

Fourteen Smarandache-Coman sequences of primes

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Abstract. In this paper I define the "Smarandache-Coman sequences" as "all the sequences of primes obtained from the Smarandache concatenated sequences using basic arithmetical operations between the terms of such a sequence, like for instance the sum or the difference between two consecutive terms plus or minus a fixed positive integer, the partial sums, any other possible basic operations between terms like $a(n) + a(n+2) - a(n+1)$, or on a term like $a(n) + S(a(n))$, where $S(a(n))$ is the sum of the digits of the term $a(n)$ etc.", and I also present few such sequences.

Definition:

We name "Smarandache-Coman sequences" all the sequences of primes obtained from the Smarandache concatenated sequences using basic arithmetical operations between the terms of such a sequence, like for instance the sum or the difference between two consecutive terms plus or minus a fixed positive integer, the partial sums, any other possible basic operations between terms like $a(n) + a(n+2) - a(n+1)$, or on a term like $a(n) + S(a(n))$, where $S(a(n))$ is the sum of the digits of the term $a(n)$ etc.

Note: The Smarandache concatenated sequences are well known for the very few terms which are primes; on the contrary, many Smarandache-Coman sequences can be constructed that probably have an infinity of terms (primes, by definition).

Examples:

Note: I shall use the notation $a(n)$ for a term of a Smarandache concatenated sequence and $b(n)$ for a term of a Smarandache-Coman sequence.

SEQUENCE I

Starting from the Smarandache consecutive numbers sequence (defined as the sequence obtained through the concatenation of the first n positive integers, see A007908 in OEIS), we define the following Smarandache-Coman sequence: $b(n) = a(n+1) - a(n) - 2$ if the last digit of the term $a(n+1)$ is even and $b(n) = a(n+1) - a(n) + 2$ if the last digit of the term $a(n+1)$ is odd.

We have:

```
:      123 - 12 + 2 = 113, prime;
:      1234 - 123 - 2 = 1109, prime;
:      12345 - 1234 + 2 = 11113, prime;
:      123456 - 12345 - 2 = 111109, prime;
:      12345789 - 12345678 + 2 = 111111113, prime;
:      12345678910 - 123456789 - 2 = 12222222119,
prime;
:      123456789101112 - 1234567891011 - 2 =
122222221210099, prime;
      (...)
```

The SEQUENCE I contains the following terms:
113, 1109, 11113, 111109, 111111113, 12222222119,
122222221210099 (...)

Note: I conjecture that this sequence has an infinity
of terms (primes, by definition).

SEQUENCE II

Starting from the Smarandache concatenated odd sequence
(defined as the sequence obtained through the
concatenation of the first n odd numbers, see A019519 in
OEIS), we define the following Smarandache-Coman
sequence: $b(n) = a(n+1) + a(n) - S(a(n+1)) - S(a(n)) + 2$,
where $S(a(n))$ is the sum of the digits of the term $a(n)$.

We have:

```
:      1 + 13 - 1 - 4 + 2 = 11, prime;
:      13 + 135 - 4 - 9 + 2 = 137, prime;
:      1357 + 13579 - 16 - 25 + 2 = 14897, prime;
:      13579 + 1357911 - 25 - 36 + 2 = 1371431, prime;
:      135791113 + 13579111315 - 49 - 64 + 2 =
13714902317, prime;
      (...)
```

Note the interesting fact that $135 + 1357 - 9 - 16 + 2 = 1469 = 13 \cdot 113$ (a semiprime with the property that $113 - 13 + 1 = 101$, prime) and $1357911 + 135791113 - 36 - 49 + 2 = 137148941 = 431 \cdot 318211$ (a semiprime with the property that $318211 + 431 - 1 = 318641$, prime).

The SEQUENCE II contains the following terms:
11, 137, 14897, 1371431, 13714902317 (...)

Note: I conjecture that this sequence has an infinity
of terms (primes, by definition).

SEQUENCE III

Starting from the Smarandache concatenated even sequence (defined as the sequence obtained through the concatenation of the first n even numbers, see A019520 in OEIS), we define the following Smarandache-Coman sequence: $b(n) = a(n+1) + a(n) - S(a(n+1)) - S(a(n)) + 1$, where $S(a(n))$ is the sum of the consecutive even numbers which form the term $a(n)$; for instance, $S(246810) = 2 + 4 + 6 + 8 + 10 = 30$.

We have:

```
:      2 + 24 - 2 - 6 + 2 = 19, prime;
:      246 + 2468 - 12 - 20 + 1 = 2683, prime;
:      2468 + 246810 - 20 - 30 + 1 = 249229, prime;
:      24681012 + 2468101214 - 42 - 56 + 1 =
:      2492782129, prime;
:      (...)
```

The SEQUENCE III contains the following terms:
19, 2683, 249229, 2492782129 (...)

