

Another infinite sequence based on mar function that abounds in primes and semiprimes

Marius Coman
email: mariuscoman13@gmail.com

Abstract. In one of my previous paper, namely "The mar reduced form of a natural number", I introduced the notion of mar function, which is, essentially, nothing else than the digital root of a number, and I also presented, in another paper, a sequence based on mar function that abounds in primes. In this paper I present another sequence, based on a relation between a number and the value of its mar reduced form (of course not the intrinsic one), sequence that seem also to abound in primes and semiprimes.

Let's consider the sequence of numbers $m(n)$, where n is odd and m is equal to $x*y + n$, where $x = 2*\text{mar } n + 2$ and $y = 2*\text{mar } n - 2$, if $y \neq 0$, respectively m is equal to $x + n$, where $x = 2*\text{mar } n + 2$, if $y = 0$:

: for $n = 1$, we have $\text{mar } n = 1$, $x = 4$ and $y = 0$ so $m = 5$;
: for $n = 3$, we have $\text{mar } n = 3$, $x = 8$ and $y = 4$ so $m = 35$;
: for $n = 5$, we have $\text{mar } n = 5$, $x = 12$ and $y = 8$ so $m = 101$;
: for $n = 7$, we have $\text{mar } n = 7$, $x = 16$ and $y = 12$ so $m = 199$;
: for $n = 9$, we have $\text{mar } n = 9$, $x = 20$ and $y = 16$ so $m = 329$;
: for $n = 11$, we have $\text{mar } n = 2$, $x = 6$ and $y = 2$ so $m = 23$;
: for $n = 13$, we have $\text{mar } n = 4$, $x = 10$ and $y = 6$ so $m = 73$;
: for $n = 15$, we have $\text{mar } n = 6$, $x = 14$ and $y = 10$ so $m = 155$;
: for $n = 17$, we have $\text{mar } n = 8$, $x = 18$ and $y = 14$ so $m = 269$;
: for $n = 19$, we have $\text{mar } n = 1$, $x = 4$ and $y = 0$ so $m = 23$;
: for $n = 21$, we have $\text{mar } n = 3$, $x = 8$ and $y = 4$ so $m = 53$;
: for $n = 23$, we have $\text{mar } n = 5$, $x = 12$ and $y = 8$ so $m = 119$;
: for $n = 25$, we have $\text{mar } n = 7$, $x = 16$ and $y = 12$ so $m = 217$;
: for $n = 27$, we have $\text{mar } n = 9$, $x = 20$ and $y = 16$ so $m = 347$;
: for $n = 29$, we have $\text{mar } n = 2$, $x = 6$ and $y = 2$ so $m = 41$;
: for $n = 31$, we have $\text{mar } n = 4$, $x = 10$ and $y = 6$ so $m = 91$;

```

:   for n = 33, we have mar n = 6, x = 14 and y = 10 so m =
173;
:   for n = 35, we have mar n = 8, x = 18 and y = 14 so m =
287;
:   for n = 37, we have mar n = 1, x = 4 and y = 0 so m = 41;
:   for n = 39, we have mar n = 3, x = 8 and y = 4 so m = 71;
:   for n = 41, we have mar n = 5, x = 12 and y = 8 so m =
137;
:   for n = 43, we have mar n = 7, x = 16 and y = 12 so m =
235;
:   for n = 45, we have mar n = 9, x = 20 and y = 16 so m =
365;
:   for n = 47, we have mar n = 2, x = 6 and y = 2 so m = 59;
:   for n = 49, we have mar n = 4, x = 10 and y = 6 so m =
109;
:   for n = 51, we have mar n = 6, x = 14 and y = 10 so m =
191;
:   for n = 53, we have mar n = 8, x = 18 and y = 14 so m =
305;
:   for n = 55, we have mar n = 1, x = 4 and y = 0 so m = 59;
:   for n = 57, we have mar n = 3, x = 8 and y = 4 so m = 89;
:   for n = 59, we have mar n = 5, x = 12 and y = 8 so m =
155;
:   for n = 61, we have mar n = 7, x = 16 and y = 12 so m =
253;
:   for n = 63, we have mar n = 9, x = 20 and y = 16 so m =
383;
:   for n = 65, we have mar n = 2, x = 6 and y = 2 so m = 77;
:   for n = 67, we have mar n = 4, x = 10 and y = 6 so m =
127;
:   for n = 69, we have mar n = 6, x = 14 and y = 10 so m =
209;
:   for n = 71, we have mar n = 8, x = 18 and y = 14 so m =
323;
:   for n = 73, we have mar n = 1, x = 4 and y = 0 so m = 77;
:   for n = 75, we have mar n = 3, x = 8 and y = 4 so m =
107;
:   for n = 77, we have mar n = 5, x = 12 and y = 8 so m =
137;
:   for n = 79, we have mar n = 7, x = 16 and y = 12 so m =
271;
:   for n = 81, we have mar n = 9, x = 20 and y = 16 so m =
401;
:   for n = 83, we have mar n = 2, x = 6 and y = 2 so m = 95;
:   for n = 85, we have mar n = 4, x = 10 and y = 6 so m =
145;
:   for n = 87, we have mar n = 6, x = 14 and y = 10 so m =
227;
:   for n = 89, we have mar n = 8, x = 18 and y = 14 so m =
341;
:   for n = 91, we have mar n = 1, x = 4 and y = 0 so m = 95;

