

# Scale independent unified quark physics

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**Abstract:** With reference to particle data group, up quark mass is  $2.15 \text{ MeV}/c^2$  and down quark mass is  $4.7 \text{ MeV}/c^2$ . Up and down quark mass ratio is 0.46(5). In the earlier published paper titled: “Super symmetry in strong and weak interactions”, the authors proposed a simple method for estimating the six quark masses. With reference to the earlier proposed results, up quark mass is  $4.4 \text{ MeV}/c^2$  and down quark mass is  $9.473 \text{ MeV}/c^2$ . By considering the up and down quark rest masses, in this paper the authors proposed very simple methods for estimating the nuclear stability and nuclear binding energy and successfully extended the quark model to low energy scale nuclear physics. Topics covered are: 1) Hint of final unification and Up and Down quark rest masses, 2) Strong coupling constant, 3) Nucleon rest masses, 4) Nuclear stability, 5) Nuclear binding energy, 6) Strong interaction range, 7) Nuclear charge radius, 8) Rms charge radius of proton, 9) Rms charge radii of atomic nuclei and 10) Magic numbers. At the stable mass number of  $Z$ , nuclear binding energy seems to be proportional to the sum of  $2Z$  up quarks rest energy and  $Z$  down quarks rest energy. Interesting point to be noted is that, binding energy near to stable mass number seems to be independent of the mass number. Finally an attempt is made to fit the SEMF energy coefficients. With further research and analysis, strong coupling constant and gravitational constant both can successfully be implemented in low energy nuclear binding energy scheme.

**Keywords:** Strong interaction, Gravitational constant/Atomic gravitational constant, Up and Down quarks, Strong coupling constant, Nuclear stability, Nuclear binding energy, Nuclear charge radii, Magic numbers.

## 1. Introduction

Denis Lacroix says [1]: “Two fundamental questions of present days nuclear physics are: (i) How to understand the very rich structure of atomic nuclei in terms of interaction between nucleons? (ii) How to relate the strong nuclear interaction to the underlying fundamental Chromodynamics (QCD) that governs the physics of quarks and gluons. These two questions illustrate the many facets of nucleon-nucleon interactions. “Low energy” nuclear scientists mainly address (i) and often consider the strong interaction as a “fundamental” interaction and nucleons as elementary (often point like) particles. From the “High energy” nuclear physics point of view, nucleons, being formed by quarks and gluons, can obviously not be considered as elementary particles and the strong interaction itself should be understood from the more fundamental Standard Model. The nuclear force is at the crossroad of these two visions.

Recently, important progresses have been made in the understanding the nuclear strong interaction directly from QCD. Conjointly, new experimental results have pointed out our lack of knowledge of the interaction in the nuclear medium”. Finally he concludes that, “Recent achievements in nuclear forces theory open new perspectives for the next decade of low energy nuclear physics, bringing together people from very different communities. Although many developments remain to be done, the possibility to directly use QCD to describe nuclear system is a major challenge that is within reach”.

With reference to particle data group [2], up quark mass is  $m_u \cong 2.15 \text{ MeV} / c^2$  and down quark mass is  $m_d \cong 4.7 \text{ MeV} / c^2$ . Up and down quark mass ratio is 0.46(5). In the earlier published paper [3] titled: “Super symmetry in strong and weak interactions”, the authors proposed a simple method for estimating the six quark masses. With reference to the earlier proposed results, up quark mass is  $m_u \cong 4.4 \text{ MeV} / c^2$  and down quark mass is  $m_d \cong 9.473 \text{ MeV} / c^2$ . By considering the up and down quark rest masses, in this paper the authors proposed very simple methods for estimating the nuclear stability and nuclear binding energy [4] and successfully extended the quark model to low energy scale nuclear physics. Topics covered are: 1) Hint of final unification and Up and Down quark rest masses, 2) Strong coupling constant, 3) Nucleon rest masses, 4) Nuclear stability, 5) Nuclear binding energy, 6) Strong interaction range, 7) Nuclear charge radius, 8) Rms charge radius of proton, 9) Rms charge radii of atomic nuclei and 10) Magic numbers.

## 2. Role of Down and Up quark masses in understanding nuclear stability

Despite the protons’ mutual electromagnetic repulsion, a stronger attractive force was postulated to explain how the atomic nucleus was bound together. This hypothesized force was called the ‘strong force’, which was believed to be a fundamental force that acted on nucleons: the protons and neutrons that make up the nucleus. It was later discovered that protons and neutrons were not fundamental particles, but were made up of constituent particles called ‘quarks’ [5]. In particle physics, the ‘strong interaction’ is the mechanism responsible for the strong nuclear force. It is approximately 100 times stronger than electromagnetism, a million times stronger than the weak force interaction. It ensures the stability of ordinary nuclear matter that constitutes observable neutrons and protons. It is also believed that, ‘residual strong force’ plays a key role in the context of binding protons and neutrons together to form atoms. Clearly speaking, it is the residuum of the strong interaction between the up and down quarks that make up the protons and neutrons.

