International J.Math. Combin. Vol.2 (2010), 68-80

New Families of Mean Graphs

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Abstract: Let $G(V,E)$ be a graph with p vertices and q edges. A vertex labeling of G is an assignment $f: V(G) \to \{1,2,3,\ldots,p+q\}$ be an injection. For a vertex labeling f, the induced *Smarandachely edge m-labeling* f_S^* for an edge $e = uv$, an integer $m \geq 2$ is defined by

$$f_S^*(e) = \left\lceil \frac{f(u)+f(v)}{m} \right\rceil.$$

Then f is called a *Smarandachely super m-mean labeling* if $f(V(G)) \cup \{f^*(e) : e \in E(G)\} = \{1,2,3,\ldots,p+q\}$. Particularly, in the case of $m = 2$, we know that

$$f^*(e) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even;} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd.} \end{cases}$$

Such a labeling is usually called a *super mean labeling.* A graph that admits a Smarandachely super mean m-labeling is called *Smarandachely super m-mean graph*, particularly, *super mean graph* if $m = 2$. In this paper, we discuss two kinds of constructing larger mean graphs. Here we prove that $(P_m; C_n)m \geq 1$, $n \geq 3$, $(P_m; Q_3)m \geq 1$, $(P_{2n}; S_m)m \geq 3$, $n \geq 1$ and for any $n \geq 1$ $(P_n; S_1)$, $(P_n; S_2)$ are mean graphs. Also we establish that $[P_m; C_n]m \geq 1$, $n \geq 3$, $[P_m; Q_3]m \geq 1$ and $[P_m; C_n^{(2)}]m \geq 1$, $n \geq 3$ are mean graphs.

Key Words: Labeling, mean labeling, mean graphs, Smarandachely edge m-labeling, Smarandachely super m-mean labeling, super mean graph.

AMS(2000): 05C78

§1. Introduction

Throughout this paper, by a graph we mean a finite, undirected, simple graph. Let $G(V,E)$ be a graph with p vertices and q edges. A path on n vertices is denoted by P_n and a cycle on n vertices is denoted by C_n. The graph $P_2 \times P_2 \times P_2$ is called the cube and is denoted by Q_3. For notations and terminology we follow [1].

[1]Received March 26, 2010. Accepted June 18, 2010.

A vertex labeling of G is an assignment $f : V(G) \to \{1, 2, 3, \ldots, p+q\}$ be an injection. For a vertex labeling f, the induced *Smarandachely edge m-labeling* f_S^* for an edge $e = uv$, an integer $m \geq 2$ is defined by

$$f_S^*(e) = \left\lceil \frac{f(u) + f(v)}{m} \right\rceil.$$

Then f is called a *Smarandachely super m-mean labeling* if $f(V(G)) \cup \{f^*(e) : e \in E(G)\} = \{1, 2, 3, \ldots, p+q\}$. Particularly, in the case of $m = 2$, we know that

$$f^*(e) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even;} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd.} \end{cases}$$

Such a labeling is usually called a *super mean labeling.* A graph that admits a Smarandachely super mean m-labeling is called *Smarandachely super m-mean graph*, particularly, *super mean graph* if $m = 2$. The mean labeling of the Petersen graph is given in Figure 1.

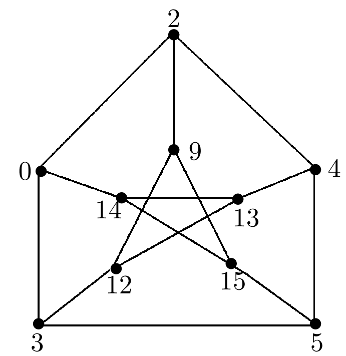

Figure 1

A super mean labeling of the graph $K_{2,4}$ is shown in Figure 2.

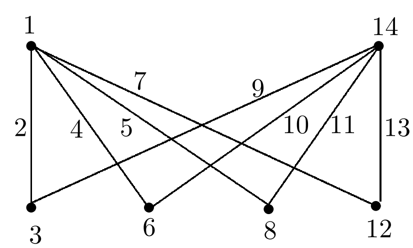

Figure 2

The concept of mean labeling was first introduced by Somasundaram and Ponraj [2] in the year 2003. They have studied in [2-5,8-9], the meanness of many standard graphs like $P_n, C_n, K_n (n \leq 3)$, the ladder, the triangular snake, $K_{1,2}, K_{1,3}, K_{2,n}$, $K_2 + mK_1$, $K_n^c + 2K_2$, $S_m + K_1$, $C_m \cup P_n (m \geq 3, n \geq 2)$, quadrilateral snake, comb, bistars $B(n)$, $B_{n+1,n}, B_{n+2,n}$, the carona of ladder, subdivision of central edge of $B_{n,n}$, subdivision of the star $K_{1,n} (n \leq 4)$, the friendship graph $C_3^{(2)}$, crown $C_n \odot K_1$, $C_n^{(2)}$, the dragon, arbitrary super subdivision of a path etc. In addition, they have proved that the graphs $K_n (n > 3), K_{1,n} (n > 3)$, $B_{m,n} (m > n + 2)$, $S(K_{1,n}) n > 4$, $C_3^{(t)} (t > 2)$, the wheel W_n are not mean graphs.

