# Minimal Math Structures needed for E8 Physics 

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This is a rough outline of the minimal math structure needed for my E8 Physics model. It makes clear that a major barrier to understanding it is the amount of not-well-known mathematics of Real Clifford Algebras, Lie Algebras, Bounded Complex Domains, etc. ... No details are given here as they can be found in viXra 1312.0036 and my other viXra papers and my web sites at tony 5 m 17 h. net and valdostamuseum.com/hamsmith/ As to why anyone should expend the effort to understand the necessary math, the payoff is the substantially realistic results of E8 Physics calculations set out at the end of this paper. The calculations are mostly tree-level with a few first-order results so further calculation work might bring even closer correspondence with observations.
$\mathrm{Cl}(8)$ contains 52 -dim F4 = 8-dim vector +28 -dim bivector +16 -dim full spinor
F4 has the basic structure of a realistic physics model:
8-dim M4 x CP2 Kaluza-Klein spacetime $28=16-\operatorname{dim} U(2,2)$ Conformal Gravity + 12-dim $S U(3) x S U(2) x U(1)$ Standard Model 8 first-generation fermion +half-spinor particles 8 first-generation fermion -half-spinor antiparticles
but
F4 does not have complex domain structure or spacetime momentum structure or detailed component structure. To get all that structure you must go beyond F4 to 248-dim E8.

E8 is naturally contained in the 8-Periodicity tensor product $\mathrm{Cl}(8) \times \mathrm{Cl}(8)=\mathrm{Cl}(16)$ as $\mathrm{E} 8=120$-dim bivector of $\mathrm{Cl}(16)+$
+128 -dim half-spinor of $\mathrm{Cl}(16)=$
$=8 \times 8$ of $\mathrm{Cl}(8) \times \mathrm{xl}(8)+1 \times 28$ of $\mathrm{Cl}(8) \times \mathrm{Cl}(8)+28 \times 1$ of $\mathrm{Cl}(8) \times \mathrm{xl}(8)+$
$+8 x 8 \mathrm{Cl}(8)+$ half-spinor $x \mathrm{Cl}(8)+$ half-spinor $+8 \times 8 \mathrm{Cl}(8)$-half-spinor $\times \mathrm{Cl}(8)$-half-spinor

Local 28 -dim Spin(8) symmetry gives Complex Bounded Domain structure by 8 -complex-dim Spin(10) / Spin(8)xU(1) with 8-real-dim RP1 x S7 Shilov Boundary.

Triality among $8 \mathrm{Cl}(8)+$ half-spinor and $8 \mathrm{Cl}(8)$-half-spinor and $8 \mathrm{Cl}(8)$ vector extends that Complex Bounded Domain and RP1 x S7 Shilov Boundary structure to fermion representation spaces.

Gauge boson representation space Complex Bounded Domain structures are:
local D4 and D3=A3 subalgebra B2
with B3 / B2xU(1) and Shilov RP1xS4
local D4 and A3 subalgebras A2 and A1 and $\mathrm{A} 0=\mathrm{U}(1)$
with A3 / A2xU(1) and Shilov S5
and with B2 / A1xU(1) and Shilov RP1xS2
and with $\mathrm{AO}=\mathrm{U}(1)$ and trivial Shilov
The $8 \times 8$ of $\mathrm{Cl}(8) \times \mathrm{Cl}(8)$ has $8 \times 8$ spacetime position x momentum structure. 8-dim Kaluza-Klein has
CP2 = A2 / A1xA0 Internal Symmetry Space and M4 Physical Spacetime. M4 has symmetry for each force gauge group:

S4 = B2 / D2 for B2 Gravity
CP2 = A2 / A1xS0 for A2 of Color Force
S2 x S2 = A1/A0 x A1/AO for A1 Weak Force
S1 x S1 x S1 x S1 = A0 xA0 xA0 xA0 for A0 ElectroMagnetism
The 64-dim ++half-spinors have 8 components for each of the 8 fermion particles. The 64-dim - - half-spinors have 8 components for each of the 8 fermion antiparticles.