Note: I conjecture that this sequence has an infinity of terms (primes, by definition).

SEQUENCE IV

Starting from the concatenated odd square sequence (defined as the sequence obtained through the concatenation of the first n odd squares, see A016754 in OEIS), we define first the following Smarandache type sequence: $a(n)$ is obtained through the concatenation of two squares of consecutive odd integers (19, 925, 2549, 4981, 81121, 121169, ...) and then the following Smarandache-Coman sequence: $b(n) = a(n) - k$, where k is equal to the even number between the two consecutive odd integers which squares form through concatenation the term $a(n)$. Example: $b(1) = 19 - 2 = 17$.

We have:

```
:      19 - 2 = 17, prime;
:      2549 - 6 = 2543, prime;
:      4981 - 8 = 4973, prime;
:      121169 - 12 = 121157, prime;
:      289361 - 18 = 289343, prime;
:      361441 - 20 = 361421, prime;
:      841961 - 30 = 841931, prime;
:      (...)
```

The SEQUENCE IV contains the following terms:
17, 2543, 4973, 121157, 289343, 361421, 841931 (...)

Note: I conjecture that this sequence has an infinity of terms (primes, by definition).

SEQUENCE V

Starting from the concatenated even square sequence (defined as the sequence obtained through the concatenation of the first n even squares, see A016742 in OEIS), we define first the following Smarandache type sequence: $a(n)$ is obtained through the concatenation of two squares of consecutive even integers (416, 1636, 3664, 64100, 100144, 144196, ...) and then the following Smarandache-Coman sequence: $b(n) = a(n) + k$, where k is equal to the odd number between the two consecutive even integers which squares form through concatenation the term $a(n)$. Example: $b(1) = 416 + 3 = 419$.

We have:

: 416 + 3 = 419, prime;
: 3664 + 7 = 3671, prime;
: 64100 + 9 = 64109, prime;
: 196256 + 15 = 196271, prime;
: 324400 + 19 = 324419, prime;
(...)

The SEQUENCE V contains the following terms:
419, 3671, 64109, 196271, 324419 (...)

Note: I conjecture that this sequence has an infinity of terms (primes, by definition).

SEQUENCE VI

Starting from the "n concatenated n times" sequence (defined as the sequence obtained concatenating n times the number n , see A000461 in OEIS), we define the following Smarandache-Coman sequence: $b(n) = a(n) + a(m) + 333$, where $a(n)$ and $a(m)$ are two even (not necessarily distinct) terms of the "n concatenated n times" sequence.

We have:

: 22 + 4444 + 333 = 4799, prime;
: 4444 + 4444 + 333 = 9221, prime;
: 22 + 666666 + 333 = 667021, prime;
: 4444 + 666666 + 333 = 671443, prime;
: 666666 + 88888888 + 333 = 89555887, prime (...)

The SEQUENCE VI contains the following terms:
4799, 9221, 667021, 671443, 89555887 (...)

Note: The sequence $b(n) = a(n) + a(m) - 333$, in the same conditions, also can be considered (e.g. $22 + 4444 - 333 = 4133$, prime, or $4444 + 666666 - 333 = 670777$, prime). Also $a(n) + a(m) - a(k)$, where $a(k)$ is an odd term of the "n concatenated n times" sequence.

Note: I conjecture that this sequence has an infinity of terms (primes, by definition).

SEQUENCE VII

Starting from the back concatenated odd sequence (A038395 in OEIS), we define the following Smarandache-Coman sequence: $b(n) = 2*a(n) - 1$.

We have:

: $2*31 - 1 = 61$, prime;
: $2*531 - 1 = 1061$, prime;
: $2*7531 - 1 = 15061$, prime;
: $2*131197531 - 1 = 262395061$, prime (...)

The SEQUENCE VII contains the following terms:
61, 1061, 15061, 262395061 (...)

Note: I conjecture that this sequence has an infinity of terms (primes, by definition).

SEQUENCE VIII

Starting from the back concatenated even sequence (A038396 in OEIS), we define the following Smarandache-Coman sequence: $b(n) = a(n) - 1$.

We have:

: $42 - 1 = 41$, prime;
: $642 - 1 = 641$, prime;
: $8642 - 1 = 8641$, prime;
: $18161412108642 - 1 = 18161412108641$, prime (...)

The SEQUENCE VIII contains the following terms:
41, 641, 8641, 18161412108641 (...)

Note: I conjecture that this sequence has an infinity of terms (primes, by definition).

SEQUENCE IX

Starting from the back concatenated odd square sequence (defined as the sequence obtained through the back concatenation of the first n odd squares), we define first the following Smarandache type sequence: $a(n)$ is obtained through the back concatenation of two squares of consecutive odd integers (91, 259, 4925, 8149, 12181, 169121, ...) and then the following Smarandache-Coman sequence: $b(n) = a(n) - 2$.

