Numeric Formula for the Masses of Baryons, Tetraquaks and Pentaquarks

(The "Alpha-12" Mass Formula)

The formula presented in this paper predicts the masses of 17 baryons, one lepton and 6 other particles. The formula uses non-consecutive quantum numbers. The maximum accuracy of the formula is 3 decimal places and the minimum is 2. This paper predicts a new particle with a rest mass between $4557.2\ MeV/c^2$ *and* $4658.2\ MeV/c^2$ *. The formula suggests the existence of a more general formula for the masses of all known particles, which is yet, to be discovered.*

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1. Introduction

In a previous article [1] I published the formula for the mass of the proton

$$m_p = \frac{m_e}{A(1-A)} \tag{1.1}$$

In that paper I showed that if we adopt the value of $6.671\,614\,932 \times 10^{-11} m^3 Kg^{-1} S^{-2}$ for Newton's gravitational constant, *G*, the formula yields the exact mass of the proton accurate to 9 decimal places. In this article I shall generalize this formula so that the modified formula , which I shall call *the Alpha-12 formula,* yields the masses of a number of particles (including all baryons, one lepton and other relatively heavy unknown particles). The price we have to pay for this generalization is a reduction in accuracy. In other words the generalized formula that I shall introduce in the next section is not as accurate as formula (1.1). Appendix 1 shows how to convert the rest mass of a particle into rest energy and viceversa.

2. The Alpha-12 Mass Formula for the Masses of Baryons

The modification consists in raising the parenthesis, $(1 - A)$, of equation (1.1) to the power of *n,* where *n* represents non-consecutive or "allowed" quantum numbers. While formula (1.1) is exact, provided we use the theoretical value of the gravitational constant proposed in the above-mentioned article, the Alpha-12 generalized formula is numeric. The Alpha-12 mass formula in terms of m_e and A is

$$m \approx \frac{m_e}{A(1-A)^n} \tag{2.1}$$

where

$$A = \alpha^{12}\left(\frac{M_P}{m_e}\right) \tag{2.2}$$

$$M_P = \sqrt{\frac{hc}{2\pi G}} \tag{2.3}$$

$$\alpha = \frac{e^2}{2\epsilon_0 hc} \tag{2.4}$$

The final "Alpha-12" mass formula in terms of m_e, α and M_P is

$$m \approx \frac{m_e}{\alpha^{12}\left(\frac{M_P}{m_e}\right)\left[1-\alpha^{12}\left(\frac{M_P}{m_e}\right)\right]^n} \tag{2.5}$$

where

m = predicted rest mass of a particle
m_e = electron rest mass
M_P = Planck mass
α = fine-structure constant
e = elementary charge
ϵ_0 = permittivity of vacuum
h = Planck's constant
c = speed of light in vacuum
G = Newton's gravitational constant[1]
n = non consecutive or "allowed" quantum numbers (exponent)
$F_{JeV} = 1.602\,176\,564 \times 10^{-19} \dfrac{J}{eV}$ = conversion factor
$F_{JMeV} = 1.602\,176\,564 \times 10^{-13} \dfrac{J}{MeV}$ = conversion factor

3. Results

The values equation (2.5) yields are shown on table 1. The table shows that this formula is a good approximation for the masses of baryons, the tau particle and other six unknown particles: Y(3940), Y(4140), Y(4260), Z(4430), Zb1(10610) and Zb2(10650). It is worthy to remark that the quantum number *n* is a non-consecutive quantum number.

(see next page)

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(1) *The value of the gravitational constant used in this paper is the value published by NIST 2010:*
$G_{NIST} = 6.673\,84 \times 10^{-11}\, m^3\, Kg^{-1}\, S^{-2}$