```

```

:   for n = 93, we have mar n = 3, x = 8 and y = 4 so m =
:   125;
:   for n = 95, we have mar n = 5, x = 12 and y = 8 so m =
:   191;
:   for n = 97, we have mar n = 7, x = 16 and y = 12 so m =
:   289;
:   for n = 99, we have mar n = 9, x = 20 and y = 16 so m =
:   419;
:   for n = 101, we have mar n = 2, x = 6 and y = 2 so m =
:   113;
:   for n = 103, we have mar n = 4, x = 10 and y = 6 so m =
:   163;
:   for n = 105, we have mar n = 6, x = 14 and y = 10 so m =
:   245;
:   for n = 107, we have mar n = 8, x = 18 and y = 14 so m =
:   359;
:   for n = 109, we have mar n = 1, x = 4 and y = 0 so m =
:   149;
:   for n = 111, we have mar n = 3, x = 8 and y = 4 so m =
:   143;
:   for n = 113, we have mar n = 5, x = 12 and y = 8 so m =
:   209;
:   for n = 115, we have mar n = 7, x = 16 and y = 12 so m =
:   307;
:   for n = 117, we have mar n = 9, x = 20 and y = 16 so m =
:   437;
:   for n = 119, we have mar n = 2, x = 6 and y = 2 so m =
:   131;
:   for n = 121, we have mar n = 4, x = 10 and y = 6 so m =
:   181;
:   for n = 123, we have mar n = 6, x = 14 and y = 10 so m =
:   263;
:   for n = 125, we have mar n = 8, x = 18 and y = 14 so m =
:   377;
:   for n = 127, we have mar n = 1, x = 4 and y = 0 so m =
:   131;
:   for n = 129, we have mar n = 3, x = 8 and y = 4 so m =
:   161;
:   for n = 131, we have mar n = 5, x = 12 and y = 8 so m =
:   227;
:   for n = 133, we have mar n = 7, x = 16 and y = 12 so m =
:   325;
:   for n = 135, we have mar n = 9, x = 20 and y = 16 so m =
:   455;
:   for n = 137, we have mar n = 2, x = 6 and y = 2 so m =
:   149;
:   for n = 139, we have mar n = 4, x = 10 and y = 6 so m =
:   199;
:   for n = 141, we have mar n = 6, x = 14 and y = 10 so m =
:   281.

```

So the sequence $m(n)$ is:

5, 35, 101, 199, 329, 23, 73, 155, 269, 23, 53, 119, 217,
347, 41, 91, 173, 287, 41, 71, 137, 235, 365, 59, 109,
191, 305, 59, 89, 155, 253, 383, 77, 127, 209, 323, 77,
107, 137, 271, 401, 95, 145, 227, 341, 95, 125, 191, 289,
419, 113, 163, 245, 359, 149, 143, 209, 307, 437, 131,
181, 263, 377, 131, 161, 227, 325, 455, 149, 199, 281
(...)

Comment:

It is notable that, from the first 71 terms of this sequence, 44 are primes, 24 are semiprimes and 2 are products of two distinct prime factors ($245 = 5 \cdot 7^2$ and $325 = 13 \cdot 5^2$)!

Conjectures:

- (1) The sequence above has an infinity of terms which are distinct primes.
- (2) The sequence above has an infinity of terms which are squares of distinct primes ($289 = 17^2, \dots$).
- (3) The sequence above has an infinity of terms which are cubes of distinct primes ($125 = 5^3, \dots$).
- (4) The sequence above has an infinity of terms which are products of twin primes ($35 = 5 \cdot 7, 323 = 17 \cdot 19, 143 = 11 \cdot 13, \dots$).
- (5) The sequence above has an infinity of terms which are products of a Sophie Germain prime and a safe prime ($253 = 11 \cdot 23, \dots$).
- (6) The sequence above has an infinity of terms which are products of a prime p and a prime $q = k \cdot p - (k - 1)$, such, for instance: $91 = 7 \cdot 13$ ($13 = 2 \cdot 7 - 1$), $217 = 7 \cdot 31$ ($31 = 5 \cdot 7 - 4$), $365 = 5 \cdot 73$ ($73 = 18 \cdot 5 - 17$), $305 = 5 \cdot 61$ ($61 = 15 \cdot 5 - 14$), $145 = 5 \cdot 29$ ($29 = 7 \cdot 5 - 6$).
- (7) The sequence above has an infinity of terms which are products of a prime p and a prime $q = k \cdot p - (k + 1)$, such, for instance: $329 = 7 \cdot 47$ ($47 = 8 \cdot 7 - 9$), $119 = 7 \cdot 17$ ($17 = 3 \cdot 7 - 4$), $287 = 7 \cdot 41$ ($41 = 7 \cdot 7 - 8$), $209 = 11 \cdot 19$ ($19 = 2 \cdot 11 - 3$), $95 = 5 \cdot 19$ ($19 = 5 \cdot 5 - 6$), $161 = 7 \cdot 23$ ($23 = 4 \cdot 7 - 5$).