The concept of super mean labeling was first introduced by R. Ponraj and D. Ramya [6]. They have studied in [6-7] the super mean labeling of some standard graphs. Also they determined all super mean graph of order ≤ 5. In [10], the super meanness of the graph C_{2n} for $n \geq 3$, the H-graph, Corona of a H-graph, 2-corona of a H-graph, corona of cycle C_n for $n \geq 3$, mC_n-snake for $m \geq 1, n \geq 3$ and $n \neq 4$, the dragon $P_n(C_m)$ for $m \geq 3$ and $m \neq 4$ and $C_m \times P_n$ for $m = 3, 5$ are proved.

Let C_n be a cycle with fixed vertex v and $(P_m; C_n)$ the graph obtained from m copies of C_n and the path $P_m : u_1u_2 \ldots u_m$ by joining u_i with the vertex v of the i^{th} copy of C_n by means of an edge, for $1 \leq i \leq m$.

Let Q_3 be a cube with fixed vertex v and $(P_m; Q_3)$ the graph obtained from m copies of Q_3 and the path $P_m : u_1u_2 \ldots u_m$ by joining u_i with the vertex v of the i^{th} copy of Q_3 by means of an edge, for $1 \leq i \leq m$.

Let S_m be a star with vertices $v_0, v_1, v_2, \ldots, v_m$. Define $(P_{2n}; S_m)$ to be the graph obtained from $2n$ copies of S_m and the path $P_{2n} : u_1u_2 \ldots u_{2n}$ by joining u_j with the vertex v_0 of the j^{th} copy of S_m by means of an edge, for $1 \leq j \leq 2n$, $(P_n; S_1)$ the graph obtained from n copies of S_1 and the path $P_n : u_1u_2 \ldots u_n$ by joining u_j with the vertex v_0 of the j^{th} copy of S_1 by means of an edge, for $1 \leq j \leq n$, $(P_n; S_2)$ the graph obtained from n copies of S_2 and the path $P_n : u_1u_2 \ldots u_n$ by joining u_j with the vertex v_0 of the j^{th} copy of S_2 by means of an edge, for $1 \leq j \leq n$.

Suppose $C_n : v_1v_2 \ldots v_nv_1$ be a cycle of length n. Let $[P_m; C_n]$ be the graph obtained from m copies of C_n with vertices $v_{1_1}, v_{1_2}, \ldots, v_{1_n}, v_{2_1}, \ldots, v_{2_n}, \ldots, v_{m_1}, \ldots, v_{m_n}$ and joining v_{i_j} and v_{i+1_j} by means of an edge, for some j and $1 \leq i \leq m-1$.

Let Q_3 be a cube and $[P_m; Q_3]$ the graph obtained from m copies of Q_3 with vertices $v_{1_1}, v_{1_2}, \ldots, v_{1_8}, v_{2_1}, v_{2_2}, \ldots, v_{2_8}, \ldots, v_{m_1}, v_{m_2}, \ldots, v_{m_8}$ and the path $P_m : u_1u_2 \ldots u_m$ by adding the edges $v_{1_1}v_{2_1}, v_{2_1}v_{3_1}, \ldots, v_{m-1_1}v_{m_1}$ (i.e) $v_{i_1}v_{i+1_1}, 1 \leq i \leq m-1$.

Let $C_n^{(2)}$ be a friendship graph. Define $[P_m; C_n^{(2)}]$ to be the graph obtained from m copies of $C_n^{(2)}$ and the path $P_m : u_1u_2 \ldots u_m$ by joining u_i with the center vertex of the i^{th} copy of $C_n^{(2)}$ for $1 \leq i \leq m$.

In this paper, we prove that $(P_m; C_n)m \geq 1, n \geq 3, (P_m; Q_3)m \geq 1, (P_{2n}; S_m)m \geq 3, n \geq 1$, and for any $n \geq 1(P_n; S_1), (P_n; S_2)$ are mean graphs. Also we establish that $[P_m; C_n]m \geq 1$, $n \geq 3$, $[P_m; Q_3]m \geq 1$ and $[P_m; C_n^{(2)}]m \geq 1, n \geq 3$ are mean graphs.

§2. **Mean Graphs** $(P_m; G)$

Let G be a graph with fixed vertex v and let $(P_m; G)$ be the graph obtained from m copies of G and the path $P_m : u_1u_2 \ldots u_m$ by joining u_i with the vertex v of the i^{th} copy of G by means of an edge, for $1 \leq i \leq m$.

For example $(P_4; C_4)$ is shown in Figure 3.

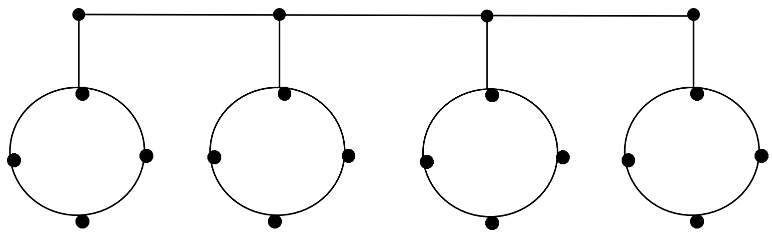

Figure 3

Theorem 2.1 *$(P_m; C_n)$ is a mean graph, $n \geq 3$.*

Proof Let $v_{i_1}, v_{i_2}, \ldots, v_{i_n}$ be the vertices in the i^{th} copy of C_n, $1 \leq i \leq m$ and $u_1, u_2, \ldots, u_m$ be the vertices of P_m. Then define f on $V(P_m; C_n)$ as follows:

