Union/intersection vs. alternative/conjunction - defining posterior hypotheses in C2 systems

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Abstract – This paper discusses the problem of application of the logical operators while defining the posterior hypotheses, that is hypotheses which are not directly created upon the sensor data.
In the authors' opinion the application of the logical sentence operators is constrained to some specific cases, where the sets operators may be applied as well. On the other hand, the sets operators enable to provide much more adequate posterior hypotheses, which results in higher precision of the fusion final decision.
In order to present that an analysis has been made and some examples, related to attribute information fusion in C2 systems have been delivered.

Keywords: Classification, observation, soft-decision fusion, union, intersection, alternative, conjunction, evidence, *DST*, *DSmT*.

1 Introduction

One of the most important requirements imposed on maritime Command & Control (*C2*) systems is to elaborate the so called Common Operational Picture (*COP*). In order to achieve that, the *C2* systems must be equipped with specific information fusion techniques, which enable to integrate both precise information as well as uncertain, incomplete or even conflicting information.
When the information is vague, sophisticated reasoning processes are performed to elaborate the final optimal decision [1], [2], [3]. In such a case it is very important to create hypotheses upon the gathered evidence as much adequate as possible. Some of them (primary hypotheses) result directly from the sensors' resolution and may be regarded as the basis for constructing the frame of discernment. Others (posterior hypotheses) are created upon the primary hypotheses with usage of logical relations.
The definition of the posterior hypotheses is very important and should be a subject of the serious considerations. Certainly, one can imagine the posterior hypotheses may be regarded as completely different classes. However, that implies two problems: The first problem is evaluation of these new hypotheses, based on observation data, which is necessary if these hypotheses are intended to be utilised effectively. The second problem is that the reasoning system, which performs the fusion of the gathered information, does not have a 'notion' of the meaning of these new hypotheses unless they are defined based on the hypotheses that already exist.
Among the specialists there have been developed two contradict opinions related to definitions of the secondary hypotheses. The first view, found typically in the military literature [4], [6], [7], suggests that the posterior hypotheses should be created using the logical operations of conjunction and alternative. The second one, found mostly in the fusion and approximate reasoning science literature [1], [2], [3], [5] indicates the sets operations of intersection and union as much more adequate.
In the next sections these two opinions are going to be discussed in details and illustrated with examples, typical for the marine *C2* systems purposes with particular emphasis on merits and drawbacks of each of these views.

2 Process of observation

It is suggested to the reader to consider the following example of the target threat classification, performed upon observation by the *C2* reconnaissance means.
The ground truth is that the target performs a friendly vessel, acting as hostile for the exercise purposes. Additionally, it is assumed that the sensors used for the classification enable to distinguish only between two classes: FRIEND (*F*) and HOSTILE (*H*).
In terms of the NATO STANAGs [4], [6], [7] the described vessel is defined as FAKER (*K*).
In order to find the formalism for the class of FAKER with respect to the logical operations, mentioned in the introduction, the following four expressions should be taken into account:

$$K = F \vee H \tag{1}$$
$$K = F \wedge H \tag{2}$$
$$K = F \cup H \tag{3}$$
$$K = F \cap H \tag{4}$$

Possible interpretations of these formalisms are going to be discussed in the following section. Whereas this section aims to scrutinise the process of observation, which may be performed by sensors as well as by the human being.

In the typical reconnaissance process target classification is based upon distinctive features of the target. That means the target classification performs a sort of conclusion related to the set of the observed target features. This, apparently obvious fact, is the key argument for the thesis that classification hypotheses should be created with usage of the set operators of union and intersection but not with alternative nor conjunction.

Figure 1 shows the example of the maritime observation process with a particular emphasis on recognising the distinctive features of the target.

Notice, that some of these features (e.g. target hull number) may not be accessible for the recognition due to the possible difficulties that may occur during the observation process.

