The Hellenistic and Alexandrian Influences on Johannes' Kepler Work

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Abstract

We investigate the close relationships between the astronomical and scientific work of the famous great astronomer Johannes Kepler and the major scientific in general, and astronomical in particular, achievements that took place within the Hellenistic period and the Alexandrian epoch that followed immediately, that is from the Battle of Actium until the death of the scholar and polymath Hypatia. We propose that the Hellenistic Philosophy and Science, as being developed by Giants of Mind, such as Hipparhus of Rhodes, Archimedes of Syracuse, Aristarchus of Samos, Apollonius of Perga, Menelaus of Alexandria, Euclid of Alexandria, Ptolemy of Alexandria, and by philosophers such as Plotinus and Proclus, as well as all the other Neoplatonic philosophers of this era, instills and permeates Kepler's philosophical view of Cosmos, and at the same time sets the guidelines and paves the way of his lifelong scientific endeavor upon discovering and determining the true laws of planetary motion within a Heliocentric Universe.

Johannes' Kepler work is deeply rooted within these epochs, his philosophical views, his religious beliefs, his physical and astronomical insights about the Universe, all of them can be traced already in the Hellenistic and Alexandrian science. Although the time interval between his era and the great achievements of the Hellenistic and Alexandrian scientists, mathematicians and philosophers spans many centuries, measured according to the flow of historical time, in reality the spirit of these eras irrigated continuously all of the great civilizations which inherited the Hellenistic tradition, that is the Byzantine, the Arabic and Islamic, and the European civilization, giving birth to the Renaissance of the 12^{th} century and to the Renaissance of the 14^{th} century. Kepler, by following his own scientific routes of inquiring, has to be confronted with this tradition, either by accepting some parts of it, or by rejecting specific dogmas and conceptions about the nature and the structure of Cosmos entailed in this tradition, in order to proceed to the theoretical astronomic endeavor of a logically comprehensible, and beautifully ordered Universe, which is subject to unifying physical principles of symmetry, harmony, proportion and analogy, as well as physical laws. Kepler makes the decisive step towards

the New Physical Philosophy, and towards the Mechanization of the World Picture, while always remaining deeply influenced by the Hellenistic and Alexandrian spirit that continued to be influential up to Kepler's era.

1. General Remarks about the Framework of Kepler's Work

Johannes Kepler is a multidimensional personality, therefore his work exhibits the synthesis of many elements, combining the mystic Christian Neoplatonism with the most modern, advanced and innovative Worldview of his age [1]. This unique mixture of often seemingly antithetic elements determines the fabric of his work, which belongs simultaneously to the pre – Newtonian age in Physical Philosophy and to the Novel post-Newtonian age, an epoch of massive Paradigm shifts in all Natural and Social Sciences, the precursor of an epoch which shall alter the landscape and the map of the European and eventually of the Global Thought drastically, and forever, and lead to the Scientific Revolution, the Industrial Revolution and the Age of Sailing [2].

Kepler does not only contribute greatly into the field of Astronomy, his rich imagination and his unusual mathematical talent guides him into various routes of discovery. Kepler stays always a Pioneer, in every field he investigates. Among his achievements belong the foundation and the formulation of the modern science of Optics, the invention of refractive Keplerian telescope, the Kepler's Conjecture about the tightest arrangement and packing of the spheres in the three dimensional space, the extensive use of the calculus of logarithms, as introduced by John Napier, the study of the Kepler regular polyhedra, the geometry of the snowflakes, and the Kepler's monster tiling.

The Universe, according to the dogmatic orthodox teachings of the Church at the era of Kepler, obeys strictly and irrefutably the Aristotelian cosmological model, which was held to be inspired by God and stayed in strict accordance with the Holy Scriptures, as embedded within the Ptolemaic astronomical Paradigm. This Universe, according to the empirical astronomical evidence available to all great astronomers during the pre-telescopic era of Astronomy, consists only of the five, visible by naked eye, planets, that is Mercury, Venus, Mars, Jupiter and Saturn, without their satellites, the Earth-Moon system, the Sun, as well as the sphere which carries the fixed stars. This outer sphere demarcates the limits of this, finite in extent, Universe. According to this cosmological system the Earth is placed at the center of the Universe, and stays motionless, while the planets are orbiting around this geostatic Earth. Observations, such as the detection of supernovas, the passing of comets, or of meteoroids entering Earth's atmosphere, cannot find their correct scientific explanation within the realm of this model, although there exists a thorough scientific treatment about the nature, the regularities and the characteristics of the lunar and solar eclipses, as well as about the orbital motions of the planets [3, 4, 5, 6, 33].

Also, as regards the science of Astronomy, even today we are confronted with the following epistemological constraint: The empirical evidence obtained does not stem by performing various experiments in the laboratory, its fountain is purely observational, and relies upon the quality of the available types of astronomical instruments, as used and developed by successive generations of polymaths, scholars and astronomers.

The notion of the physical law also differed in Kepler's era, since the Zeitgeist in Kepler's epoch was strongly influenced by concepts of Reality, such as the prevalent position of the five Platonic solids within the Creation of Cosmos, or the basic musical harmonies produced by the motion of the planets, as they move on their orbits, that is the Music of the Spheres. These basic "modulators of physical reality" are regarded to play the role of the physical laws, which determine the Universe and the totality of the physical phenomena, according to the eternal inherited ideas of the Demiurge. We can already detect the degree of difference between this conception about the nature of the Physical law, and our contemporary understanding about it [6, 7, 10, 22, 24].