Take $n = \begin{cases} 2k & \text{if } n \text{ is even} \\ 2k+1 & \text{if } n \text{ is odd.} \end{cases}$

Then $f(u_i) = \begin{cases} (n+2)(i-1) & \text{if } i \text{ is odd} \\ (n+2)i - 1 & \text{if } i \text{ is even} \end{cases}$

Label the vertices of v_{i_j} as follows:

Case (i) n is odd

When i is odd,

$$f(v_{i_j}) = (n+2)(i-1) + 2j - 1, 1 \leq j \leq k+1$$
$$f(v_{i_{k+1+j}}) = (n+2)i - 2j + 1, 1 \leq j \leq k, 1 \leq i \leq m.$$

When i is even,

$$f(v_{i_j}) = (n+2)i - 2j, 1 \leq j \leq k,$$
$$f(v_{i_{k+j}}) = (n+2)(i-1) + 2(j-1), 1 \leq j \leq k+1, 1 \leq i \leq m$$

Case (ii) n is even

When i is odd,

$$f(v_{i_j}) = (n+2)(i-1) + 2j - 1, 1 \leq j \leq k+1$$
$$f(v_{i_{k+1+j}}) = (n+2)i - 2j, 1 \leq j \leq k-1, 1 \leq i \leq m$$

When i is even,

$$f(v_{i_j}) = (n+2)i - 2j, 1 \leq j \leq k+1$$
$$f(v_{i_{k+1+j}}) = (n+2)(i-1) + 2j + 1, 1 \leq j \leq k-1, 1 \leq i \leq m$$

The label of the edge $u_i u_{i+1}$ is $(n+2)i, 1 \leq i \leq m-1$.

The label of the edge $u_i v_{i_1}$ is $\begin{cases} (n+2)(i-1)+1 & \text{if } i \text{ is odd,} \\ (n+2)i - 1 & \text{if } i \text{ is even} \end{cases}$

and the label of the edges of the cycle are

$(n+2)i-1, (n+2)i-2, \ldots, (n+2)i-n$ if i is odd,

$(n+2)i-2, (n+2)i-3, \ldots, (n+2)i-(n+1)$ if i is even.

For example, the mean labelings of $(P_6; C_5)$ and $(P_5; C_6)$ are shown in Figure 4. □

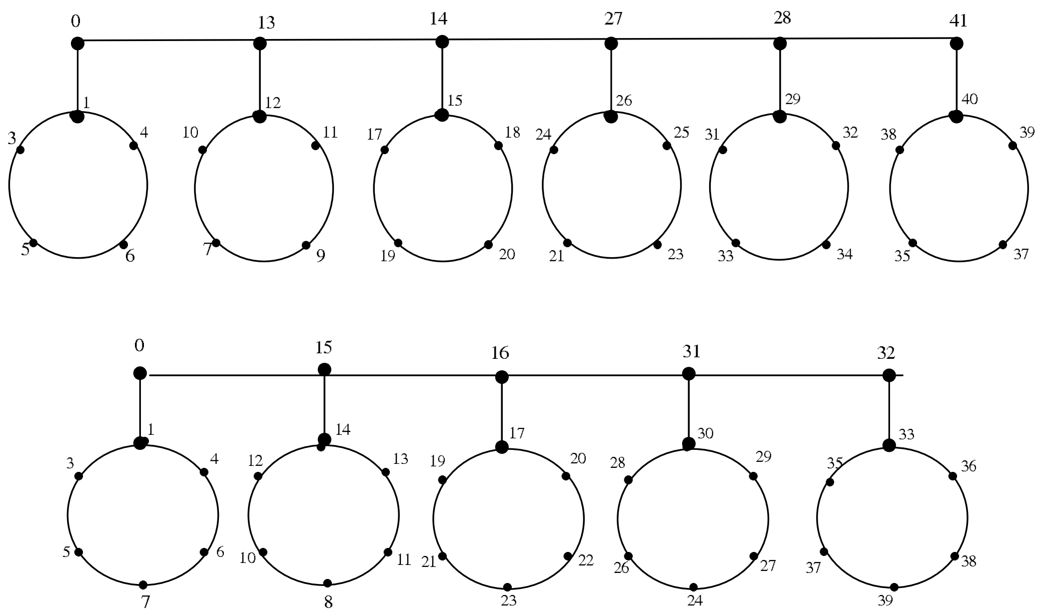

Figure 4

Theorem 2.2 *$(P_m; Q_3)$ is a mean graph.*

Proof For $1 \leq j \leq 8$, let v_{i_j} be the vertices in the i^{th} copy of $Q_3, 1 \leq i \leq m$ and $u_1, u_2, \ldots, u_m$ be the vertices of P_m.