In the semi empirical mass formula [4], by maximizing  $B(A, Z)$  with respect to  $Z$ , we find the number of protons  $Z$  of the stable nucleus of atomic weight  $A$  as,

$$Z \approx \frac{A}{1 + (a_c / 2a_a) A^{2/3}}. \quad (1)$$

This is roughly  $A/2$  for light nuclei, but for heavy nuclei there is an even better agreement with nature. By substituting the above value of  $Z$  back into  $B$  one obtains the binding energy as a function of the atomic weight,  $B(A)$ . Maximizing  $B(A)/A$  with respect to  $A$  gives the nucleus

which is most strongly bound and most stable. Considering the up and down quark masses, and without considering the semi empirical mass formula it is also possible to show that [4],

$$A_{stable} \cong 2Z + \frac{(Z\alpha_s)^2}{2} \quad (2)$$

where,  $\alpha_s$  is the strong coupling constant [6] and  $A_{stable}$  represents the estimated stable mass numbers of  $Z$ . It is possible to guess that,

$$\frac{1}{\alpha_s} \cong \exp\left(\frac{m_d}{m_u}\right) \cong 8.90 \rightarrow \alpha_s \cong 0.11236 \quad (3)$$

where  $m_u \cong 2.15 \text{ MeV}/c^2$  and  $m_d \cong 4.7 \text{ MeV}/c^2$ . This value of the strong coupling constant can be compared with the current estimates of  $\alpha_s \cong 0.1186$ . For the proposed nuclear stability data see columns 1 and 2 of table-1.

### 3. Role of Down and Up quark masses in understanding nuclear binding energy

In this section the authors proposed a very simple method for understanding the nuclear binding energy with one energy constant. One important point to be noted here is that, as per the quark model, proton constitutes 3 quarks as  $(U, U, D)$ . For  $Z$  numbers of participating protons, number of participating up quarks can be  $2Z$  and number of participating down quarks can be  $Z$ . At the stable mass number, it is possible to show that,

$$(B)_{Stable} \cong kZ [2m_u c^2 + m_d c^2] \quad (4)$$

where  $k$  is a coefficient that seems to be related with proton number and the down and up quark mass ratio. For the observed data it can be suggested that,

$$\left. \begin{array}{l} \text{Case-1: } Z \cong 2 \text{ to } 30, k \cong \left(\frac{Z}{30}\right)^{\frac{1}{6}} \left(\frac{m_d}{m_u}\right) \\ \text{Case-2: } Z \geq 30, k \cong \left(\frac{m_d}{m_u}\right) \cong 2.186 \end{array} \right\} \quad (5)$$

Here interesting point to be noted is that, binding energy near to stable mass number is practically independent of mass number! See the following table-1 for nuclear binding energy close to the stable mass number of  $Z$ . Above and below the stable mass number, to a very good approximation, nuclear binding energy can be expressed in the following way.

$$\left. \begin{array}{l} \text{If } (A > A_{Stable}), (B)_A \cong \left(\frac{A}{A_{Stable}}\right)^{\frac{2}{3}} (B)_{Stable} \cong \left(\frac{A}{A_{Stable}}\right)^{\frac{2}{3}} kZ [2m_u c^2 + m_d c^2] \\ \text{If } (A < A_{Stable}), (B)_A \cong \left(\frac{A}{A_{Stable}}\right)^{\frac{4}{3}} (B)_{Stable} \cong \left(\frac{A}{A_{Stable}}\right)^{\frac{4}{3}} kZ [2m_u c^2 + m_d c^2] \end{array} \right\} \quad (6)$$

Obtained nuclear binding can be compared with results of the standard form of the following semi empirical formula:

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} \pm \frac{a_p}{\sqrt{A}} \quad (7)$$

where,  $\begin{cases} a_v \cong 15.78 \text{ MeV}, a_s \cong 16.34 \text{ MeV}, a_c \cong 0.71 \text{ MeV}, \\ a_a \cong 23.21 \text{ MeV}, a_p \cong 12.0 \text{ MeV}. \end{cases}$

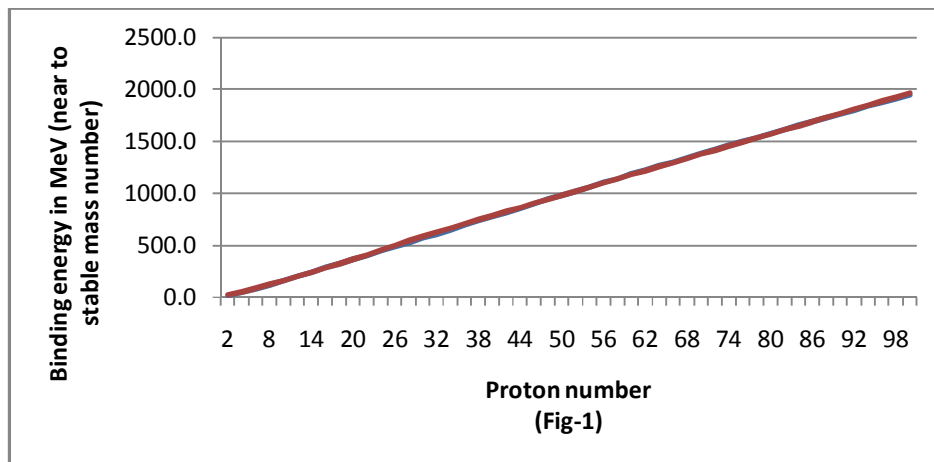
In the following table-1, column-1 represents proton number, column-2 represents stable estimated stable mass number with the proposed relations (2, 3), column-3 represents the estimated value of  $k$  from relation (5), column-4 represents the neutron number, column-5 represents the mass number, column-6 represents binding energy calculated from SEMF relation (7) with current energy coefficients and column-7 represents binding energy calculated with proposed relations (4,5).