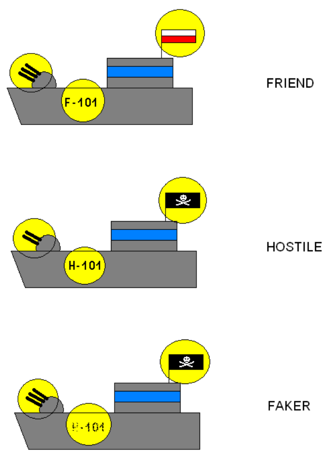

Figure 1 Observation of targets with emphasised specific features

Figure 1 shows that FAKER may have some properties of FRIEND and some of HOSTILE. Nevertheless, it is extremely improbable (and almost impossible in practice) for the target to have all the features[1] of both classes.

Therefore, in the authors' opinion the most accurate formalism will be the equation (4).

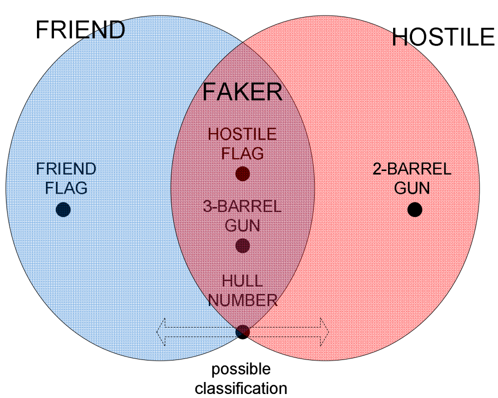

Figure 2 Venn's diagram of the observed target features

[1] In such case 'all' means all accessible for observation.

Figure 2 shows the Venn's diagram of the observed situation where the subset of FAKER contains the common features of FRIEND and HOSTILE target classes. Notice the position of the hull number attribute, which after scrutiny may be moved either towards FRIEND or HOSTILE when the identification data occur.

3 Interpretation of acquired information

Considering the adequacy of the rest of the formalisms regarding to the ground truth situation, it is suggested to examine possible interpretations of these equations.

The formalism based on the logical sentence operators seems to be very categorical. If one wishes to use it to reflect another situation, where two closely-spaced targets FRIEND and HOSTILE are being tracked, the statements (1) and (3) are equivalent[2] in such a case.

However, in the considered observation example, any attempt of defining FAKER class with usage of the logical sentence operators leads inevitably to the serious degradation of information related to the observed target. The statement (2) applied to this very case is incorrect, due to the fact, the observed target is neither fully friendly nor hostile. Accepting such a formalism may have a negative impact during the stage of processing of this information.

The key problem is the precise interpretation of the hypotheses defined with usage of the conjunction operation. Classical interpretation of (2) says that FAKER performs FRIEND and HOSTILE simultaneously. According to the binary logic as FRIEND, it possesses all the properties of the friendly target and as HOSTILE all the properties of the hostile target. That, on the other hand, leads to the conclusion that FAKER may be performed as the union of FRIEND and HOSTILE as well. This is very dangerous due to the fact it may be confused with two closely-spaced targets, as mentioned above.

This very problem also emerges while calculating the respective belief functions for each of these hypotheses. Namely, in case of definitions based on the conjunction operator (2), the belief function referring to the hypothesis of FRIEND is determined only by the mass of FRIEND. Whereas the belief function referring to FAKER is supported by FAKER itself and FRIEND hypothesis since FAKER aggregates all the features of FRIEND and HOSTILE classes.

$$Bel(F) = m(F) \tag{5}$$

$$Bel(K) = m(K) + m(F) \tag{6}$$

This view contradicts the common sense, due to the fact that the degree of belief of the hypotheses FAKER is successively increased with the mass referring to FRIEND.

[2] The equivalence is defined here in terms of informational income.

