore uncertain. Because there are continuous families of smooth structures on R^4, the uncertainties for outcomes are uncountably many. Generically, the trajectories of particles are non-smooth. Motion based on a smooth structure appears to be non-smooth in a different smooth structure since the structure that generates the motion would appear wrinkled. Such properties are reminiscent of properties of quantum mechanical uncertainties. Field equations of motion will be non-unique also and the related uncertainties should be related to second quantization. We speculate that this is the source of quantum mechanical uncertainties. It has a flavor of a many-worlds interpretation of quantum uncertainty without many worlds. The different smooth structures would drive different temporal evolutions, they nevertheless represent the same space-time. Since within each smooth structure the equations of motion remain classical, there may not be infinities that have be dealt with using renormalization in the usual quantum formulation. In our hypothetical theory (call it "Q theory"), invariance of space-time under continuous transformations is in conflict with

1 See for example, "4-Manifolds and Kirby Calculus", Robert E. Gompf and Andras I. Stipsicz, Graduate Studies in Mathematics, Volume 20, American Mathematical Society, 1999.

deterministic smooth equation of motion. We thus have the progression of theories, each enlarging the invariance group of space-time, as in Table 1.

Table 1. Structures of Theories

Theory	invariance	Space-time structure
Newtonian	Time translation, space translation and rotation	Time × Euclidean Space
Special Relativity	Linear Lorentz transformation	4D Linear space
General Relativity	diffeomorphisms	Smooth 4D manifold
"Q Theory"	homeomorphisms	4D topological manifold

Four dimensional manifolds have unique richness of structures. If our speculation is correct, not only we would be able to explain the nature of quantum uncertainties, but also we would be able to explain why the space-time is four dimensional because all R^n, except R^4, have unique smooth structure and therefore cannot support the uncertainties of equations of motion. The possibility of relation of smooth structures on R^4 to quantum gravity has been discussed before.[2] Our proposal here is from a different perspective. Instead of regarding the smooth structures as extra degrees of freedom in a path integral formulation of space-time geometries or regarding a particular smooth structure as representing space-time, we regard the smooth structures as the possible mechanism of genesis of quantum uncertainties. Instead of regarding quantization as an external procedure to convert classical physics to quantum physics, our conjecture is that the smooth structures correspond to different, equally valid classical physics. The time evolutions of these different classical physics lead to the quantum uncertainties. Thus, the mechanism should explain non-gravitational quantum phenomena as well. For dimensions less or equal to 6, one can also define piecewise linear (PL) structures and show that

2 See for example, T. Asselmeyer-Maluga, C.H. Brans, Exotic Smoothness and Physics, World Scientific, Singapore 2007 and reference therein.

the classifications are equivalent to smooth structures. This obviously applies to R^4, which suggests that non-gravitational physics can be "quantized" based on the PL structures.

What implication does our conjecture have on the quantization of gravity? In our conjecture, the quantum uncertainties are the result of the existence of inequivalent smooth structures on space-time. It is natural that field equations of general relativity are inequivalent and therefore evolve with uncertainties. Our suggestion is that the theory should be completed by a measurement theory utilizing the reference frames defined by the smooth structures and there is no need to "sum over" the possible paths.

In conclusion, we have conjectured that quantum uncertainties can be explained by non-uniqueness of smooth structures on space-time, which explains the uniqueness of 4 as the dimension of space-time and unifies gravity and quantum mechanics.