2. Kepler enters Astronomy: "Mysterium Cosmographicum"

Kepler enters the scenery of the cultural and social interregnum, the epoch between the 16^{th} and 17^{th} century, and into the realm of the Physical Philosophy and Astronomy, with the publication of his first monumental work *Mysterium Cosmographicum*. Here, Kepler outlines the basic philosophical doctrines, and his intense religious beliefs, together with his physical and astronomical speculations, which guide him on his struggle for the discovery of the blueprint of Cosmos according to the will and plan of the Demiurge, especially as presented within the Platonic spirit, and according to the teachings of Proclus and his exegesis about the Platonic doctrines.

Within this work, he proposes to identify the structure of the Universe, and also to provide solid physical laws, which in turn support the underlying structure of Creation, which is assumed to be Neoplatonic, and also justifies the Heliocentric Copernican astronomical model. His major goal is to define strictly, and establish the foundations of the Heliocentric Copernican paradigm, based completely on physical reasoning, combined by mathematical imagery, as well on the available empirical evidence [4, 6, 8, 16].

Kepler begins to ask fundamental physical questions which arise naturally within the context of the Copernican Paradigm. His scientific conquest involves the physical reasoning about the number, the size, and the motion of the spheres, which carry the planetary orbits. He also tries to define, that is to derive from the available data, a concrete pattern among the proportions of the planetary distances with respect to the Sun. On the other hand, these methods by alone cannot provide a sufficient reason for limiting the number of the planets to its correct value, that is a sufficient reason which explains the existence of only six planets within the Universe. This comes due to the fact that the space outside Saturn's orbit and within Mercury's orbit could still be divided, and hold hypothetical "infinitesimal" planets, or celestial objects not visible by naked eye. Also, within the Copernican astronomical framework, there existed huge interplanetary spaces between each of the six planets, and the physical reason for their determination had also to be explained and justified [4, 6 -8, 10, 16].

Kepler achieves this second goal by the introduction of a structure which again involves the five Platonic solids, with these placed in a specific succession, in an arrangement constituted by the intercalation of these five Platonic solids between the imaginery spheres that carry the orbits of the planets. According to Kepler's first astronomical model, first we encounter the sphere of the fixed stars, and then, at a large

distance in space, and towards the Sun, that is the center of the Universe, comes the sphere which carries the orbit of Saturn. A hexahedron is inscribed within this sphere, and another sphere is inscribed within the hexahedron, which corresponds to the orbit of Jupiter. Following the same procedure, we obtain the following order of the other Platonic solids: This is comprised by the dodecahedron, the icosahedron and the octahedron, corresponding accordingly to the orbits of Mars, Earth, Venus and Mercury. Within an acceptable margin of error, as deduced by the available at hand astrometrical data of his time, Kepler also determined an amazing fact, that the ratios among the radii of the circular orbits of the planets could correspond to the ratios among the radii of the inscribed spheres.

Here, we encounter another eminent feature of Kepler's way of thinking and doing science, that is the abandoning of the concept of the mean Sun, a notion which steadily occurs and used within the framework of the Ptolemaic, Copernican and Tychonic astronomical models. The hypothesis of the mean Sun corresponds to an abstract mathematical point, which has no physical relevance, a point whose function cannot be justified within the context of a truly dynamical astronomical Paradigm.

The distances within Copernicus' astronomical Paradigm were derived via the assumption of the mean Sun, while using the mean Sun as a center renders the sphere of the Earth to be dimensionless, since the distance between the Mean Sun and the Earth remains always equidistant. The dodecahedron and the icosahedron within Kepler's geometrical arrangement would touch the same sphere, a fact that is mathematically acceptable, but not physically relevant, since the science of Astronomy according to Kepler belongs to the realm of the Physical Philosophy, and not to the domain of mathematical constructions which simply "save the phenomena" [8 -9, 16].

It is amazing that Kepler derives one of the most elegant and beautiful astronomical theories, which is at the same time is wrong. Kepler, within his lifelong investigation into the Laws of the celestial objects, shall reject some of the promises of his first astronomical Theory, but shall follow the very same paths into the unknown, the Terrae Incognitae lying outside the horizon of the Aristotelian Physics and Metaphysics, and of the boundary of the Ptolemaic Astronomical Model.

3. The Major Breakthrough: "Astronomia Nova" and "Harmonice Mundi"

Kepler, after the publication of the *Mysterium Cosmographicum*, continues its endeavor for determining the number of the planets, their proportions and the law of their motions. After a long struggle, both in his personal, as well as in his professional life, Kepler finally succeeds to formulate the first two of his eponymous laws of planetary motion, that is the Law of the ellipses and the Law of equal areas, and continues on his investigation for unraveling the blueprint of the Demiurge, up to the statement of his third, or Harmonic law, as presented in his most famous *Harmonice Mundi*. Kepler, by these attempts, expands further the Heliocentric Copernican model, and discovers the Harmony of the Spheres produced by the celestial dance of the planets, in a purely dynamical way, which involves also the introduction of some kind of "gravito-magnetic

force" exerted by the Sun, and on the planets, as they orbit along their elliptical paths, that is a precursor theoretical model of Newton's theory of Gravitation [4, 8, 12, 23].