For comparison: in case of definitions based on the sets operations, the hypothesis (4) supports the hypothesis of FRIEND, when calculating the belief function *Bel(F)*.

$$Bel(F) = m(F) + m(K) \tag{7}$$

$$Bel(K) = m(K) \tag{8}$$

That means that presuming the target is FAKER, it is possible to increase the degree of belief that the target is friendly. However the converse is not true, since it is unjustified to increase the degree of belief of the hypothesis FAKER with the mass referring to FRIEND, which meets the reality.

Does it mean that the logical operators do not have any applications in creating the posterior hypotheses?

It is difficult to answer this question unequivocally. The logical operators do have their application, when aggregating the target distinctive features, in order to evaluate the gathered information, and to classify the target.

Table 1 shows example definitions[3] of different vessel types, so called platforms, in terms of their activities and the equipment onboard. According to this table each type may be defined as the set of its distinctive features using the logical operators.

Example 1:

Table 1 Example definitions of surface platforms

Transporter	AUX ∧ AIR ∧ D ∧ TRAN
Command	AUX ∧ S&MCAL ∧ AIR ∧ C2
Cruiser	S&MCAL ∧ LCAL ∧ SSC ∧ AIR ∧ E ∧ INF ∧ C2
Destroyer	S&MCAL ∧ SSC ∧ AIR ∧ E ∧ ATTC ∧ REC
Patrolcraft Escort	GUN ∧ E ∧ SUP
Hydrofoil	GUN ∧ E
Support	SUP

where:

AUX – auxiliary vessel;
S&MCAL – equipped with artillery of small and medium calibre;
LCAL – equipped with artillery of large calibre;
SSC – destroys surface, subsurface and coastal targets;
E – escort vessel;
INF – influences to tactical and operational situation;
AIR – against the air targets;
D – performs landing operations;
C2 – command & control;
SUP – support;
GUN – equipped with artillery of small calibre or machine guns;
TRAN – transport of landing forces;

With reference to the example above, the following statement may be formulated: The logical operators may have the application in creating posterior hypotheses if the primary hypotheses refer to the observable features of the target, and additionally, they are possible to be verified in the binary manner. For instance: a *Patrolcraft Escort* may be defined with respect to its features as a vessel that possesses three features: performs an escort, performs a support and it is equipped with a gun. Nevertheless, similarly as in case of FAKER, a certain danger resides in such a definition, which may lead to the misclassification. Namely, the *Patrolcraft Escort(PE)* encompasses all the (tactical) features of a *hydrofoil (HF)* and any *support* (*SUP*) vessel, as Table 1 shows. Thus one could define the *Patrolcraft Escort* as:

$$PE = HF \wedge SUP \tag{9}$$

which would be commonly interpreted as an absurd object that performs the *hydrofoil* and the *support* vessel at the same time.

On the other hand defining the Patrolcraft Escort in terms of sets algebra would result in:

$$PE = HF \cap SUP \tag{10}$$

which would be interpreted properly as a vessel that encompasses the common features of the respective: *hydrofoil* and *support* entities.

Up to now there has been presented a spread range of application of the sets operators and a potential application of the logical operators under the assumption the hypotheses do refer to the directly observable features of the target. What happens if they do not? Is it possible to use the logical operators then?

In most of the cases when the posterior hypotheses are created for the purpose of the target classification, there is a need to take into account factors which are not directly subjected to the observation process. For example, the mentioned *threat* attribute classification may be performed upon the observation only to some extent. The value of this very attribute is so important that an additional plausibility analysis is often performed to assess, whether it is possible to identify FRIEND, HOSTILE or FAKER target, according to some external sources of information e.g. data bases, intelligence, general knowledge, etc.

Abandoning the observability assumption, in the authors' opinion, it is possible to utilise the logical operators, however, under another assumption, related to the exclusivity of the primary hypotheses. Namely, if the primary hypotheses do not contradict each other the logical operators may be used to create the posterior hypotheses in order to emphasise the completeness of the particular primary hypotheses.