Then define f on $V(P_m; Q_3)$ as follows:

$$f(u_i) = \begin{cases} 14i - 14 & \text{if } i \text{ is odd} \\ 14i - 1 & \text{if } i \text{ is even.} \end{cases}$$

When i is odd,

$$f(v_{i_1}) = 14i - 13, \quad 1 \leq i \leq m$$
$$f(v_{i_j}) = 14i - 13 + j, \quad 2 \leq j \leq 4, 1 \leq i \leq m$$
$$f(v_{i_5}) = 14i - 5, \quad 1 \leq i \leq m$$
$$f(v_{i_j}) = 14i - 9 + j, \quad 6 \leq j \leq 8, 1 \leq i \leq m$$

when i is even,

$$f(v_{i_j}) = 14i - 1 - j, \quad 1 \leq j \leq 3, 1 \leq i \leq m$$
$$f(v_{i_4}) = 14i - 6, 1 \leq i \leq m$$
$$f(v_{i_j}) = 14i - 5 - j, 5 \leq j \leq 7, 1 \leq i \leq m$$
$$f(v_{i_8}) = 14i - 14, 1 \leq i \leq m$$

The label of the edges of P_m are $14i$, $1 \leq i \leq m - 1$.

The label of the edges of $u_i v_{i_1} = \begin{cases} 14i - 13, & \text{if } i \text{ is odd} \\ 14i - 1, & \text{if } i \text{ is even} \end{cases}$

The label of the edges of the cube are

$14i - 1, 14i - 2, \ldots, 14i - 12$ if i is odd,

$14i - 2, 14i - 3, \ldots, 14i - 13$ if i is even.

For example, the mean labeling of $(P_5; Q_3)$ is shown in Figure 5. □

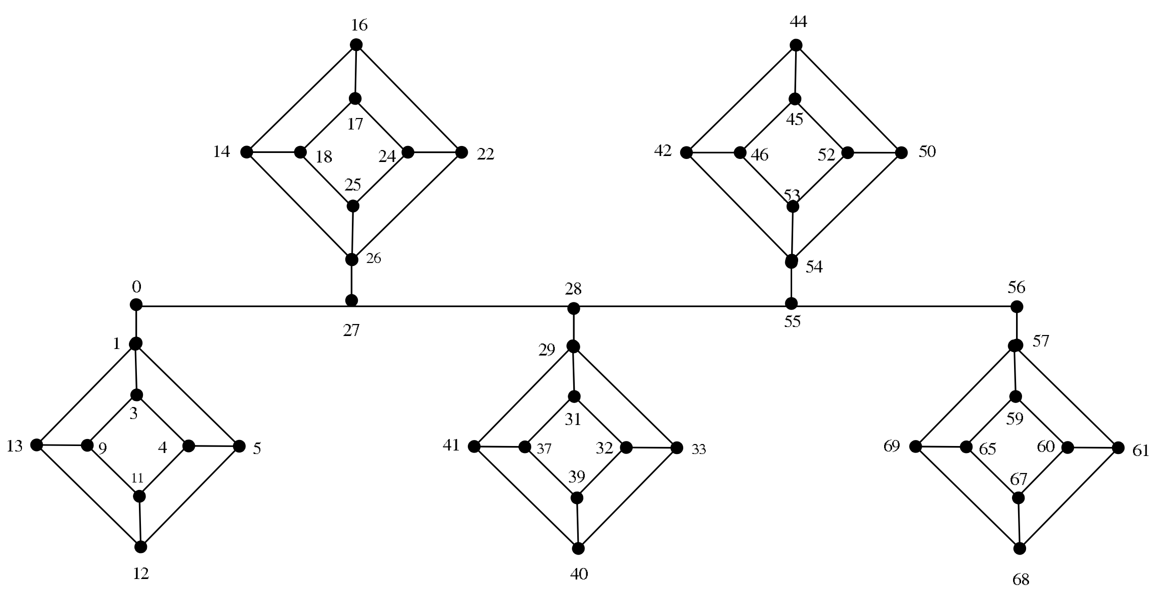

Figure 5

Theorem 2.3 *$(P_{2n}; S_m)$ is a mean graph, $m \geq 3, n \geq 1$.*

Proof Let $u_1, u_2, \ldots, u_{2n}$ be the verfices of P_{2n}. Let $v_{0_j}, v_{1_j}, v_{2_j}, v_{3_j}, \ldots, v_{m_j}$ be the vertices in the j^{th} copy of $S_m, 1 \leq j \leq 2n$.

Label the vertices of $(P_{2n}; S_m)$ as follows:

$$f(u_{2j+1}) = (2m+4)j, \quad 0 \le j \le n-1,$$
$$f(u_{2j}) = (2m+4)j - 1, \quad 1 \le j \le n,$$
$$f(v_{0_{2j+1}}) = (2m+4)j + 1, \quad 0 \le j \le n-1,$$
$$f(v_{0_{2j}}) = (2m+4)j - 2, \quad 1 \le j \le n,$$
$$f(v_{i_{2j+1}}) = (2m+4)j + 2i, \quad 0 \le j \le n-1, 1 \le i \le m$$
$$f(v_{i_{2j}}) = (2m+4)(j-1) + 2i + 1, \quad 1 \le j \le n, 1 \le i \le m$$

The label of the edge u_ju_{j+1} is $(m+2)j, 1 \le j \le 2n-1$
The label of the edge $u_jv_{0_j}$ is

$$\begin{cases} (m+2)(j-1)+1, & \text{if } j \text{ is odd} \\ (m+2)j-1, & \text{if } j \text{ is even} \end{cases}$$

The label of he edge $v_{0_j}v_{i_j}$ is

$$\begin{cases} (m+2)(j-1)+i+1, & \text{if } j \text{ is odd}, 1 \le i \le m \\ (m+2)(j-1)+i, & \text{if } j \text{ is even}, 1 \le i \le m \end{cases}$$