**Table-1: To fit the nuclear binding energy near the estimated mean stable mass number of  $Z$**

| Proton number | Estimated stable mass number | Estimated value of $k$ | Neutron number | Nucleon number | Binding energy in MeV form SEMF Relation(7) | Proposed binding energy in MeV Relation(4) |
|---------------|------------------------------|------------------------|----------------|----------------|---|--|
| 2             | 4                            | 1.3920                 | 2              | 4              | 22.0  | 25.1                                       |
| 4             | 8                            | 1.5625                 | 4              | 8              | 52.9  | 56.2                                       |
| 6             | 12                           | 1.6717                 | 6              | 12             | 87.4  | 90.3                                       |
| 8             | 16                           | 1.7538                 | 8              | 16             | 123.2                                       | 126.3                                      |
| 10            | 21                           | 1.8203                 | 11             | 21             | 167.5                                       | 163.8                                      |
| 12            | 25                           | 1.8764                 | 13             | 25             | 204.7                                       | 202.7                                      |
| 14            | 29                           | 1.9252                 | 15             | 29             | 241.6                                       | 242.6                                      |
| 16            | 34                           | 1.9686                 | 18             | 34             | 290.8                                       | 283.5                                      |
| 18            | 38                           | 2.0076                 | 20             | 38             | 327.2                                       | 325.2                                      |
| 20            | 43                           | 2.0432                 | 23             | 43             | 371.6                                       | 367.8                                      |
| 22            | 47                           | 2.0759                 | 25             | 47             | 407.5                                       | 411.0                                      |
| 24            | 52                           | 2.1062                 | 28             | 52             | 454.6                                       | 454.9                                      |
| 26            | 56                           | 2.1345                 | 30             | 56             | 489.6                                       | 499.5                                      |
| 28            | 61                           | 2.1610                 | 33             | 61             | 532.5                                       | 544.6                                      |
| 30            | 66                           | 2.1864                 | 36             | 66             | 577.9                                       | 590.2                                      |
| 32            | 70                           | 2.1864                 | 38             | 70             | 611.7                                       | 629.6                                      |
| 34            | 75                           | 2.1864                 | 41             | 75             | 653.3                                       | 668.9                                      |
| 36            | 80                           | 2.1864                 | 44             | 80             | 697.0                                       | 708.3                                      |
| 38            | 85                           | 2.1865                 | 47             | 85             | 737.6                                       | 747.6                                      |
| 40            | 90                           | 2.1865                 | 50             | 90             | 780.2                                       | 787.0                                      |
| 42            | 95                           | 2.1865                 | 53             | 95             | 819.8                                       | 826.3                                      |
| 44            | 100                          | 2.1866                 | 56             | 100            | 861.2                                       | 865.7                                      |
| 46            | 105                          | 2.1866                 | 59             | 105            | 899.8                                       | 905.0                                      |
| 48            | 111                          | 2.1866                 | 63             | 111            | 947.7                                       | 944.4                                      |
| 50            | 116                          | 2.1867                 | 66             | 116            | 987.5                                       | 983.7                                      |

|     |     |        |     |     |        |        |
|-----|-----|--------|-----|-----|--------|--------|
| 52  | 121 | 2.1867 | 69  | 121 | 1024.6 | 1023.1 |
| 54  | 126 | 2.1867 | 72  | 126 | 1063.4 | 1062.4 |
| 56  | 132 | 2.1868 | 76  | 132 | 1108.7 | 1101.7 |
| 58  | 137 | 2.1868 | 79  | 137 | 1144.4 | 1141.1 |
| 60  | 143 | 2.1868 | 83  | 143 | 1188.5 | 1180.4 |
| 62  | 148 | 2.1869 | 86  | 148 | 1225.3 | 1219.8 |
| 64  | 154 | 2.1869 | 90  | 154 | 1268.2 | 1259.1 |
| 66  | 159 | 2.1869 | 93  | 159 | 1302.1 | 1298.5 |
| 68  | 165 | 2.1870 | 97  | 165 | 1343.9 | 1337.8 |
| 70  | 171 | 2.1861 | 101 | 171 | 1385.1 | 1377.2 |
| 72  | 177 | 2.1861 | 105 | 177 | 1425.7 | 1416.5 |
| 74  | 183 | 2.1861 | 109 | 183 | 1465.7 | 1455.9 |
| 76  | 188 | 2.1861 | 112 | 188 | 1499.2 | 1495.2 |
| 78  | 194 | 2.1861 | 116 | 194 | 1538.2 | 1534.6 |
| 80  | 200 | 2.1861 | 120 | 200 | 1576.7 | 1573.9 |
| 82  | 206 | 2.1861 | 124 | 206 | 1614.6 | 1613.3 |
| 84  | 213 | 2.1861 | 129 | 213 | 1657.5 | 1652.6 |
| 86  | 219 | 2.1861 | 133 | 219 | 1694.3 | 1692.0 |
| 88  | 225 | 2.1861 | 137 | 225 | 1730.7 | 1731.3 |
| 90  | 231 | 2.1861 | 141 | 231 | 1766.5 | 1770.7 |
| 92  | 237 | 2.1861 | 145 | 237 | 1801.9 | 1810.0 |
| 94  | 244 | 2.1862 | 150 | 244 | 1843.4 | 1849.4 |
| 96  | 250 | 2.1862 | 154 | 250 | 1877.8 | 1888.7 |
| 98  | 257 | 2.1862 | 159 | 257 | 1916.5 | 1928.1 |
| 100 | 263 | 2.1862 | 163 | 263 | 1949.9 | 1967.4 |