[3] In the majority of cases the description enables to identify the particular platform unambiguously. Therefore it is suggested to accept that as definition for the illustrative purposes.

Example 2:

Table 2 Example of mine warfare platforms, based on [4]

MHO	Mine Hunter Ocean
MSO	Mine Sweeper Ocean
MHSO	Mine Hunter/Sweeper Ocean

According to [4] MHSO stands for a vessel, which is capable to perform mine-hunting activities as well as mine-sweeping. However, it is neither a pure MHO nor MSO. The best fitting formalism would probably be (11) which indicates that, to some extent, MHSO has functionalities of MHS and MHO.

$$MHSO = MHO \cap MSO \tag{11}$$

The larger the degree (of MHO as well as MSO) is, the closer to $MHO \wedge MSO$ MHSO is, which may be formalised as (12).

$$\forall_{\substack{MHO \to fullMHO \\ MHS \to fullMHS}} MHSO = MHO \wedge MSO \tag{12}$$

Figure 3 MHSO - the completeness of MHO and MSO functionalities

Although this example seems to be similar to the FAKER case, it is worth noticing that, on the contrary to FAKER, the concept of maximal completeness of the particular primary hypotheses does not contradict the common sense. The idea of existence of the target which possesses all the functionalities of the typical mine-hunter and all the functionalities of the typical mine-sweeper simultaneously is reasonable, however improbable. That is due to the fact the primary hypotheses do not refer to exclusive concept as it was in the FAKER case.

4 Operators and soft-decision fusion

Before deciding about application of the exact operators for creating the posterior hypotheses, it is very important to consider their potential influence on the performance of information fusion modules. Particularly, the operators may impact the adequacy of the elaborated decisions due to the fact the processing of information is based on logical operations (e.g. combination, conditioning).

As it was mentioned in the previous sections, application of the logical operators leads to introduction of categorical hypotheses, which do not reflect the partial affiliation of the considered targets to the particular classes, defined by the primary hypotheses. Such a view is typical for hard-decision fusion approach.

However, while resolving conflicting information fusion problems the usability of soft-decision fusion, where instead of decisions declarations (evaluated hypotheses) are considered [5], is much greater. Such approach enables to elaborate much more adequate decisions due to the fact the processing of information is not interfered by any categorical assumptions. For this reason, application of the sets operators, which enable to keep the hypotheses flexibility, seems to be justified.

Figure 4 and Figure 5 show the models of the target threat evaluation. The process of evaluation of the defined hypotheses is another very extensive problem which has been discussed in [3] and [8] in details. Although the models have been also presented in this paper for the reader to be aware that the completeness of the particular primary hypotheses of FRIEND and HOSTILE is up to the evaluation in the next stage of processing of information. Thus, in the authors' opinion it performs another argument for the application of the sets operators.

For basic description: the activity-oriented model, in the first measurement stage, resolves whether (according to observed target's activity) the target seems to be more like FRIEND or HOSTILE. In the following stages the degrees of belief for each of these two hypotheses are defined by the observation of the real target.

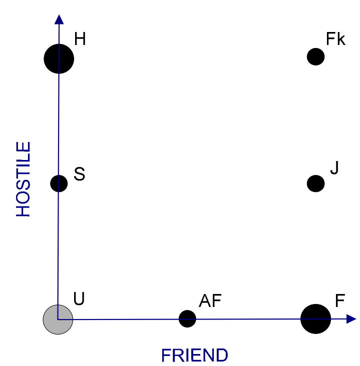

Figure 4 Activity-oriented model

where:

primary hypotheses:

- F – friend,
- H – hostile,
- U – unknown (specific class – represents the ignorance about the target)

posterior hypotheses:

- AF – assumed friend, $AF = F \cap U$
- S – suspect, $S = H \cap U$
- J – joker, $J = F \cap H \cap U$
- Fk – faker, $Fk = K = F \cap H$

The threat-oriented model in the first measurement stage resolves the real threat of the target. In the following stages, the degree of belief (whether the target act as SUSPECT or HOSTILE) are defined according to the current target's activity.