For example, the mean labeling of $(P_6; S_5)$ is shown in Figure 6. □

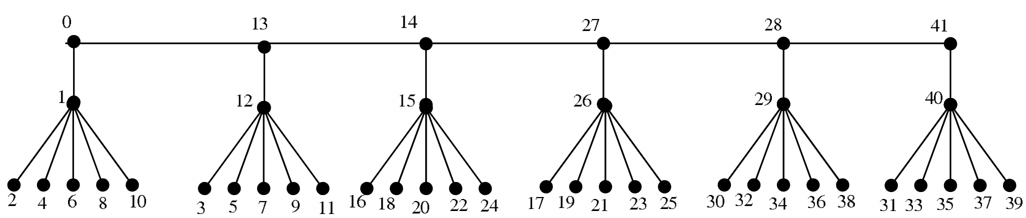

Figure 6

Theorem 2.4 *$(P_n; S_1)$ and $(P_n; S_2)$ are mean graphs for any $n \ge 1$.*

Proof Let $u_1, u_2, \ldots, u_n$ be the vertices of P_n. Let $v_{o_1}, v_{0_2}, \ldots, v_{0_n}$ and $v_{1_1}, v_{1_2}, \ldots, v_{1_n}$ be the vertices of S_1.

Label the vertices of $(P_n; S_1)$ as follows:

$$f(u_j) = \begin{cases} 3j-3 & \text{if } j \text{ is odd}, 1 \le j \le n \\ 3j-1 & \text{if } j \text{ is even}, 1 \le j \le n \end{cases}$$
$$f(v_{0_j}) = 3j-2, \quad 1 \le j \le n$$
$$f(v_{1_j}) = \begin{cases} 3j-1 & \text{if } j \text{ is odd}, 1 \le j \le n \\ 3j-3 & \text{if } j \text{ is even}, 1 \le j \le n \end{cases}$$

The label of the edges of P_n are $3j, 1 \leq j \leq n-1$.

The label of the edges $u_j v_{0_j}$ is $\begin{cases} 3j-2, & \text{if } j \text{ is odd} \\ 3j-1, & \text{if } j \text{ is even} \end{cases}$

The label of the edges $v_{0_j} v_{1_j}$ is $\begin{cases} 3j-1, & \text{if } j \text{ is odd} \\ 3j-2, & \text{if } j \text{ is even} \end{cases}$

Let $v_{0_1}, v_{0_2}, \ldots, v_{0_n}, v_{1_1}, v_{1_2}, \ldots, v_{1_n}$ and $v_{2_1}, v_{2_2}, \ldots, v_{2_n}$ be the vertices of S_2.
Label the vertices of $(P_n; S_2)$ as follows:

$$f(u_j) = \begin{cases} 4j-4 & \text{if } j \text{ is odd, } 1 \leq j \leq n \\ 4j-1 & \text{if } j \text{ is even, } 1 \leq j \leq n \end{cases}$$

$$f(v_{0_j}) = 4j-2, \quad 1 \leq j \leq n$$

$$f(v_{1_j}) = \begin{cases} 4j-3 & \text{if } j \text{ is odd, } 1 \leq j \leq n, \\ 4j-4 & \text{if } j \text{ is even, } 1 \leq j \leq n, \end{cases}$$

$$f(v_{2_j}) = \begin{cases} 4j-1 & \text{if } j \text{ is odd, } 1 \leq j \leq n, \\ 4j-3 & \text{if } j \text{ is even, } 1 \leq j \leq n, \end{cases}$$

The label of the edges of P_n are $4j, 1 \leq j \leq n-1$

The label of the edges $u_j v_{0_j}$ is $\begin{cases} 4j-3, & \text{if } j \text{ is odd} \\ 4j-1 & \text{if } j \text{ is even} \end{cases}$

The label of the edges $v_{0_j} v_{1_j}$ is $\begin{cases} 4j-2, & \text{if } j \text{ is odd} \\ 4j-3, & \text{if } j \text{ is even} \end{cases}$

The label of the edges $v_{0_j} v_{2_j}$ is $\begin{cases} 4j-1, & \text{if } j \text{ is odd} \\ 4j-2, & \text{if } j \text{ is even} \end{cases}$

For example, the mean labelings of $(P_7; S_1)$ and $(P_6; S_2)$ are shown in Figure 7. □

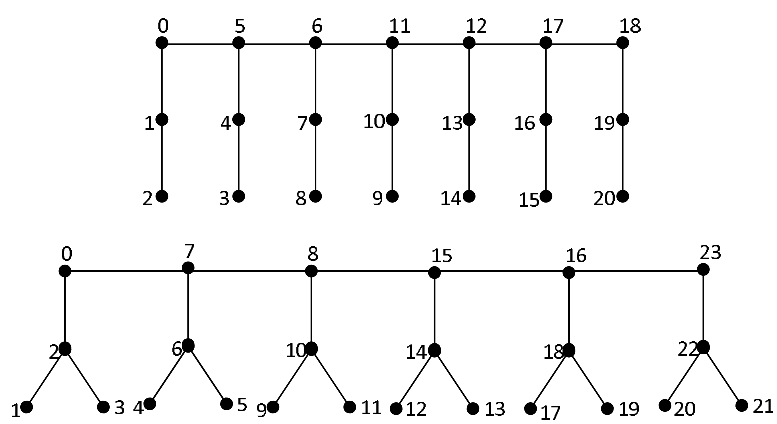

Figure 7

§3. Mean Graphs $[P_m; G]$

Let G be a graph with fixed vertex v and let $[P_m; G]$ be the graph obtained from m copies of G by joining v_{i_j} and v_{i+1_j} by means of an edge, for some j and $1 \leq i \leq m-1$.