See the following figure-1 for nuclear binding energy near to estimated stable mass number. In the figure-1, red line represents the estimated binding energy and blue line represents the nuclear binding energy calculated from the semi empirical mass formula with the current recommended energy coefficients.

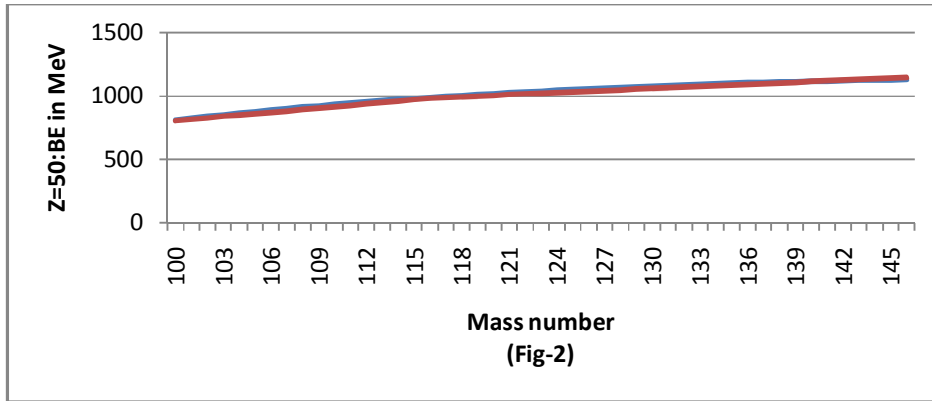


Estimated stable mass number of  $Z = 50$  is  $A_{Stable} = 116$ . With this stable mass number- an attempt is made to fit the nuclear binding energy with relations (5 and 6). See the following table-2 and related figure-2. In the figure, red curve represents the estimated binding energy from relation (6) and blue curve represents the nuclear binding energy calculated from the semi empirical mass formula with the current recommended energy coefficients (7).

**Table-2: To fit the SEMF binding energy of  $Z = 50$**

| Proton number | Stable mass number | Assumed value of $k$ | Neutron number | Mass number | Binding energy in MeV from SEMF Relation (7) | Binding energy in MeV calculated with relation (6) |
|---------------|--------------------|----------------------|----------------|-------------|--|--|
| 50            | 116                | 2.187                | 50             | 100         | 809.3  | 807.1  |
| 50            | 116                | 2.187                | 51             | 101         | 822.3  | 817.9  |
| 50            | 116                | 2.187                | 52             | 102         | 837.2  | 828.7  |
| 50            | 116                | 2.187                | 53             | 103         | 849.2  | 839.5  |
| 50            | 116                | 2.187                | 54             | 104         | 863.2  | 850.4  |
| 50            | 116                | 2.187                | 55             | 105         | 874.5  | 861.3  |
| 50            | 116                | 2.187                | 56             | 106         | 887.6  | 872.3  |
| 50            | 116                | 2.187                | 57             | 107         | 898.1  | 883.3  |
| 50            | 116                | 2.187                | 58             | 108         | 910.4  | 894.3  |
| 50            | 116                | 2.187                | 59             | 109         | 920.1  | 905.4  |
| 50            | 116                | 2.187                | 60             | 110         | 931.8  | 916.5  |
| 50            | 116                | 2.187                | 61             | 111         | 940.7  | 927.6  |
| 50            | 116                | 2.187                | 62             | 112         | 951.6  | 938.7  |
| 50            | 116                | 2.187                | 63             | 113         | 960.0  | 949.9  |
| 50            | 116                | 2.187                | 64             | 114         | 970.2  | 961.2  |
| 50            | 116                | 2.187                | 65             | 115         | 977.9  | 972.4  |
| 50            | 116                | 2.187                | 66             | 116         | 987.5  | 983.7  |
| 50            | 116                | 2.187                | 67             | 117         | 994.6  | 989.3  |
| 50            | 116                | 2.187                | 68             | 118         | 1003.6                                       | 995.0  |
| 50            | 116                | 2.187                | 69             | 119         | 1010.1                                       | 1000.6   |
| 50            | 116                | 2.187                | 70             | 120         | 1018.5                                       | 1006.2   |
| 50            | 116                | 2.187                | 71             | 121         | 1024.4                                       | 1011.8   |
| 50            | 116                | 2.187                | 72             | 122         | 1032.3                                       | 1017.3   |
| 50            | 116                | 2.187                | 73             | 123         | 1037.8                                       | 1022.9   |
| 50            | 116                | 2.187                | 74             | 124         | 1045.1                                       | 1028.4   |
| 50            | 116                | 2.187                | 75             | 125         | 1050.1                                       | 1033.9   |
| 50            | 116                | 2.187                | 76             | 126         | 1056.9                                       | 1039.5   |
| 50            | 116                | 2.187                | 77             | 127         | 1061.4                                       | 1044.9   |
| 50            | 116                | 2.187                | 78             | 128         | 1067.8                                       | 1050.4   |
| 50            | 116                | 2.187                | 79             | 129         | 1071.8                                       | 1055.9   |