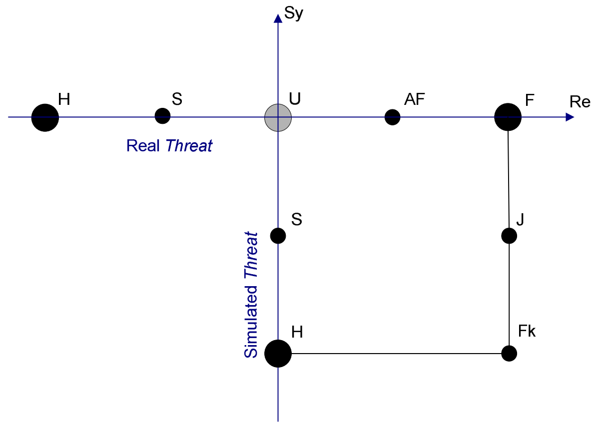

Figure 5 Threat-oriented model

where:

- primary hypotheses:
  - F – friend,
  - H – hostile,
  - U – unknown (specific class – represents the ignorance about the target)
- posterior hypotheses:
  - AF – assumed friend, $AF = F \cap U$
  - S – suspect, $S = H_{real} \cap U$
  - J – joker, $J = F \cap H_{sim} \cap U$
  - Fk – faker, $Fk = K = F \cap H_{sim}$

The detailed definitions (based on observations) of each of the target threat values have been shown in Table 3, where $\Delta\theta_i(hyp)$ denotes the distance metric for the hypothesis *hyp* in the i-th measurement step of the three-step measurement observation [3].

Table 3 Target threat models comparison

Threat	**Activity-oriented model**	**Threat-oriented model**
FRD	$\begin{bmatrix}\Delta\theta_I(F)\\ \Delta\theta_{II}(U)\\ \Delta\theta_{III}(F)\end{bmatrix}$	$\begin{bmatrix}\Delta\theta_I(F)\\ \Delta\theta_{II}(U)\\ \Delta\theta_{III}(F)\end{bmatrix}$
HOS	$\begin{bmatrix}\Delta\theta_I(H)\\ \Delta\theta_{II}(H)\\ \Delta\theta_{III}(U)\end{bmatrix}$	$\begin{bmatrix}\Delta\theta_I(H)\\ \Delta\theta_{II}(H)\\ \Delta\theta_{III}(U)\end{bmatrix}$
UNK	$\begin{bmatrix}\Delta\theta_I(H)-\Delta\theta_I(F)\\ \Delta\theta_{II}(U)\\ \Delta\theta_{III}(U)\end{bmatrix}$	$\begin{bmatrix}\Delta\theta_I(H)-\Delta\theta_I(F)\\ \Delta\theta_{II}(U)\\ \Delta\theta_{III}(U)\end{bmatrix}$
FAK	$\begin{bmatrix}\Delta\theta_I(F)-\Delta\theta_I(H)\\ \Delta\theta_{II}(H)\\ \Delta\theta_{III}(F)\end{bmatrix}$	$\begin{bmatrix}\Delta\theta_I(F)\\ \Delta\theta_{II}(H)\\ \Delta\theta_{III}(U)\end{bmatrix}$
JOK	$\begin{bmatrix}\Delta\theta_I(F)\\ \Delta\theta_{II}(H)-\Delta\theta_{II}(U)\\ \Delta\theta_{III}(F)\end{bmatrix}$	$\begin{bmatrix}\Delta\theta_I(F)\\ \Delta\theta_{II}(H)-\Delta\theta_{II}(U)\\ \Delta\theta_{III}(U)\end{bmatrix}$
SUS	$\begin{bmatrix}\Delta\theta_I(H)\\ \Delta\theta_{II}(H)-\Delta\theta_{II}(U)\\ \Delta\theta_{III}(U)\end{bmatrix}$	$\begin{bmatrix}\Delta\theta_I(H)\\ \Delta\theta_{II}(H)-\Delta\theta_{II}(U)\\ \Delta\theta_{III}(U)\end{bmatrix}$
AFR	$\begin{bmatrix}\Delta\theta_I(F)\\ \Delta\theta_{II}(U)\\ \Delta\theta_{III}(U)-\Delta\theta_{III}(F)\end{bmatrix}$	$\begin{bmatrix}\Delta\theta_I(F)\\ \Delta\theta_{II}(U)\\ \Delta\theta_{III}(U)-\Delta\theta_{III}(F)\end{bmatrix}$