Kepler's *Astronomia Nova* includes these first two Laws of planetary motion, and Kepler enters the relevant discussion about these with the statement of these laws, together with their order of appearance during his own theoretical struggle. Kepler always follows the general schemes and the scientific guidelines, as well as his own heuristic methods, already encountered within his first major work, the *Mysterium Cosmographicum* [4, 10]. Within the corpus of the text of *Astronomia Nova*, Kepler establishes several fundamental facts about the structure and the dynamical laws of Cosmos, and also presents clearly his religious beliefs, his philosophical inclinations, together with the statement of the two of his eponymous laws. Also, Kepler introduces the notion of the true Sun, a fact which is strongly interconnected and with his own theological and philosophical beliefs, and then determines the fact that the planetary orbits obtain the shape of an ellipse, with the true Sun placed at one of the foci of the ellipse. He also discovers that the angular velocity of the planets vary with the passage of time, so that at perihelion they accelerate, and at aphelion they decelerate, always following the Law of equal areas [4, 9, 11].

The issue about the description of the planetary orbits in terms of combined perfect circular motions, and not by the use of ellipses, is very interesting. Greek astronomers, like Apollonius of Perga, the founder of the theory of the *Conics,* that is the study of the conic sections, already use the deferent – epicycle system in order to reproduce the planetary orbits. The Giants of the Hellenistic and Alexandrian astronomy never bothered to use the shape of the ellipse in order to represent the motions of the celestial objects, probably because they preferred to use combinations of perfect circular motions, that is motions that can be easily defined, drawn, and even reproduced by the use of complicated gear mechanisms, as we observe them to function in the ancient mechanical Cosmos, known as the Antikythera Mechanism [3, 33]. The description of the planetary orbits via the use of epicycles is akin to the use of the Fourier series. Here, we encounter again the great Ionian and Hellenistic decisive step towards the understanding of the logically ordered Universe, the Cosmos, where Nature is being described according to the Laws of Physics by the exact use of Mathematics, that is the Laws of Physics are expressible by exact mathematical relations, thus reproducing and predicting the natural phenomena, a long tradtion and a concrete World Picture starting already from the epoch of the great Pythagoras. The science and the technology, as well as the tradition of the exact modeling is born within this scientific spirit, and implemented mechanically by the use of gear mechanisms, as we encounter in the Antikythera Mechanism. This mechanism can be used, among its other functions, for the prediction of the lunar and solar eclipses, the phases of the Moon, and possibly for the positions of the then known planets [3, 6, 13-14, 17 -18, 33 -36].

Near the end of his life, Kepler states in his monumental work *Harmonice Mundi* his third Law, namely the relation between the cube of the semi-major axis of the elliptical orbit for each planet, which stays always proportional to the square of the orbital period of the planet, something that holds for all the six planets, known at the age of Kepler, and was also confirmed to hold for the satellites of Jupiter and Saturn. Kepler states his amazement about this discovery, of fundamental physical character, only within a small passage within the text of the whole corpus of the *Harmonice Mundi.* It seems

that Kepler himself traveled through his novel, long and extended, mathematical and musical inquiry about the Harmony of the Spheres, and that the Harmonic law constituted perhaps only a by-product of his, more general in character, attempt. It is astonishing that Kepler tries to derive the Celestial Cantata produced by the dance of the planets, to define and describe the Harmony of the Spheres in a quantitative manner, this is the major goal, that is the determination of the musical scales attributed to every planet, as he moves around its orbit, but also the music produced by the collective "song of the planets". Kepler's attempt is surely founded upon a Neoplatonic Cosmos, as exposed for example in the teachings of the great Neoplatonic phisopher Proclus [4, 9, 12].

4. The Hellenistic Astronomy and Hellenistic Mathematics: Influences on Kepler's Thought

Johannes' Kepler work, which is the result of the harvest of a long scientific tradition, and not of parthenogenesis, that is the Hellenistic and the Alexandrian tradition in Astronomy and Mathematics. This tradition continues its route beyond its cradle, the Hellenistic and Alexandrian epoch, and up to the Byzantine, Arabic and Islamic, and the European civilization [3, 13, 14, 17, 30, 31 -33]. Within the work of Johannes Kepler we encounter, directly or indirectly, a whole list of references to the work and the achievements of Hellenistic astronomers and polymaths, who contributed greatly to the science of Astronomy, Cosmology, and even Astrology, a branch of social discourse which had already obtained a symbiotic relationship with Astronomy, at least up to the age of Johannes Kepler, Galileo Galilei, and Isaac Newton. We shall mention the main key influence on Kepler's work, namely the legacy of Aristarchus of Samos and Seleucus of Seleucia, of Archimedes of Syracuse, of Apollonius of Perga and of Menelaus of Alexandria, of Hipparchus of Rhodes and Ptolemy of Alexandria, and finally of Euclid of Alexandria. Kepler creates his whole "New Universe", partly by accepting and using freely the elements and parts of this tradition, and partly by rejecting them [13, 14, 17 - 18].

According to the mathematical part of Kepler's work, we may find a host of influences within the whole corpus of his work, achievements which appear for the first time in the Hellenistic period, as an offspring of the Greek tradition in mathematics. The great Astronomer continuously faces several hard physical and mathematical problems, which have to be solved, as he moved on towards his personal astronomical inquiry. Already, Kepler tries to answer several fundamental questions in his *Mysterium Cosmopgraphicum,* and these questions repeat to appear continuously as Leitmotiv within his thought and throughout his scientific life.