|    |     |       |    |     |        |        |
|----|-----|-------|----|-----|--------|--------|
| 50 | 116 | 2.187 | 80 | 130 | 1077.7 | 1061.3 |
| 50 | 116 | 2.187 | 81 | 131 | 1081.4 | 1066.8 |
| 50 | 116 | 2.187 | 82 | 132 | 1086.9 | 1072.2 |
| 50 | 116 | 2.187 | 83 | 133 | 1090.1 | 1077.6 |
| 50 | 116 | 2.187 | 84 | 134 | 1095.2 | 1083.0 |
| 50 | 116 | 2.187 | 85 | 135 | 1098.0 | 1088.4 |
| 50 | 116 | 2.187 | 86 | 136 | 1102.7 | 1093.8 |
| 50 | 116 | 2.187 | 87 | 137 | 1105.1 | 1099.1 |
| 50 | 116 | 2.187 | 88 | 138 | 1109.4 | 1104.4 |
| 50 | 116 | 2.187 | 89 | 139 | 1111.5 | 1109.8 |
| 50 | 116 | 2.187 | 90 | 140 | 1115.5 | 1115.1 |
| 50 | 116 | 2.187 | 91 | 141 | 1117.2 | 1120.4 |
| 50 | 116 | 2.187 | 92 | 142 | 1120.8 | 1125.7 |
| 50 | 116 | 2.187 | 93 | 143 | 1122.3 | 1131.0 |
| 50 | 116 | 2.187 | 94 | 144 | 1125.6 | 1136.2 |



#### 4. Discussion

With reference to quark soup, understanding nuclear binding energy is very critical and very interesting at fundamental. In this critical moment, proposed relations connected with strong interaction and nuclear binding energy scheme can be recommended for further research. If one is willing to consider the up quark mass estimate as  $m_u \approx 4.4 \text{ MeV} / c^2$  and down quark mass estimate as  $m_d \approx 9.473 \text{ MeV} / c^2$  it is possible to show that, **starting from  $Z = 30$ , near to the stable mass number, proportionality coefficient being “ $(m_d/2m_u)$ ”, nuclear binding energy is equal to the sum of rest energies of  $(2Zm_u c^2)$  and  $(Zm_d c^2)$ .** See the following relations for nuclear binding energy with suggested up and down quark masses.

$$(B)_{\text{Stable}} \cong kZ [2m_u c^2 + m_d c^2] \quad (8)$$

where  $k$  is a coefficient that seems to be related with proton number and the ratio of ‘one down quark mass’ and ‘two up quark masses’. Here  $m_u \cong 4.4 \text{ MeV} / c^2$  and  $m_d \cong 9.473 \text{ MeV} / c^2$ . For the observed data, now it can be suggested that,

$$\left. \begin{array}{l} \text{Case-1: } Z \cong 2 \text{ to } 30, k \cong \left( \frac{Z}{30} \right)^{\frac{1}{6}} \left( \frac{m_d}{2m_u} \right) \\ \text{Case-2: } Z \geq 30, k \cong \left( \frac{m_d}{2m_u} \right) \cong 1.0765 \end{array} \right\} \quad (9)$$

Above and below the stable mass number, to a very good approximation, nuclear binding energy can be expressed in the following way.

$$\left. \begin{array}{l} \text{If } (A \geq A_{Stable}), (B)_A \cong \left( \frac{A}{A_{Stable}} \right)^{\frac{2}{3}} (B)_{Stable} \cong \left( \frac{A}{A_{Stable}} \right)^{\frac{2}{3}} kZ [2m_u c^2 + m_d c^2] \\ \text{If } (A < A_{Stable}), (B)_A \cong \left( \frac{A}{A_{Stable}} \right)^{\frac{4}{3}} (B)_{Stable} \cong \left( \frac{A}{A_{Stable}} \right)^{\frac{4}{3}} kZ [2m_u c^2 + m_d c^2] \\ \text{where } A_{stable} \cong 2Z + \left[ \left( \frac{m_u}{m_d} \right) (Z\alpha_s)^2 \right] \end{array} \right\} \quad (10)$$

The authors are working in this new direction and are confident that very soon a very simplified formula can be developed for nuclear binding energy with Up and Down quark masses.