PARTICLE NAME	SYMBOL	QUARK CONTENT OR LEPTON	OBSERVED REST MASS (*MeV/c^2*)	OBSERVED REST MASS (*Kg*)	"ALLOWED" QUANTUM NUMBERS (*n*)	PREDICTED REST MASS *m* (*Kg*) (Equation 2.5)
proton	p	uud	938.272	1.6726217771E-27	1	1.6729005220E-27
neutron	n	udd	939.5654	1.674927292E-27	3	1.6747248825E-27
Lambda 0	Λ0	uds	1115.683	1.988885514E-27	318	1.9883694870E-27
Sigma +	Σ+	uus	1189.37	2.120244517E-27	436	2.1204357199E-27
Sigma 0	Σ0	uds	1192.642	2.126077387E-27	441	2.1262214850E-27
Sigma −	Σ−	dds	1197.449	2.134646642E-27	448	2.1343480878E-27
Delta ++	Δ++	uuu	1232	2.196239391E-27	501	2.1968944476E-27
Delta +	Δ+	uud	1232	2.196239391E-27	501	2.1968944476E-27
Delta 0	Δ0	udd	1232	2.196239391E-27	501	2.1968944476E-27
Delta −	Δ−	ddd	1232	2.196239391E-27	501	2.1968944476E-27
Xi 0	Ξ0	uss	1314.86	2.343950752E-27	619	2.3428107754E-27
Xi −	Ξ−	dss	1321.71	2.356161985E-27	630	2.3568973737E-27
Omega −	Ω−	sss	1672	2.980610603E-27	1061	2.9809106232E-27
Tau [2]	τ	lepton	1776.821	3.167471000E-27	1172	3.1667970325E-27
Y1	Y(3940)	?quark	3940	7.023687664E-27	2633	7.0211298528E-27
Y2	Y(4140)	?quark	4140	7.380220033E-27	2725	7.3821250031E-27
Y3	Y(4260)	?quark	4260	7.594139454E-27	2777	7.59431590343E-27
Y4	Y(4380)	Pentaquark?	4380	7.808058876E-27	2827	7.804095291E-27
T1	T(4430)	Tetraquark?	4430	7.897191968E-27	2849	7.8982247074E-27
P1	P(4450)	Pentaquark?	4450	7.932845205E-27	2857	7.932734297E-27
X1 (predicted)	X1(4500)	Pentaquark	4500	8.022E-27	(2878)	(8.024041186E-27)
Xi0b [3]	Ξ_b^0	usb	5791.80	1.032480871E-26	3340	1.0321396374E-26
Xi'-b [4] (state with spin 1/2)	$\Xi_b^{\prime -}$	dsb	5935.02	1.058013370E-26	3386	1.058341074E-26
Xi*-b [5] (state with spin 3/2)	Ξ_b^{*-}	dsb	5955.33	1.061633956E-26	3391	1.0612288360E-26
Omega b − [4]	Ω_b^-	ssb	6165	1.099011027E-26	3456	1.099494644E-26
Zb1	Zb1(10610)	?quark	10610	1.891404216E-26	4451	1.890989841136E-26
Zb2	Zb2(10650)	?quark	10650	1.898534864E-26	4458	1.898211618911E-26

Table 1: *The last column of the table shows the predicted mass for seventeen baryons, one lepton (the tau particle) and six unknown particles. The digits (including the decimal places) shown in either blue or white*

indicate that these digits coincide with the corresponding digits of the observed values. It is worthy to observe that the Ξ_b^0 *baryon is about 6.17 times as massive as the proton. The navy blue row corresponds to a hypothetical particle called: X1(4500). This particle is, possibly, a pentaquark. Note that I have predicted the mass of this particle based on the mass, in Mev/c^2, of observed particles and not from the model presented here. Note also that the literature could use different names for some of the particles shown here.*

3. How to Predict New Particles from the Alpha-12 Mass Formula

The formula presented in this paper allow us to predict new particles. I shall illustrate this point by predicting the mass range for a new particle that I shall call: X . Let us have a look at table 1. Firstly, let us compute the differences between the "allowed" quantum numbers corresponding to the pair of particles Y_2 and Y_1, between the pair Y_3 and Y_2 and between the pair Z_1 and Y_3. These differences are shown on table 2.

Pair number	Pair of particles	"Allowed" quantum numbers for the pair of particles of column 1	"Allowed" quantum number difference
1	Y_2 and Y_1	2725 - 2633	92
2	Y_3 and Y_2	2777 - 2725	52
3	Z_1 and Y_3	2849 - 2777	72

Table 2: *The table shows the "allowed" quantum number difference between three* Y_2 *and* Y_1*, between* Y_3 *and* Y_2*; and between* Z_1 *and* Y_3*. Note that the literature could use different names for the particles shown here.*

Secondly, let us have a look at table 2. This table shows that the minimum difference (52) between quantum numbers corresponds to pair 2. The maximum difference (92) between quantum numbers corresponds to pair 1. The mean of the differences is 72. Thirdly, based on these three differences we can postulate that it is very likely that a new particle exists between the "allowed" quantum numbers 2849 + 52 = 2901 and 2849 + 92 = 2941. Now if we use these two quantum numbers in equation (2.5) we get

$$m_X(min) = 8.125\times10^{-27}\,Kg \quad \text{for } n = 2901$$

$$m_X(max) = 8.304\times10^{-27}\,Kg \quad \text{for } n = 2941$$

the value we get with the mean quantum number of: (2901 + 2941)/2 = 2921 is

$$m_X(mean) = 8.214 \times 10^{-27}\ Kg$$

Prediction

Formula (2,5) seems to predict the existence of a new particle with a rest mass in the following mass range:

$$m_X(min) = 8.124 \times 10^{-27}\ Kg = 4557.2\ MeV/c^2$$

and

$$m_X(max) = 8.304 \times 10^{-27}\ Kg = 4658.2\ MeV/c^2 .$$

or equivalently

$$4557.2\ \frac{MeV}{c^2} \le m_X \le 4658.2\ \frac{MeV}{c^2}$$

This prediction is yet to be confirmed by the experiment.