First of all, Kepler embraces completely the Heliocentric doctrine, which can be found as already present, though not in its strictly scientific form, in the corpus of the teachings of the Pythagoreans, while it was introduced as an Astronomical Paradigm by Aristarchus of Samos, and his disciple, Seleucus of Seleucia. The idea of Heliocentrism, already existing as a rivalry Paradigm within the framework of the Hellenistic astronomy, as opposed to the idea of Geocentrism, continues its road until its triumphally reappearance through the work of Nicolaus Copernicus. Although it was practically abandoned as a sound astronomical and cosmological idea through the centuries, finally

returned with all of its strength, as presented in Copernicus' Heliocentric Universe, and then as transformed within the Keplerian Paradigm [4, 6, 7, 13 -16, 23, 34]. Nicolaus Copernicus reintroduces the concept and the function of a Heliocentric Universe in his monumental *Revolutionibus Orbium Coelestum* [14 -16], where he places at the center of the Universe the Sun, although he continues to use basic notions and methods of the Geocentric Astronomical Paradigm of Ptolemy, that is the deferent- epicycle system and the eccentric points for the determination of the planetary orbits, or even the problematic notion of the mean Sun. Kepler is totally convinced that only the Copernican Paradigm corresponds to the physical Reality, this is the operative reason for the publication of his *Mysterium Cosmographicum*. According to Kepler, the Heliocentric doctrine involves both theological and philosophical ontological postulates, as well as astronomical, physical and mathematical considerations, and can serve as the only realistic model of Nature, according to Kepler's thought this is not another astronomical model which operates only for pragmatic reasons, only just "saving the phenomena".

We already know that Nicolaus Copernicus was strongly influenced both by the Neopythagorean doctrines, as well as by the work of Aristarchus of Samos, where this great Astronomer of the Antiquity was, in his turn, strongly influenced by the teachings of Philolaus of Croton, who postulated the existence of a Central Fire, around which all the other celestial objects revolve. Archimedes of Syracuse mentions the Heliocentric Cosmos of Aristarchus of Samos within a passage of his great work *Psammites*, addressed to King Gelon of Syracuse [17-18]. There, we read that Aristarchus introduced novel hypotheses within the realm of Astronomy, where according to his astronomical views the Universe obtains an enormous size, if compared with the Aristotelian Geocentric Universe, the Sun and the fixed stars remain motionless, the Sun is placed at the center of Cosmos, and all planets revolve around the Sun. This served as a secure and intriguing scientific basis for the acceptance of the Heliocentric model in the field of Astronomy, that the Heliocentric Universe can carry a logical meaning, and correspond to the physical Reality, and is deeply rooted within the astronomical scientific praxis, although it had to wait for centuries for his reappearance as a sound alternative fundamental astronomical notion by the work of Nicolaus Copernicus. Secondly, Kepler has to determine the shape of the trajectory of Mars and of Earth, a tedious task which had to be supported both by the evaluation of the data he possessed, after having served the great Danish observational Astronomer Tycho Brahe, and by the familiarity he already possesses with the geometrical properties of the ellipse. The mathematical definition of the ellipse, along with the main geometrical properties of this curve, but also of the circle, the parabola and the hyperbola, can already be traced in the textbook of Apollonius of Perga, the *Conics*. Apollonius of Perga, one of the greatest and most fruitful and influential mathematicians of all epochs, defines and describes collectively the properties of the circle, of the ellipse, the parabola and the hyperbola, namely the curves generated by conic sections. This constitutes another great influence on Kepler's thought, since the originality of Apollonius' treatment of the conic sections provided to Kepler an indispensable mathematical tool, which he used with the outmost success in his investigations [13 -14, 19, 33 -36].

Also, the mathematical method for calculating areas, or volumes of surfaces of revolution, became a most necessary needed tool for Johannes Kepler, since he was forced to compute the change of the angular velocity of the planets, as they evolve on

their elliptic orbits, and these necessary calculations eventually led Kepler into the formulation of his second Law, the Law of equal areas. This general mathematical method is already present within the work of Archimedes of Syracuse, an eminent figure in the History of Mathematics, in general, and a brilliant representative of the Hellenistic era, in particular.

Archimedes uses the Method of Exhaustion in order to calculate the area inclosed by a parabola and a linear segment between two points of the parabola. In his *Quadrature of the Parabola* Archimedes dissects the parabolic segment into an infinite number of triangles, and then proceeds to calculate the total area, that is the area between the line and the parabolic segment, by taking the limit of this summation which involves an infinite geometric series [18 -19]. Kepler must have been surely impressed by the Method of Exhaustion, and probably this groundbreaking mathematical procedure inspired his own work.

Whereas Apollonius, as a mathematician, offers to Johannes Kepler the material for a thorough study of the ellipse, that is the needed material for the statement of his first Law of planetary motion, Archimedes, again as a mathematician, offers him, or inspires him, with a general method for calculating areas of arbitrary shape, which in turn constitutes the essence of the Law of equal areas. It is worth noting that the Method of Exhaustion used by Archimedes becomes a source of inspiration for the founders of Infinitesimal calculus, Gottfried Leibniz and Isaac Newton, where both of these Giants of Physical Philosophy in general, and Astronomy in particular, develop further the framework of the mathematical Method of Exhaustion, in order to represent motion, and calculate areas and volumes of arbitrary shape.

The always recurrent Leitmotiv within Kepler's work, that is the extensive geometrical study of the five Platonic solids, and their use as fundamental "modulators of physical reality", not only in the scales of the Microcosm, but also for the overall structure in the scales of the Macrocosm, as presented for the first time in Kepler's *Mysterium Cosmographicum*, and reappearing in triumph in his *Harmonice Mundi*, could never have existed without Euclid's monumental work, the *Elements*. Euclid of Alexandria not only provides an axiomatic definition of the Geometry we call as Euclidean, but also studies the definition, the construction, the properties and symmetries of the five convex regular polyhedra, the Platonic solids. The axiomatic deductive treatment of Geometry finds its culmination in the last book of the Elements, the Book 13, where the Platonic solids are studied extensively [7, 8, 14, 19, 20].