## 5. Semi empirical unified approach for fixing the basic nuclear physical parameters

### 5.1 To fix Up and down quark rest masses and to fix the strong coupling constant

#### Step-1: To fix Down and Up quark mass ratio

$$\begin{aligned} \exp\left(\frac{m_d c^2}{m_u c^2}\right)^2 &\cong \ln\left(\frac{\hbar c}{Gm_e^2}\right) \\ \rightarrow \frac{m_d c^2}{m_u c^2} &\cong \sqrt{\ln\left[\ln\left(\frac{\hbar c}{Gm_e^2}\right)\right]} \cong 2.1529545 \end{aligned} \quad (11)$$

where  $G$  is the gravitational constant. This relation is very complicated to understand and very simple to consider. The authors proposed this relation in 2010 [3]. This can be considered as a hint of final unification. In the published paper [9] and references therein, the authors suggested that, with reference to Schwarzschild interaction, atomic gravitational constant  $G_A$  is squared Avogadro number  $N_A^2$  times the Newtonian gravitational constant. Based on this proposal, it is possible to show that,



$$\begin{aligned}
\text{If } G_A \cong N_A^2 G, \quad \hbar \cong \left( \frac{m_u c^2}{m_d c^2} \right) \sqrt{\left( \frac{G_A m_e^2}{c} \right) \left( \frac{e^2}{4\pi\epsilon_0 c} \right)} \\
\rightarrow \frac{m_d c^2}{m_u c^2} \cong \sqrt{\left( \frac{G_A m_e^2}{\hbar c} \right) \left( \frac{e^2}{4\pi\epsilon_0 \hbar c} \right)}
\end{aligned} \tag{12}$$

From relations (11) and (12),

$$\frac{m_d c^2}{m_u c^2} \cong \sqrt{\ln \left[ \ln \left( \frac{\hbar c}{G m_e^2} \right) \right]} \cong \sqrt{\left( \frac{G_A m_e^2}{\hbar c} \right) \left( \frac{e^2}{4\pi\epsilon_0 \hbar c} \right)} \tag{13}$$

This is another way of fitting Down and Up quark mass ratio. At this juncture it is really hard to understand the physical meaning of this characteristic relation. With this relation (13) and with reference to the Avogadro number, Newtonian gravitational constant can be fixed like other physical constants. Its magnitude comes out to be  $6.673087915 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ sec}^{-2}$  and  $(m_d c^2 / m_u c^2) \cong 2.152967995$ .

### Step-2: To fix the Up and down quark masses

Ratio of Up quark rest mass and electron rest mass can be assumed as follows.

$$\ln \left( \frac{m_u c^2}{m_e c^2} \right) \cong \left( \frac{m_d c^2}{m_u c^2} \right) \cong \sqrt{\ln \left[ \ln \left( \frac{\hbar c}{G m_e^2} \right) \right]} \tag{14}$$

$$\rightarrow m_u c^2 \cong \exp \left[ \ln \left[ \ln \left( \frac{\hbar c}{G m_e^2} \right) \right] \right] m_e c^2 \cong 4.4 \text{ MeV}. \tag{15}$$

$$\Rightarrow m_d c^2 \cong \sqrt{\ln \left[ \ln \left( \frac{\hbar c}{G m_e^2} \right) \right]} m_u c^2 \cong 9.473 \text{ MeV}.$$

### Step-3: To fix the strong coupling constant

It can be fitted as follows.

$$\begin{aligned}
\frac{1}{\alpha_s} &\cong \exp \left( \frac{m_d c^2}{m_u c^2} \right) \cong \exp \left[ \ln \left[ \ln \left( \frac{\hbar c}{G m_e^2} \right) \right] \right] \\
\rightarrow \frac{1}{\alpha_s} &\cong \exp(2.1529545) \cong 8.61026 \\
\Rightarrow \alpha_s &\cong \left[ \exp(2.1529545) \right]^{-1} \cong 0.11614.
\end{aligned} \tag{16}$$

With reference to the atomic gravitational constant [9],

$$\begin{aligned}
\frac{1}{\alpha_s} &\cong \exp\left(\frac{m_d c^2}{m_u c^2}\right) \cong \exp\sqrt{\left(\frac{G_A m_e^2}{\hbar c}\right)\left(\frac{e^2}{4\pi\epsilon_0 \hbar c}\right)} \\
\rightarrow \frac{1}{\alpha_s} &\cong \exp(2.15309061) \cong 8.611432 \\
\Rightarrow \alpha_s &\cong \left[\exp(2.15309061)\right]^{-1} \cong 0.116125.
\end{aligned} \tag{17}$$

If so, relation between strong coupling constant, atomic gravitational constant and down quark mass can be expressed as follows.

$$\frac{1}{\alpha_s} \cong \ln \sqrt{\frac{4\pi\epsilon_0 G_A m_d^2}{e^2}} \tag{18}$$

## 5.2 About the neutron rest mass and neutron–proton mass difference

Qualitatively and quantitatively neutron rest energy can be interlinked with the proposed up and down quarks as expressed in the following way.