For example $[P_5; C_3]$ is shown in Figure 8.

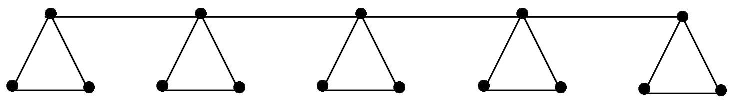

Figure 8

Theorem 3.1 *$[P_m; C_n]$ is a mean graph.*

Proof Let $u_1, u_2, \ldots, u_m$ be the vertices of P_m. Let $v_{i_1}, v_{i_2}, \ldots, v_{i_n}$ be the vertices of the i^{th} copy of $C_n, 1 \leq i \leq m$ and joining $v_{i_j} (= u_i)$ and $v_{i+1_j} (= u_{i+1})$ by means of an edge, for some j.

Case (i) $n = 4t, t = 1, 2, 3, \ldots$

Define $f : V([P_m; C_n]) \to \{0, 1, 2, \ldots, q\}$ by

$$f(v_{i_j}) = (n+1)(i-1) + 2(j-1), 1 \leq j \leq 2t+1$$
$$f(v_{i_{2t+1+j}}) = (n+1)i - 2j, 1 \leq j \leq 2t-1, 1 \leq i \leq m$$

The label of the edge $v_{i_{(t+1)}} v_{i+1_{(t+1)}}$ is $(n+1)i, 1 \leq i \leq m-1$. The label of the edges of the cycle are $(n+1)i - 1, (n+1)i - 2, \ldots, (n+1)i - n, 1 \leq i \leq m$.

For example, the mean labeling of $[P_4; C_8]$ is shown in Figure 9.

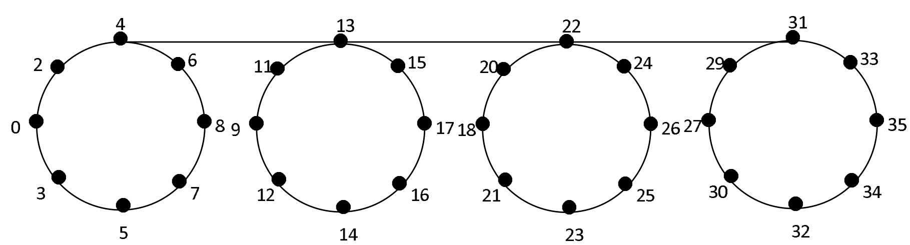

Figure 9

Case (ii) $n = 4t + 1, t = 1, 2, 3, \ldots$

Define $f : V([P_m; C_n]) \to \{0, 1, 2, \ldots, q\}$ by

$$f(v_{i_1}) = (n+1)(i-1), 1 \leq i \leq m$$
$$f(v_{i_j}) = (n+1)(i-1) + 2j - 1, 2 \leq j \leq 2t+1, 1 \leq i \leq m$$
$$f(v_{i_{(2t+1+j)}}) = (n+1)i - 2j, 1 \leq j \leq 2t, 1 \leq i \leq m$$

The label of the edge $v_{i_{(t+1)}}v_{i+1_{(t+1)}}$ is $(n+1)i, 1 \leq i \leq m-1$. The label of the edges of the cycle are $(n+1)i-1, (n+1)i-2, \ldots, (n+1)i-n, 1 \leq i \leq m$.

For example, the mean labeling of $[P_6; C_5]$ is shown in Figure 10.

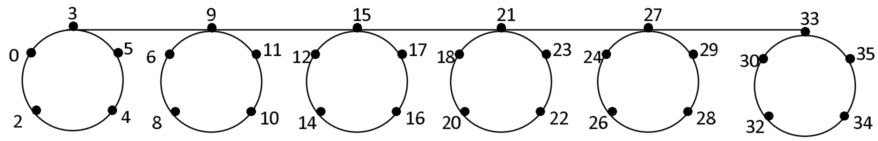

Figure 10

Case (iii) $n = 4t+2, t = 1, 2, 3, \ldots$

Define $f : V([P_m; C_n]) \to \{0, 1, 2, \ldots, q\}$ by

$$f(v_{i_1}) = (n+1)(i-1), 1 \leq i \leq m$$
$$f(v_{i_j}) = (n+1)(i-1) + 2j - 1, 2 \leq j \leq 2t+1, 1 \leq i \leq m$$
$$f(v_{i_{(2t+1+j)}}) = (n+1)i - 2j + 1, 1 \leq j \leq 2t+1, 1 \leq i \leq m$$

The label of the edge $v_{i_{(t+1)}}v_{i+1_{(t+1)}}$ is $(n+1)i, 1 \leq i \leq m-1$. The label of the edges of the cycle are $(n+1)i-1, (n+1)i-2, \ldots, (n+1)i-n, 1 \leq i \leq m$.

For example, the mean labeling of $[P_5; C_6]$ is shown in Figure 11.