We have:

```
:      91 - 2 = 89, prime;
:      259 - 2 = 257, prime;
:      8149 - 2 = 8147, prime;
:      225169 - 2 = 225167, prime;
:      441361 - 2 = 441359, prime;
:      841729 - 2 = 841727, prime (...)
```

The SEQUENCE IX contains the following terms:
89, 257, 8147, 225167, 441359, 841727 (...)

Note: I conjecture that this sequence has an infinity of terms (primes, by definition).

SEQUENCE X

Starting from the back concatenated square sequence, we define first the following sequence: $a(n)$ is obtained through the concatenation to the left of the square of the number 8 (i.e. 64) with a square of an odd number (164, 964, 2564, 4964, 8164, 12164, 16964 ...) and then the following Smarandache-Coman sequence: $b(n) = a(n)/4$.

We have:

```
:      164/4 = 41, prime;
:      964/4 = 241, prime;
:      2564/4 = 641, prime;
:      12164/4 = 3041, prime;
:      16964/4 = 4241, prime;
:      22564/4 = 5641, prime;
:      36164/4 = 9041, prime;
:      52964/4 = 13241, prime (...)
```

The SEQUENCE X contains the following terms:
41, 241, 641, 3041, 4241, 5641, 9041, 13241 (...)

Note: I conjecture that this sequence has an infinity of terms (primes, by definition).

SEQUENCE XI

Starting from the Smarandache $n2*n$ sequence (the n -th term of the sequence is obtained concatenating the numbers n and $2*n$, see A019550 in OEIS), we define first the following sequence: $a(n)$ is obtained through the concatenation of two consecutive terms of the sequence mentioned (1224, 2436, 3648, 48510, 510612 ...) and then the following Smarandache-Coman sequence: $b(n) = a(n) - 1$.

We have:

```
:      1224 - 1 = 1223, prime;
:      510612 - 1 = 510611, prime;
:      612714 - 1 = 612713, prime;
:      9181020 - 1 = 9181019, prime;
:      14281530 - 1 = 14281529, prime (...)
```

The SEQUENCE XI contains the following terms:
1223, 510611, 612713, 9181019, 14281529 (...)

Note: I conjecture that this sequence has an infinity of terms (primes, by definition).

SEQUENCE XII

Starting again from the Smarandache $n2*n$ sequence, we define first the following sequence: $a(n)$ is obtained through the concatenation of three consecutive terms of the sequence mentioned (122436, 243648, 3648510, 48510612, 510612714 ...) and then the following Smarandache-Coman sequence: $b(n) = a(n)/6 + 1$.

We have:

```
:      122436/6 + 1 = 20407, prime;
:      243648/6 + 1 = 40609, prime;
:      612714816/6 + 1 = 102119137, prime (...)
```

The SEQUENCE XII contains the following terms:
20407, 40609, 102119137 (...)

Note: I conjecture that this sequence has an infinity of terms (primes, by definition).

Comment: inspired by the sequence above I also conjecture that there exist an infinity of primes formed by concatenation in the following way: $n0(n+2)0(n+5)$, where n is an even number; the sequence of these numbers is: 20407, 40609, 608011, 12014017, 16018021, 24026029, 26028031, 28030033 (...)

SEQUENCE XIII

Starting from the Smarandache nn^2 sequence (the n -th term of the sequence is obtained concatenating the numbers n and n^2 , see A053061 in OEIS), we define the following Smarandache-Coman sequence: $b(n) = a(n) + n + 1$.

We have:

```
:      11 + 1 + 1 = 13, prime;
:      39 + 3 + 1 = 43, prime;
:      416 + 4 + 1 = 421, prime;
:      636 + 6 + 1 = 643, prime;
:      749 + 7 + 1 = 757, prime;
:      981 + 9 + 1 = 991, prime;
:      10100 + 10 + 1 = 10111, prime;
:      12144 + 12 + 1 = 12157, prime;
:      15225 + 15 + 1 = 15241, prime;
:      13169 + 13 + 1 = 13183, prime (...)
```

The SEQUENCE XIII contains the following terms:

13, 43, 421, 643, 757, 991, 10111, 12157, 15241, 13183
(...)

Note: I conjecture that this sequence has an infinity of terms (primes, by definition).

SEQUENCE XIV

Starting again from the Smarandache nn^2 sequence, we define the following Smarandache-Coman sequence: $b(n)$ is obtained concatenating to the right the terms $a(n)$ with the number 11.

We have:

```
:      2411 is prime;
:      3911 is prime;
:      41611 is prime;
:      52511 is prime;
:      63611 is prime;
:      1419611 is prime;
:      1522511 is prime;
:      1728911 is prime (...)
```

The SEQUENCE XIV contains the following terms:

2411, 3911, 41611, 52511, 63611, 1419611, 1522511, 1728911
(...)

Note: I conjecture that this sequence has an infinity of terms (primes, by definition).