$$\begin{aligned}
m_n c^2 &\cong \left(\frac{1}{\alpha} + \frac{1}{\alpha_s}\right) \sqrt{m_u m_d} c^2 \cong \left(\frac{\alpha_s + \alpha}{\alpha_s \alpha}\right) \sqrt{m_u m_d} c^2 \cong 940.27 \text{ MeV} \\
\rightarrow \frac{m_n c^2}{\sqrt{m_u m_d} c^2} &\cong \left(\frac{1}{\alpha} + \frac{1}{\alpha_s}\right) \cong \left(\frac{\alpha_s + \alpha}{\alpha_s \alpha}\right)
\end{aligned} \tag{19}$$

By considering this relation (19), strong coupling constant can also be fitted. Neutron and proton rest energy difference can be fitted in the following way.

$$\begin{aligned}
\frac{\sqrt{m_u m_d} c^2}{m_n c^2 - m_p c^2} &\cong \ln\left(\frac{1}{\alpha} + \frac{1}{\alpha_s}\right) \cong \ln\left(\frac{\alpha_s + \alpha}{\alpha_s \alpha}\right) \\
\rightarrow m_n c^2 - m_p c^2 &\cong \left[\ln\left(\frac{1}{\alpha} + \frac{1}{\alpha_s}\right)\right]^{-1} \sqrt{m_u m_d} c^2 \cong 1.296 \text{ MeV}
\end{aligned} \tag{20}$$

## 5.3 About the Semi empirical mass formula energy coefficients

The semi empirical mass formula energy coefficients can be fitted and understood as follows. For understanding purpose, data fitting purpose and for simplicity,  $\sqrt{m_u m_d} c^2$  is taken as a reference. Interested researchers may consider any other characteristic mass unit that connected with up and down quark masses.

### Step-1: Volume energy coefficient

It can be fitted as follows.

$$a_v \cong \ln \sqrt{\frac{m_p c^2}{\sqrt{m_u m_d} c^2}} \cdot \sqrt{m_u m_d} c^2 \cong 16.07 \text{ MeV} \cong 16.1 \text{ MeV} \quad (21)$$

Thus it can be suggested that,

$$\frac{a_v}{\sqrt{m_u m_d} c^2} \cong \ln \sqrt{\frac{m_p c^2}{\sqrt{m_u m_d} c^2}} \quad (22)$$

### Step-2: Surface energy coefficient

It can be fitted as follows.

$$a_s \cong \sqrt{\frac{1}{\alpha_s}} \cdot \sqrt{m_u m_d} c^2 \cong 18.94 \text{ MeV} \quad (23)$$

Thus it can be suggested that,

$$\frac{a_s}{\sqrt{m_u m_d} c^2} \cong \sqrt{\frac{1}{\alpha_s}} \quad (24)$$

### Step-3: Coulombic energy coefficient

It can be fitted as follows.

$$a_c \cong \alpha_s \sqrt{m_u m_d} c^2 \cong 0.75 \text{ MeV} \quad (25)$$

Thus it can be suggested that,

$$\frac{a_c}{\sqrt{m_u m_d} c^2} \cong \alpha_s \quad (26)$$

With this Coulombic energy coefficient ‘ending of strong interaction range’ or ‘beginning of coulombic repulsion range’ can be expressed as follows.

$$R_s \cong \frac{e^2}{4\pi\epsilon_0 a_c} \cong \left(\frac{1}{\alpha_s}\right) \frac{e^2}{4\pi\epsilon_0 \sqrt{m_u m_d} c^2} \cong 1.92 \text{ fm} \quad (27)$$

With this relation (23) it is noticed that,

$$\left. \begin{aligned} R_{up} &\cong \left(\frac{1}{\alpha_s}\right) \frac{e^2}{4\pi\epsilon_0 m_u c^2} \cong 2.818 \text{ fm}, \\ R_{down} &\cong \left(\frac{1}{\alpha_s}\right) \frac{e^2}{4\pi\epsilon_0 m_d c^2} \cong 1.309 \text{ fm} \\ \sqrt{R_{up} R_{down}} &\cong 1.92 \text{ fm}, \quad \frac{R_{up}^2}{R_{down}} \cong 0.61 \text{ fm}, \\ \frac{2R_{up}^2}{R_{down}} &\cong 1.216 \text{ fm} \quad \text{and} \quad \frac{\sqrt{2}R_{up}^2}{R_{down}} \cong 0.86 \text{ fm} \end{aligned} \right\} \quad (28)$$

Here, 1.216 fm can be compared with the characteristic ‘nuclear charge radius’ and 0.86 fm can be compared with the ‘rms radius’ of proton. With  $(R_{up}^2/R_{down}) \cong 0.61$  fm, rms charge radii of atomic nuclei [10] can be fitted as follows.

$$R_{(N,Z)} \cong \left[ (Z+2) + (N+2) \right]^{\frac{1}{3}} (R_{up}^2/R_{down}) \quad \left. \vphantom{R_{(N,Z)}} \right\} \quad (29)$$

$$\cong \left[ (Z+2) + (N+2) \right]^{\frac{1}{3}} 0.61 \text{ fm, where } N = A - Z$$

#### Step-4: Asymmetry energy coefficient

Asymmetry energy coefficient can be fitted as follows.

$$a_a \cong \left[ \left( \frac{m_d}{m_u} \right)^2 - 1 \right] \sqrt{m_u m_d} c^2 \cong 23.47 \text{ MeV} \quad (30)$$