Among the other influences on Kepler's work, we can also mention the work of Menelaus of Alexandria, titled as the *Sphaerica,* that is the Alexandrian mathematician who founded and studied extensively the Spherical Geometry, and also provided the fundamental theorems of Spherical Trigonometry, a most valuable tool for the needs of theoretical and observational Astronomy, especially for the conversions between different astronomical coordinate systems, which is in use for astronomical purposes up to our times.

To this list of the great Hellenistic and Alexandrian personalities we may add the most obvious influences, always referring to the realm of Astronomy, which include Aristotle and his Cosmological Paradigm, as being supported by the Ptolemaic astronomical model, and founded upon the Aristotelian Metaphysics and Aristotle's Causation. Kepler has to work with the main astronomical theoretical tools available in

the Ptolemaic system, and study it with the outmost detailed examination. Kepler departs on his own route first by applying in his calculations the notions and the mathematical machinery of the astronomical Paradigm of Ptolemy of Alexandria, and only after this attempt he renders them obsolete within the framework of his "New Astronomy". Johannes Kepler follows a long tradition, rooted both in the Physical Philosophy the Astronomy and the Mathematics of the Hellenistic Epoch, the tradition of Heliocentrism, as being proposed and developed by Aristarchus of Samos, and his follower, Seleucus of Seleucia, and at the same time it surpasses its barriers and paves the path towards the Modern Physical Philosophy [9, 15, 16, 21].

5. The Neoplatonic Philosophy of Nicolaus Cusanus and Kepler's Worldview

There are numerous influences on Kepler's work which can be classified into the philosophical and theological aspects of his thought, always having in mind that these aspects, the theological, the philosophical, the physical, the astronomical and the mathematical comprise an organic Whole, so that the one element of this "noetic structure" cannot be separated from all the others.

Among the most important spiritual influences on Kepler's thought, except the one exerted by his spiritual father and mentor Michael Maestlin, one of Kepler's professors in the University of Tuebingen, when Kepler was a student, and a most important proponent of the Copernican Paradigm in general, we can list without any doubt the name of Nicolaus Cusanus. Within the philosophical treatises written by Nicolaus Cusanus it becomes evident the foundation of a novel form of Metaphysics, and a novel form of Kinematics and Dynamics within the Universe, strongly opposed to the Aristotelian doctrines, and always strongly influenced by the teachings of Proclus, as well as by Pseudo-Dionysius the Aeropagite. According to Cusanus, the Heliocentric doctrine stands as a metaphysical Principle underlying the physical reality. Nicolaus Cusanus offers and introduces several novel conceptions and severe modifications about the fundamental cosmological and physical aspects Cosmos, such as about the nature of space and time, the nature of motion and change, the notion of the system of reference of a material observer within the Universe, or about the knowability and the quantitative description of the physical phenomena, which stay always in contrast with the Paradigmatic doctrines of his era, and inspire, but also shape, at the greatest degree the thought of Johannes Kepler [22, 23, 25 -29].

The Neoplatonic tradition in Philosophy, as introduced by the work of Plotinus, Porphyry, Zethos, Iamblichus, Proclus, Pseudo-Dionysius the Aeropagite, and Boethius, shaped and gave essence and substance to the work of Nicolaus Cusanus, and provided him with several sources of inspiration. Within the intellectual tradition and atmosphere of Nicolaus Cusanus, one of the most prominent figures of the European philosophical tradition, Johannes Kepler dives deeply, and finds support for his personal astronomical investigations [7, 10, 22 -23, 28 -29]. Kepler finds his Ariadne's Thread out of the Labyrinth he enters, not only by resorting to the physical and astronomical works of the Hellenistic and Alexandrian period, but also to the mystic Christian Neoplatonism of Nicolaus Cusanus.

The most basic and fundamental notion of Heliocentrism, a main scientific guideline and a governing physical principle, can also be regarded as a specific meta-principle of physical reality, which reflects the structure of Cosmos. This governing principle has to be combined with the kinematical and dynamical characteristics of Cosmos, in order for us to obtain a Picture of it. The role of the Sun within a heliocentric Universe is introduced not only because of physical reasoning, that is according to the principles of Economy and Parsimony of the astronomical model at hand, but also according to the belief into ontological reasoning about the correct place of the Sun within Cosmos, a Cosmos which operates according to a certain self-consistent and evasive Metaphysics [23].

Nicolaus Cusanus, one of the most influential Thinkers of the European tradition, ecclesiastical reformer, administrator and cardinal, as well as a self –taught in Theology and Philosophy, exposes his own idiosyncratic, but also systematic, mystic Christian Neoplatonism, a platform and a fountain which serves as the philosophical and theological foundation of Kepler's work. As we already mentioned, Nicolaus Cusanus introduces novel concepts and key ideas within the realm of Astronomy, Mathematics, Cosmology and the Natural Sciences, and especially to the empirical and experimental counterpart of each scientific modeling of Cosmos. Within his extended work the "divine Cusanus", as Kpelr calls him, the Ontology, the Philosophy of Mind, the Epistemology and the Theology built a solid, unified corpus of knowledge [7, 10, 22-24, 28 -29].