These two models show that considering the posterior hypotheses as subsets (composite hypotheses) in target threat classification problem is necessary. That is due to the fact the pieces of information related to the target's hostility and uncertainty are measured and effectively used for calculation of the resulting masses for each of the posterior hypotheses.

Simplification of the posterior hypotheses to logical sentences implies 'cutting off the details' and disables to utilise these partial information, which leads to serious problems with evaluation of these posterior hypotheses.

Essentially, discussing the problem of determinism/ indeterminism in terms of creating categorical/flexible hypotheses for the purpose of attribute information fusion there is a need to distinguish two possible stages of this determinism/ indeterminism. The soft-decision fusion approach is indeterministic in itself and this aims to be the first stage of the indeterminism (see Figure 6). The indeterminism of the soft-decision approach comes directly from the definition of this approach, where none of hard-decision is made but declarations (the statement of all possible and evaluated hypotheses).

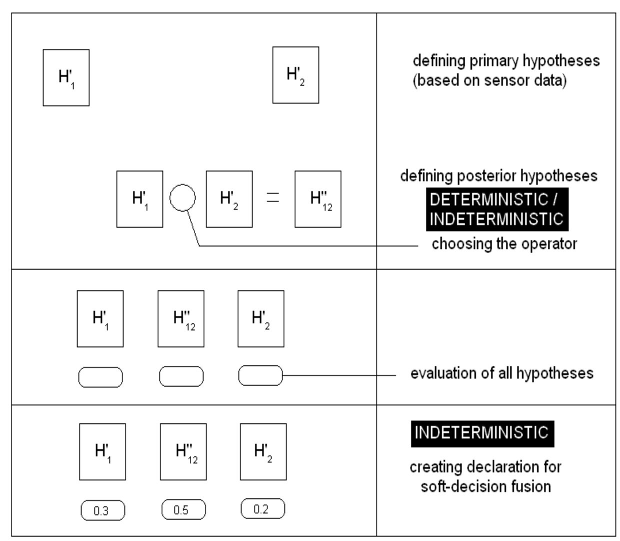

Figure 6 Determinism/indeterminism in soft-decision fusion

Creating the posterior hypotheses is another problem which, depending on the used operators, may be more or less deterministic. In case of usage the sets operators the whole process of information fusion, including the creation of the posterior hypotheses, stays fully indeterministic. In case of usage the logical operators one interferes the typically indeterministic processing of information with not fully justified deterministic assumptions that come from the categorical definitions of the posterior hypotheses.

5 Conclusion

The aim of this paper was to discuss the possible applications of the logical operators as well as the sets operators in creating the posterior hypotheses for the purpose of attribute information fusion in C2 systems.
In the authors' opinion the application of the sets operators is common, intuitive and in many cases necessary. While the application of the logical operators is merely plausible and constrained to several cases.
It is suggested by the authors to choose the logical operators for aggregation of the target features or if the primary hypotheses are derived directly from the observations and additionally they may be assessed in binary manner.
Another possible application of the logical operators is if the primary hypotheses do not contradict each other and the completeness of the particular primary hypotheses, included in the posterior hypotheses are confirmed.
However, the utility of the sets operators is incomparably greater and covers all the cases of the applications of the logical operators.

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