With World-Lines regarded as Strings, E8 Physics can be represented as 26D Bosonic String Theory with $8+8=16$ dimensions Orbifolded to represent Fermions and 26-16 $=10$ dimensions representing 4-dim CP2 Internal Symmetry Space and 6-dim D3 = A3 Conformal Spacetime that effectively reduces to Minkowski M4. Each cell of its local 26D Lorentz Leech lattice structure has Monster Group symmetry.

These structures, along with a MacDowell-Mansouri mechanism of Gravity, the Dark Energy of D3 / B2, the emergence of second and third generation fermions from formation of (4+4)-dim Kaluza-Klein spacetime, and a Mayer mechanism Higgs as fermion (predominantly Truth quark) condensate allow the construction of a realistic E8 Physics Local Lagrangian associated with $\mathrm{Cl}(16)$ with calculation, based on Schwinger Sources and Hua/Wyler geometry, of particle masses, force strengths, K-M parameters, and the ratio Dark Energy : Dark Matter : Ordinary Matter
and
a realistic Algebraic Quantum Field Theory by using Real Clifford 8-Periodicity to construct the completion of the union of all tensor products of $\mathrm{Cl}(16)$ thus producing a generalization of the type II1 hyperfinite von Neumann factor algebra.

Within that AQFT, Creation and Annihilation Operators are described by the Maximal Contraction of E8 = semidirect product H92 x A7 where

H92 is the Heisenberg Algebra with graded structure $8+28+56+1+56+28+8$ with bosonic even part 28+1+28 and fermionic odd part $8+56+0+56+8$ :

```
grade -3:8 =( by Triality ) = +half-spinor of Cl(8) = Fermion Particle
    = Creation of 8-component Neutrino Fermion = R8 in Jordan Algebra R1+R8 = J(Spin(8))
    Spin(8) = D4
+
grade -2 : 28 = Creation of Gauge Bosons = Jordan Algebra J(4,Q) with J(4,Q)O=J(3,O)
    Aut(J(3,O)) = F4
+
grade -1 : 56 = Creation of 8-component Electron / Quark Fermions = Freudenthal Algebra Fr(3,O)
    Aut(Fr(3,O)) = E6
+
grade 0:1 = R1 in Jordan Algebra R1+R8 = J(Spin(8))
+
grade +1:56 = Annihilation of 8-component Electron / Quark Fermions
+
grade +2:28=Annihilation of Gauge Bosons
+
grade +3:8 =( by Triality )= -half-spinor of Cl(8) = Fermion AntiParticle
    = Annihilation of 8-component Neutrino Fermion
```

A7 is the Lie Algebra $\mathrm{SI}(8)$ that by semidirect product goes into grade 0 of $\mathrm{H} 92 \times \mathrm{A} 7$ : grade $0: 1+A 7$ where $1=R 1$ in Jordan Algebra $R 1+R 8=J(S p i n(8)$ and 63 -dim A7 $=\mathrm{SI}(8)$ so that the dimension of $\mathrm{H} 92 \times \mathrm{A} 7=8+28+56+(1+63)+56+28+8=248$

```
Zero Grade Spacetime Algebra:
grade 0 Spacetime Position / Momentum Algebra = 1+SI(8)
```

Odd Grade Fermionic Creation Algebras:
grade -1 Electron / Quark Creation Jordan-type Algebra Fr( $3, \mathrm{O}$ )
grade -3 Neutrino Creation Jordan Algebra $J(\operatorname{Spin}(8))$ is in the $2^{\wedge} 8 \times 2^{\wedge} 8$ matrix algebra which is $\mathrm{Cl}(8) \times \mathrm{Cl}(8)=\mathrm{Cl}(16)$ the home of E 8 .