The thought of Cusanus embraces the Christian God, the Cosmos, that is the physical Universe, the Christ and the Men. The Universe, according to Cusanus, is the limited and contracted maximum, that is the Image of the Absolute Maximum, which in turn is identified with God, the Creator, the Demiurge and Architect of Cosmos. All beings and all phenomena are enfolded in the One Divine Source, and at the same time they stand as the unfolding of God in Space and Time. Nicolaus Cusanus follows the lines of specific Neoplatonic doctrines, according to which the physical Universe, the realm of human experience, of genesis and death, of constant flow, evolution, birth and decay, is enfolded within the Nous of the Demiurge, and depends strongly upon him [25 -29].

Nicolaus Cusanus pursues even further his own theological and philosophical investigations, as based on the postulate that everything contained within the Cosmos also contains everything. Each individual physical existence, according to Cusas' conception of the Universe, is an image that reflects the totality of the One, while it stays also interconnected with all other individual existences, or parts, of the Universe. All the elements and phenomena of physical reality are interconnected via mutual interrelations with all the other parts or existences, within the hierarchically stratified structureof Creation. This physical Universe is characterized, as we already mentioned, by motion and change, its nature is dynamical in character, and not static. As Cusanus remarks, the finite in extent motion or change within the Universe can be measured, but without obtaining a fixed maximum or minimum.

Nicolaus Cusanus bases his argumentation of the metaphysical grounds referring to the ontological relativity of more and of less, achieving thus the statement of remarkable comments about the nature of the Physical Universe.

The Earth cannot be conceived within the geocentric and geostatic Universe of the Ptolemaic astronomical Paradigm, because in Cusanus' Cosmos everything is in constant

motion, and cannot be placed at the center of the Universe, since this center cannot exist. The outermost sphere carrying the fixed stars does not comprise a physical boundary, but refers to a Universe which is infinite in extent. This dynamical Universe, always subject to motion and change, does not obtain a fixed center, or a concrete boundary, the spheres that carry the motion of the wandering stars, the planets, cannot retain a perfect and exact shape, as they have to, according to the Aristotelian Kinematics, the Aristotelian cosmology and the supporting Ptolemaic astronomical Paradigm [25-27]. This fundamental ideas about Cosmos continues its presence and prevalence within Johannes' Kepler work, and provide an ontological status for his eponymous Laws of planetary motion, the Law of the ellipses and the Law of equal areas.

Since, according to Nicolaus Cusanus, the Demiurge of the Universe is at the same time the exact circumference, but also the center of the Universe, there does not exist a preferable direction or point within the physical Universe, it is only a matter of the viewpoint, it depends only on the frame of reference of the observer. This viewpoint only determines the center and the circumference each time the observer determines and lists the various phenomena, and all the beings within Cosmos. In this, infinite in extent, Universe there does not exist a fixed central point, one immovable and privileged point, since according to the measurements of one observer, this point will become movable, and non – privileged, according to another independent observer. According to Nicolaus Cusanus the totality of substances of matter, and all kinds of motions, have to be non-uniform. He thinks that if any two objects are equal and similar, they also must be again equal and similar, with this process continuing ad infinitum. Since this is absurd, Cusanus states the principle that any positable entities always shall remain different. One motion cannot be equal to another, or be the measure of another motion, the act of measuring which uses an object of reference measures another quantity, but these two have always to differ, thus the description of the planetary orbits in terms of perfect circular motions becomes impossible [24 – 27, 29].

The parallels between Cusa's thought and Kepler's first two discoveries are striking, since Kepler introduces the ellipses as the correct geometrical curves which describe the planetary orbits, and not a system of perfect circular motions, since perfect motions cannot exist within the physical Universe. Secondly, Kepler postulates on the basis of the available to him data that the motion of the planets is non- uniform, as they accelerate at their perihelia and decelerate at their aphelia.

Proclus belongs to the major key figures who shape and inspire Cusanus' work, and it is Proclus, with his lifelong attempt, who introduces a new spirit, and sheds new light, on the Platonic dialogues, especially the famous *Timaeus*, the dialogue which remained the source of inspiration of numerous generations of scholars and scientists that followed, from the end of the Alexandrian era and up to the age of Kepler. The Mathematical, Physical, Theological, and Ethical exegesis provided by Proclus constitutes the fountain of Nicolaus Cusanus' world of ideas. The Platonic motto *panta en pasin* refers to an organically structured Cosmos, which obeys symmetries, harmonies, proportions and analogies, is hierarchically constructed, with all of its layers interconnected and interacting with all the other, it is a Cosmos which is comprehensible, knowable, that is measurable, at the same time [24 -29].

Proclus provides the needed World-view, or the specific framework and mentality for the inquiry into the general physical principles, out of which the fabric of Cosmos is

made of, and according to which the Universe operates. He studies Form and Matter, the nature of Space and Time, but also of Movement and Change. Among all his great philosophical achievements, Proclus reintroduces Plotinus' three hypostases of the natural Reality, that is the One, the Intellect and the Soul, paving the way for the European theology. These three substances are going to be incorporated within the Christian scriptures and influence the Worldview of the European civilization, in general, and correspond to the picture of the Keplerian universe, in particular, first by passing through the network of the thought of Nicolaus Cusanus [7, 10, 22, 28 -29].