Thus it can be suggested that,

$$\frac{a_a}{\sqrt{m_u m_d} c^2} \cong \left[ \left( \frac{m_d}{m_u} \right)^2 - 1 \right] \quad (31)$$

#### Step-5: Pairing energy coefficient

Pairing energy coefficient can be fitted as follows.

$$a_p \cong \frac{1}{2} \left[ \left( \frac{m_d}{m_u} \right)^2 - 1 \right] \sqrt{m_u m_d} c^2 \cong 11.735 \text{ MeV} \quad (32)$$

Thus it can be suggested that,

$$\frac{a_p}{\sqrt{m_u m_d} c^2} \cong \frac{1}{2} \left[ \left( \frac{m_d}{m_u} \right)^2 - 1 \right] \quad (33)$$

See table-3 for the comparison of current and proposed energy coefficients. See table-4 for the measured and estimated binding energy from SEMF with the proposed energy coefficients.

**Table-3: Current and proposed SEMF binding energy coefficients**

|                 | $a_v$ MeV | $a_s$ MeV | $a_c$ MeV | $a_a$ MeV | $a_p$ MeV |
|-----------------|-----------|-----------|-----------|-----------|-----------|
| <b>Current</b>  | 15.78     | 18.34     | 0.71      | 23.21     | 12.0      |
| <b>Proposed</b> | 16.07     | 18.944    | 0.75      | 23.47     | 11.735    |

#### 5.4 To understand the magic numbers

Considering the strong interaction that takes place in between Up-Up, Up-Down and Down-Down along with the neutron-proton as a whole, one can expect four kinds of ‘strong attractive’

forces within the nucleus. Considering the number ‘four’ as a characteristic structural feature, observed magic numbers [7,8] can be fitted in the following.

**Table-4: To fit the measured binding energy with the proposed SEMF energy coefficients**

| $Z$ | $A$ | Measured<br>$(BE)_{meas}$ in MeV | $(BE)_{cal}$ in MeV |
|-----|-----|----------------------------------|---------------------|
| 26  | 56  | 492.258                          | 490.07              |
| 44  | 100 | 861.928                          | 860.53              |
| 50  | 116 | 988.684                          | 986.0               |
| 70  | 170 | 1378.13                          | 1373.28             |
| 82  | 208 | 1636.43                          | 1619.12             |
| 92  | 238 | 1801.69                          | 1797.12             |

### Step-1: The magic plateaus’

$$P_I \cong 4[n(n+1)] + 4 \cong 4I + 4 \quad (34)$$

where  $n = 0, 1, 2, 3, \dots$  and  $I = n(n+1)$ .

Magic plateau begins at  $(P_I)_{start} \cong 4[n(n+1)]$  and extends up to  $(P_I)_{end} \cong 4[n(n+1)] + 4$ .

### Step-2: Mid of the magic plateaus

It is noticed that, mid of the magic plateau is very close to the observed magic numbers.

$$M \cong \frac{(P_I)_{start} + (P_I)_{end}}{2} \quad (35)$$

### Step-3: Mid of the ending of consecutive magic plateaus’

Mid of the ending of consecutive magic plateaus’ seems to be close to the observed semi magic numbers.

$$M \cong \frac{(P_{I1})_{end} + (P_{I2})_{end}}{2} \quad (36)$$

where  $I1 = n(n+1)$  and  $I2 = (n+1)(n+2)$ . See the following table-5 for the comparison of actual and proposed magic numbers. From the table-5, it is very clear to say that, proposed magic plateau mid is close to and proportional to the corresponding magic numbers. Magic numbers like 114, 126, 164, 174, ... etc are found to be missing from the net and authors are working on this.

**Table-5: To fit the magic numbers**

| n | I  | 4I  | 4I+4 | $[(4I)+(4I+4)]/2$ | Actual magic numbers | Mid of the ending of consecutive magic plateaus' |
|---|----|-----|------|-------------------|----------------------|--|
| 0 | 0  | 0   | 4    | 2                 | 2                    |  |
| 1 | 2  | 8   | 12   | 10                | 8                    | $(4+12)/2=8$                                     |
| 2 | 6  | 24  | 28   | 26                | 20, 28               | $(12+28)/2=20$                                   |
| 3 | 12 | 48  | 52   | 50                | 50                   | $(28+52)/2=40$                                   |
| 4 | 20 | 80  | 84   | 82                | 82                   | $(52+84)/2=68$                                   |
| 5 | 30 | 120 | 124  | 122               | (114,122,124,126)    | $(84+124)/2=104$                                 |
| 6 | 42 | 168 | 172  | 170               | (164,174,184,196)    | $(124+172)/2=148$                                |
| 7 | 56 | 224 | 228  | 226               | (236)                | $(172+228)/2=200$                                |
| 8 | 72 | 288 | 292  | 290               | (318)                | $(228+292)/2=260$                                |

## 6. Conclusion

Considering all of the above proposed relations, concepts and data fitting, in near future - a scale independent 'quark sea' model of observed and stable atomic nuclei can certainly be developed with great confidence. The five energy coefficients of the famous semi empirical mass formula can be understood in terms of up and down quarks.

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