Even Grade Bosonic Creation Algebra:
grade -2 Gauge Boson Creation Jordan Algebra $\mathrm{J}(4, \mathrm{Q})$ has $\mathrm{J}(4, \mathrm{Q}) \mathrm{O}=\mathrm{J}(3, \mathrm{O})$
The relative symmetries of the relative grades are:
grade - 2 / grade -3: $\mathrm{F} 4 / \mathrm{D} 4=24$-dim OxOxO
grade -1 / grade -2 : E6 / F4 = 26 -dim J(3,O)o
total algebra / grade -1 : E8 / E6xA0xA0 $=168$-dim $\operatorname{PSL}(2,7)=\operatorname{SL}(3,2)$

The payoff for understanding all this math is the substantially realistic E8 Physics calculation data set listed on the next page.

Here is a summary of E8 Physics model calculation results. Since ratios are calculated, values for one particle mass and one force strength are assumed. Quark masses are constituent masses. Most of the calculations are tree-level, so more detailed calculations might be even closer to observations.

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Dark Energy : Dark Matter : Ordinary Matter = 0.75 : 0.21 : 0.04
```

Fermions as Schwinger Sources have geometry of Complex Bounded Domains with Kerr-Newman Black Hole structure size about 10^(-24) cm.

| Particle/Force | Tree-Level | Higher-Order |
| :---: | :---: | :---: |
| e-neutrino | 0 | 0 for nu_1 |
| mu-neutrino | 0 | $9 \mathrm{x} 10^{\wedge}(-3) \mathrm{eV}$ for $\mathrm{nu} \mathrm{C}^{2}$ |
| tau-neutrino | 0 | $5.4 \times 10^{\wedge}(-2)$ eV for $n$ u_3 |
| electron | 0.5110 MeV |  |
| down | 312.8 MeV | charged pion $=139 \mathrm{MeV}$ |
| up | 312.8 MeV | ```proton = 938.25 MeV neutron - proton = 1.1 MeV``` |
| muon | 104.8 MeV | 106.2 MeV |
| strange | 625 MeV |  |
| charm | 2090 MeV |  |
| tauon | 1.88 GeV |  |
| beauty | 5.63 GeV |  |
| truth(low state) | 130 GeV | $\begin{aligned} & \text { truth }(\text { middle state })=174 \mathrm{GeV} \\ & \text { truth }(\text { high state })=218 \mathrm{GeV} \end{aligned}$ |
| W+ | 80.326 GeV |  |
| W- | 80.326 GeV |  |
| W0 | 98.379 GeV | $\mathrm{Z} 0=91.862 \mathrm{GeV}$ |
| Higgs VEV | 252.5 GeV (assumed) | Mplanck=1.217x10^19 GeV |
|  |  | Higgs (low state) $=126 \mathrm{GeV}$ |
|  |  | Higgs (middle state) $=182 \mathrm{GeV}$ |
|  |  | Higgs (high state) $=239 \mathrm{GeV}$ |
| Gravity Gg | 1 (assumed) |  |
| (Gg)(Mproton^2 / Mplanck^2) |  | $5 \times 10^{\wedge}(-39)$ |

EM fine structure $\quad 1 / 137.03608$

Weak Gw 0.2535
$\mathrm{Gw}\left(\operatorname{Mproton}{ }^{\wedge} 2 /\left(\mathrm{Mw}^{+} 2+\mathrm{Mw}-\wedge 2+\mathrm{Mz} 0^{\wedge} 2\right)\right) \quad 1.05 \times 10^{\wedge}(-5)$
color force at $0.245 \mathrm{GeV} 0.6286 \quad 0.106$ at 91 GeV
Kobayashi-Maskawa parameters for $W+$ and $W$ - processes are:

|  | d | s | b |
| :---: | :---: | :---: | :---: |
| u | 0.975 | 0.222 | 0.00249-0.00388i |
| c | -0.222-0.000161i | 0.974-0.0000365i | 0.0423 |
| t | 0.00698 -0.00378i | -0.0418-0.00086i | 0.999 |
| The phase angle d13 is taken to be 1 radian. |  |  |  |