Conclusions

The Hellenistic era and the Alexandrian period, they both mark a specific node, as well as a system of reference, within the collective framework of our Global Cultural Heritage. Within the Hellenistic era many scientific branches solidify, in their structure and function, and obtain their Paradigmatic scheme, that is their axiomatization, their methodology, their observational elements and their own space of inquiry, as well as the necessary scientific equipment used for observational or experimental purposes. These basic schemes, together with the structure which supports them, are going to be transmitted and imitated, let us say, in the form of cultural "Memes", always in a productive fashion, and according to the measures and the overall understanding, or Zeitgeist, of each great Civilization that stayed influenced by these achievements. There exists a constant flow of written evidence, that is of major key texts and books of reference, as well as an interchange or emigration of great polymaths and scholars, among the Byzantine, the Arabic and Islamic, and the European Civilization, throughout the ages and up to the era of the Renaissance [30-32, 34].

The realms of the Natural Sciences, and of Technology, become transformed, new Sciences are introduced for the first time, while older forms of scientific branches become enriched, both theoretically and empirically, and obtain their axiomatic formulation, such as the Ptolemaic astronomical Paradigm or the Euclidean Geometry, or obtain their first systematic form, such as Botany, Psychology and Geography.

The extensive use of mathematical methods and the deliberate empirical research within the Natural Sciences comprises the core of the Hellenistic spirit, and this Hellenistic "Light" it conveyed through the ages and adopted by the great Civilizations that followed and were influenced by it. It is evident that the conduct of Scientific research demands also a whole range of supporting social structures, and the necessary infrastructure for these social structures, a social atmosphere in which scientific researched is praised as significant, always ready and able to transform and expand its borders, and be safely delivered and passed on to the new generations. It is in these eras, and especially the Hellenistic epoch, where rich and strong patrons made possible the establishment of such "paradigmatic social structures", that is the establishment of Universities, of Research Centers, or of Technological and Scientific Institutes. Within these, Theory meets Praxis, that is the theoretical coexists and influences the practical and the technological, and vice versa. As such most famous exemplar we can mention the Museum and Library of Alexandria, or the cities of Syracuse and Rhodes, and the city of Pergamon. We can mention a full catalogue, though not exhaustive, which includes the Natural Sciences, as fully developed and within a progressive manner, in the Hellenistic

era, and the Alexandrian period: This has to include the fields of Astronomy, Fluid Mechanics, Solid Mechanics, Optics, Anatomy, Physiology, Geometry, Logic, Linguistics, Geography, Topography, Metallurgy, Textile Industry, Engineering, the Construction of Gear Mechanisms, such as the Antikythera Mechanism, and the Construction of Cybernetic Systems of first order, such as the various forms of Automata we encounter within the works of the great inventors who lived and flourished in the Museum and the Library of Alexandria [3, 13, 14, 17, 19, 33-35].

These Sciences, combined with their ways of transmission to the next generation, as well as their development and evolution, can be regarded as Autopoietic systems, and surely their roots and foundations can be traced some centuries ago, in the era of the Ionian Renaissance, a historical epoch which introduced the needed Worldview for the foundations and the general concepts we call Philosophy, and especially the Physical Philosophy, Astronomy, Physics, Geometry, Number Theory, just to mention a few of the most fundamental scientific disciplines. There are only a few fundamental principles and ways, guidelines and methods, uses of Reason and Views about Reality, for conducting Philosophy, according to the measures and standards of the Ionian Renaissance of the 6^{th} century BCE. The same holds also for the ontological establishment, the basic principles, the forms, the methodology, and the practice and development of the various scientific disciplines, ranging from Medicine to Metallurgy, and from Psychology to the construction of complicated Automata mechanisms [33 -36].

The Hellenistic era defines and shapes completely all the necessary elements for Kepler's work and achievements. It provides a solid and most successful basis for the science of Astronomy, as conceived and distributed through the passage of time, which is nothing but the Ptolemaic astronomical and cosmological Paradigm, a plethora of mathematical branches, such as the Euclidean Geometry, the Plane Trigonometry, the Spherical Geometry and the Spherical Trigonometry, mathematical innovations, such as the Stereographic projection, the study of the Conic sections, or the Method of Exhaustion, the doctrine of Heliocentrism, and the nature of a Heliocentric Universe, including without any doubt the teachings of the great Neoplatonic masters, who inspired and influenced strongly the thought of Johannes Kepler. Additionally, within the realms of these epochs Astronomy also obtains the most basic phylotypes of the observational and computing astronomical instruments, which dominated up to the begin of the telescoping era, and were used extensively up to the age of Johannes Kepler [2, 3, 9, 11, 14 -15, 19, 33- 34].

References

1. Caspar M., Kepler, New York: Courier Dover Publications, 1993.

2. Dijksterhuis E. J., The mechanization of the world picture: Pythagoras to Newton, Princeton, Princeton University Press, 1986.

3. Freeth, T.; Bitsakis, Y.; Moussas, X.; Seiradakis, J. H.; Tselikas, A.; Mangou, H.; Zafeiropoulou, M.; Hadland, R.; Bate, D.; Ramsey, A.; Allen, M.; Crawley, A.; Hockley, P.; Malzbender, T.; Gelb, D.; Ambrisco, W.; Edmunds, M. G., Decoding the ancient

Greek astronomical calculator known as the Antikythera Mechanism, *Nature,* v. 444, 30 November 2006, pp. 587-591.

4. Frisch C. (Ed.) . Ioannis Kepleri Astronomi Opera omnia, vols. 1-8, Frankfurt a.M and Erlangen, Heyder & Zimmer, 1858- 1872.

5. Jones A. Ptolemy in Perspective: Use and Criticism of his Work from Antiquity to the Nineteenth Century, New York: Series:Archimedes, Vol. 23 2010, XVI.