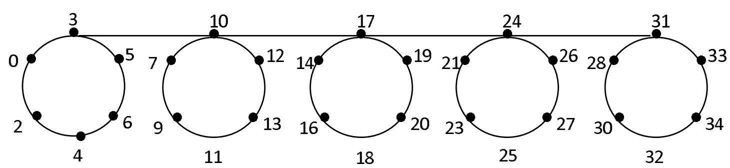

Figure 11

Case (iv) $n = 4t-1, t = 1, 2, 3, \ldots$

Define $f : V([P_m; C_n]) \to \{0, 1, 2, \ldots, q\}$ by

$$f(v_{i_j}) = (n+1)(i-1) + 2(j-1), 1 \leq j \leq 2t, 1 \leq i \leq m$$
$$f(v_{i_{(2t+j)}}) = (n+1)i - 2j + 1, 1 \leq j \leq 2t-1, 1 \leq i \leq m$$

The label of the edge $v_{i_{(t+1)}}v_{i+1_{(t+1)}}$ is $(n+1)i, 1 \leq i \leq m-1$. The label of the edges of the cycle are $(n+1)i-1, (n+1)i-2, \ldots, (n+1)i-n, 1 \leq i \leq m$.

For example, the mean labeling of $[P_7; C_3]$ is shown in Figure 12. □

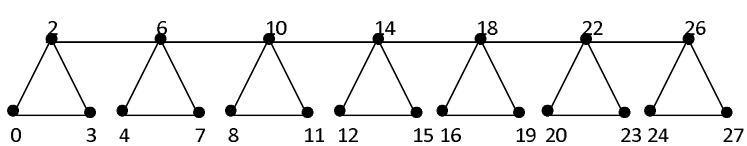

Figure 12

Theorem 3.2 *$[P_m; Q_3]$ is a mean graph.*

Proof For $1 \leq j \leq 8$, let v_{i_j} be the vertices in the i^{th} copy of $Q_3, 1 \leq i \leq m$. Then define f on $V[P_m; Q_3]$ as follows:

When i is odd.

$$f(v_{i_1}) = 13i - 13, 1 \leq i \leq m$$
$$f(v_{i_j}) = 13i - 13 + j, 2 \leq j \leq 4, 1 \leq i \leq m$$
$$f(v_{i_5}) = 13i - 5, 1 \leq i \leq m$$
$$f(v_{i_j}) = 13i - 9 + j, 6 \leq j \leq 8, 1 \leq i \leq m$$

When i is even.

$$f(v_{i_j}) = 13i - j, 1 \leq j \leq 3, 1 \leq i \leq m$$
$$f(v_{i_4}) = 13i - 5, 1 \leq i \leq m$$
$$f(v_{i_j}) = 13i - j - 4, 5 \leq j \leq 7, 1 \leq i \leq m$$
$$f(v_{i_8}) = 13i - 13, 1 \leq i \leq m$$

The label of the edge $v_{i_1}v_{(i+1)_1}$ is $13i, 1 \leq i \leq m - 1$. The label of the edges of the cube are $13i - 1, 13i - 2, \ldots, 13i - 12, 1 \leq i \leq m$.

For example the mean labeling of $[P_4; Q_3]$ is shown in Figure 13. □

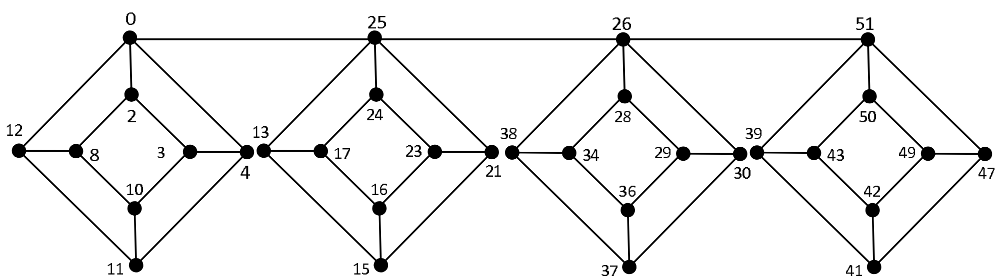

Figure 13

Theorem 3.3 *$[P_m; C_n^{(2)}]$ is a mean graph.*

Proof Let $u_1, u_2, \ldots, u_m$ be the vertices of P_m and the vertices $u_i, 1 \leq i \leq m$ is attached with the center vertex in the i^{th} copy of $C_n^{(2)}$. Let $u_i = v_{i_1}$ (center vertex in the i^{th} copy of $C_n^{(2)}$).

Let v_{i_j} and v'_{i_j} for $1 \leq i \leq m, 2 \leq j \leq n$ be the remaining vertices in the i^{th} copy of $C_n^{(2)}$.