4. How to Predict New Particles from the Rest Energy Values (without the Alpha-12 Mass Formula)

We can also predict the existence of new particles by observing patterns in the masses of table 1. For example, let us consider the following three rows of table 1:

Y2	Y(4140)	?quark	4140	7.380220033E-27	2725	7.3821250031E-27
Y3	Y(4260)	?quark	4260	7.594139454E-27	2777	7.59431590343E-27
Y4	Y(4380)	Pentaquark	4380	7.808058876E-27	2827	7.804095291E-27

The value for the mass of Y2 is 4140 MeV/c^2 . If we add 120 MeV/c^2 to this value we get the mass of Y3. If we, now, add 120 MeV/c^2 to the mass of Y3 we get the mass of Y4: 4380 MeV/c^2 . Therefore, we could extrapolate and add 120 MeV/c^2 to the mass of Y4. Thus we obtain a mass of 4500 MeV/c^2 . This mass could correspond to a new particle that I have called X1 (see the navy blue row with yellow labels of table 1)

5. Conclusions

The equation introduced in this paper predicts the masses of 17 baryons, one lepton (the tau particle), and eight additional heavy particles: Y(3940), Y(4140), Y(4260), Y(4380), T(4430), P(4450), Zb1(10610) and Zb2(10650) (possibly heavy “baryons”: tetraquaks and pentaquarks). The minimum accuracy of the formula is 2 decimal places and the maximum is 3. Additionally, this paper predicts the existence of two new particles:
The first one should have a mass in the following range:

$$4557.2 \frac{MeV}{c^2} \leq m_X \leq 4658.2 \frac{MeV}{c^2}$$

The second one should have a mass of, approximately

$$4500 \frac{MeV}{c^2}$$

It is very likely that these two particles, if they exits, to be pentaquaks. We could use a new name to refer to particles made of 4, 5 or more quarks. Perhaps: bary-baryons (meaning heavy baryons) or heavy “baryons” could be an appropriate names.

A potential strength of this numeric formula is that if we knew all the quantum numbers “allowed” by Nature, then we could predict the existence of new particles that the Standard Model does not predicts. Using quantum numbers rather than the masses of particles seems to simplify the problem of finding the masses of undiscovered particles. Another point to observe is that between the proton and the neutron there is only one possible quantum number, which is 2. However since no particles have been observed with a mass between the mass of the proton and that of the neutron, the quantum number 2 seems to be forbidden. This indicates that the “granularity” of the formula is correct meaning that we shall not miss out any particle if we increase any quantum number by 1. Having said that I must say that the problem of finding all the “allowed” quantum numbers might not have a solution or might be extremely difficult to solve. We simply don't know. However it would be a very rewarding work to be able to solve it. Finally, the formulation presented in this paper suggests the existence of a more general formula for the masses of all known particles.

Appendix 1
How to Convert the Rest Mass of a particle into Rest Energy and Viceversa

a) How to Convert the Rest Mass of a particle in *Kg* into Rest Energy in *MeV*

The conversion formula is

$$E_0(MeV) = \frac{m_0(Kg) \times c^2}{1.602\ 176\ 564 \times 10^{-13} \dfrac{J}{MeV}} \qquad \text{(A1)}$$

Example

To convert the neutron rest mass, m_n , to energy, in *MeV*, we apply the above formula (A1)

$$E_0(MeV) = \frac{1.674\ 927\ 292 \times 10^{-27}\, Kg \times \left(299\ 792\ 458 \frac{m}{S}\right)^2}{1.602\ 176\ 564 \times 10^{-13} \frac{J}{MeV}} = 939.565\ 379\ 1\ MeV$$

Usually the rest mass of a particle (in this case for the neutron rest mass) is expressed as

$$m_n(MeV/c^2) = 939.565\ 379\ 1\ \frac{MeV}{c^2}$$

b) How to Convert the Rest Energy of a particle in *MeV* into Rest Mass in *Kg*

The conversion formula is

$$m_0(Kg) = \frac{E_0(MeV) \times 1.602\ 176\ 564 \times 10^{-13} \frac{J}{MeV}}{c^2} \qquad \text{(A2)}$$

Example

To convert the rest energy, in *MeV*, of a neutron to its equivalent rest mass, m_n , in *Kg* we apply the above formula (A2)

$$m_0(Kg) = \frac{939.565\ 379\ 1\ MeV \times 1.602\,176\,564 \times 10^{-13} \frac{J}{MeV}}{\left(299\ 792\ 458 \frac{m}{S}\right)^2} = 1.674\ 927\ 351 \times 10^{-27}\, Kg$$

which is the rest mass of the neutron in *Kg*.

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