6. Dreyer J. L. E. A History of Astronomy from Thales to Kepler, Dover Publications Inc, 1967.

7. Dillon J., Lloyd P. G. Neoplatonic Philosophy. Introductory Readings, translations of portions of the works of Plotinus, Porphyry, Iamblichus, and Proclus, Indianapolis:Hackett, 2004.

8. Kepler J. Mysterium Cosmographicum: The secret of the universe, (Trans. Duncan A., M., Introduction and Commentary by Aiton E. J., with a Preface by Cohen I.B.), New York: Abaris Books, 1981.

9. Stephenson B. Kepler's Physical Astronomy [Studies in the History of Mathematics and Physical Sciences 13], New York, Berlin: Springer Verlag, 1987.

10. Vlastos G. Plato's Universe*,* Seattle: University of Washington Press, 1975.

11. Voelkel J. R. Johannes Kepler and the New Astronomy, New York and Oxford: Oxford University Press, 1999.Kepler J. Harmonice mundi libri V, The Harmony of the World, (Trans. Aiton, E. J., Duncan A.M., Field J. V.), Philadelphia: American Philosophical Society (Memoirs of the American Philosophical Society), 1997.

12. Wilson C. Horrocks, Harmonies, and the Exactitude of Kepler's Third Law, *Science and History*: Studies in Honor of Edward Rosen, [Studia Copernicana 16] Wrocław (Ossolineum), pp. 235–59, 1978.

13. Sarton G. A history of science I. Ancient science through the Golden Age of Greece. II. Hellenistic science and culture in the last three centuries B.C. Norton, 1970.

14. Russo L. The Forgotten Revolution. How Science Was Born in 300 BC and Why it Had to be Reborn, Springer, 2004.

15. Kuhn T. The Copernican Revolution: Planetary Astronomy in the Development of Western Thought, Cambridge Mass: Harvard University Press, 1957.

16. Copernicus N. On the Revolutions, (trans. Rosen E.), Baltimore: The Johns Hopkins University Press (originally published as volume 2 of Nicholas Copernicus: Complete Works*,* Warsaw: Polish Scientific Publishers, 1978), 1992.

17. Spandagos E., Spandagou R. Οι Αστρονόμοι της Αρχαίας Ελλάδος [The Astronomers of Ancient Greece], in Greek, Αίθρα, 1995.

18. Spandagos E. Αρχιμήδους Βίος και Έργον, [Life and Work of Archimedes], in Greek, Αίθρα, 2011.

19. Filli C. Οι Αρχαιοελληνικές Καταβολές των Συγχρόνων Μαθηματικών, [The Ancient Greek Roots of Modern Mathematics], in Greek, Α. Παπασωτηρίου & ΣΙΑ Ο.Ε., Athens, 2010.

20. Mueller I. Euclid's Elements and the Axiomatic Method. *The British Journal for the Philosophy of Science.* v. 20, no. 4, Dec. 1969, 289 – 309.

21. Koestler A. The Watershed: a Biography of Johannes Kepler, Garden City: Doubleday, 1960

22. Steel C. "Neoplatonism" in Encyclopedia of Philosophy, Borchert D. M. (Ed.), Detroid: Macmillan Reference USA, vol. 6, col. 546-557.

23. E. Theodossiou, E. Danezis, V. Manimanis, E.- M. Kalyva, From Pythagoreans to Kepler: the dispute between the geocentric and the heliocentric systems, *Journal of Astronomical History and Heritage*, Vol. 5, No. 1, pp. 89 – 98, 2002.

24. Hopkins J. (Trans). Complete Philosophical and Theological Treatises of Nicholas of Cusa. Minneapolis: Banning, 2001.

25. Nicolas of Cusa. De docta ignorantia I (Ed. and Trans. Wilpert P., Senger H. G.). Philosophische Bibliothek, vol 264a, (Facing Latin and German text), 1994.

26. Nicolas of Cusa. De docta ignorantia I (Ed. and Trans. Wilpert P., Senger H. G.). Philosophische Bibliothek, vol 264b, (Facing Latin and German text), 1999.

27. Nicolas of Cusa. De docta ignorantia I (Ed. and Trans. Wilpert P., Senger H. G.). Philosophische Bibliothek, vol 264c, (Facing Latin and German text), 1999.

28. Kutash E. Ten Gifts of the Demiurge: Proclus on Plato's Timaeus, London/New York: Bristol Classical Press, 2011.

29. Yamaki K. (Ed.) .Nicholas of Cusa: A Medieval Thinker for the Modern Age, Richmond, England: Curzon, 2002.

30. Laiou A. E. Byzantium: A World Civilization, Washington, DC, : Dumbarton Oaks, 1992

31. Meri J. W. Medieval Islamic Civilization, New York: Routledge, 2004

32. Brotton J. The Renaissance: A Very Short Introduction, Oxford: Oxford University Press, 2006

33. Moussas X. Ο Μηχανισμός των Αντικυθήρων:Πίναξ, Το Πρώτο Μηχανικό Σύμπαν [The Antikythera Mechanism: Pinax, The First Mechanical Universe], (in Greek), Ένωση Ελλήνων Φυσικών, 2012.

34. Schroedinger E. Nature and the Greeks and Science and Humanism, (For. By Roger Penrose) Cambridge University Press, 1996.

35. Llyod G. E. R. Greek Science after Aristotle. New York: W. W. Norton & Co, 1973.

36. Evans J. The History and Practice of Ancient Astronomy, Oxford: Oxford University Press, 1998.