Then define f on $V[P_m, C_n^{(2)}]$ as follows:

$$\text{Take } n = \begin{cases} 2k & \text{if } n \text{ is even} \\ 2k+1 & \text{if } n \text{ is odd.} \end{cases}$$

Label the vertices of v_{i_j} and v'_{i_j} as follows:

Case (i) When n is odd

$$
\begin{aligned}
f(v_{i_1}) &= (2n+1)i-(n+1), 1 \le i \le m\\
f(v_{i_j}) &= (2n+1)i-(n+2)-2(j-2), 2 \le j \le k+2\\
f(v_{i_{k+2+j}}) &= (2n+1)i-2(n-1)+2(j-1), 1 \le j \le k-1, k \ge 2\\
f(v'_{i_j}) &= (2n+1)i-(n-1)+2(j-2), 2 \le j \le k+1\\
f(v'_{i_{k+1+j}}) &= (2n+1)i-1-2(j-1), 1 \le j \le k, 1 \le i \le m
\end{aligned}
$$

Case (ii) When n is even

$$
\begin{aligned}
f(v_{i_j}) &= (2n+1)i-(n+1)-2(j-1), 1 \le j \le k+1\\
f(v_{i_{k+1+j}}) &= (2n+1)i-2(n-1)+2(j-1), 1 \le j \le k-1, 1 \le i \le m\\
f(v'_{i_j}) &= (2n+1)i-(n-1)+2(j-2), 2 \le j \le k+1\\
f(v'_{i_{k+1+j}}) &= (2n+1)i-2-2(j-1), 1 \le j \le k-1, 1 \le i \le m
\end{aligned}
$$

The label of the edge u_iu_{i+1} is $(2n+1)i, 1 \le i \le m-1$ and the label of the edges of $C_n^{(2)}$ are $(2n+1)i-1, (2n+1)i-2, \ldots, (2n+1)i-2n$ for $1 \le i \le m$.

For example the mean labelings of $[P_4, C_6^{(2)}]$ and $[P_5, C_3^{(2)}]$ are shown in Figure 14. □

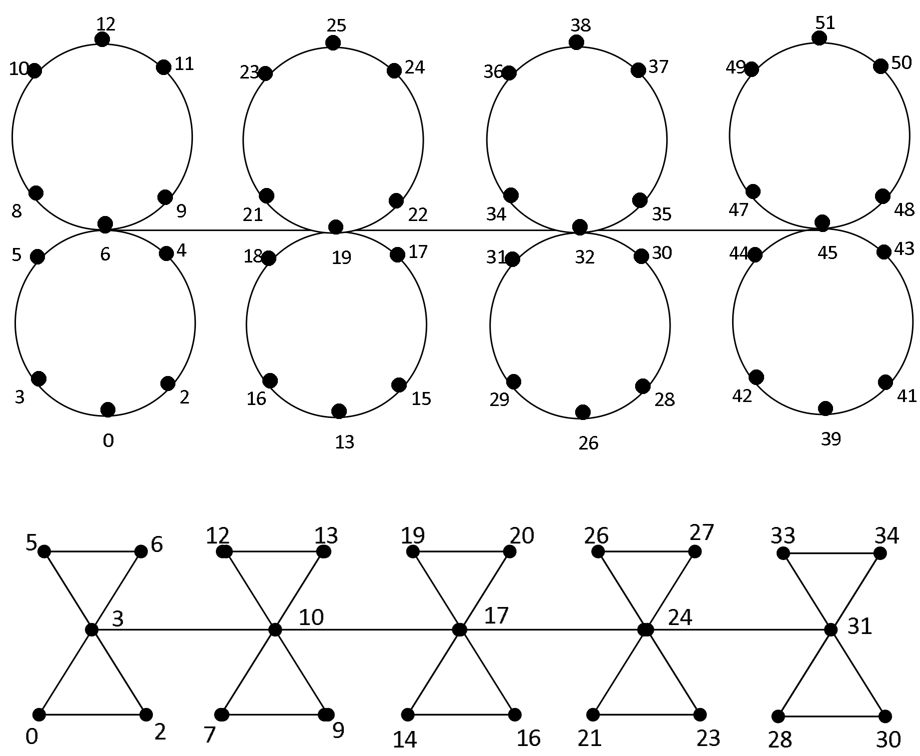

Figure 14

References

[1] R. Balakrishnan and N. Renganathan, *A Text Book on Graph Theory*, Springer Verlag, 2000.

[2] S. Somasundaram and R. Ponraj, Mean labelings of graphs, *National Academy Science letter,* **26** (2003), 210 - 213.

[3] S. Somasundaram and R. Ponraj, Non - existence of mean labeling for a wheel, *Bulletin of pure and Applied Sciences*, (Section E Maths & Statistics) **22E**(2003), 103 - 111.

[4] S. Somasundaram and R. Ponraj, Some results on mean graphs, *Pure and Applied Mathematika Sciences*, **58**(2003), 29 - 35.

[5] S. Somasundaram and R. Ponraj, On Mean graphs of order ≤ 5, *Journal of Decision and Mathematical Sciences*, **9**(1 - 3) (2004), 48 - 58.

[6] R. Ponraj and D. Ramya, Super mean labeling of graphs, Reprint.

[7 R. Ponraj and D. Ramya, On super mean graphs of order ≤ 5, *Bulletin of Pure and Applied Sciences*, (Section E Maths and Statistics) **25E** (2006), 143-148.

[8] R. Ponraj and S. Somasundaram, *Further results on mean graphs*, Proceedings of Sacoeference, August 2005. 443 - 448.

[9] R. Ponraj and S.Somasundaram, Mean labeling of graphs obtained by identifying two graphs, *Journal of Discrete Mathematical Sciences and Cryptography*, **11**(2)(2008), 239-252.

[10] R. Vasuki and A. Nagarajan, Some results on super mean graphs, *International Journal of Mathematical Combinatorics*, **3**(2009), 82-96.