Khmelnik S.I.

Mathematical Model of Dust Whirl

Abstract

The question of the source of energy in a dust whirl is considered. Atmospheric conditions cannot be the sole source of energy, as such dust whirls exist on Mars, where the atmosphere is absent. Here we show that the source of energy for the dust whirl is the energy of the gravitational field. We present a mathematical model of the sand vortex, which uses a system of Maxwell-like gravitational equations. The model explains some of the properties of the dust whirl – preservation of cylindrical vertical shape of the dust whirl, motion of the dust whirl as a whole.

Contents

1. Introduction
2. Mathematical Model
3. The Energy Flows
4. Vertical Stability
5. The Motion of the Dust Whirl
Appendix
References

1. Introduction

There exists a widely known dust dust whirl, which is an almost vertical cloud of dust – see Fig. 1.

Such a dust whirl has a vertical axis of rotation, height of a few tens of meters, diameter - a few meters, the time of existence - a few tens of seconds [1]. There are similar phenomena - water, air, ash dust whirls. The cause of their existence is assumed to be various atmospheric phenomena (wind, heating of the atmosphere). However, the very existence of the dust whirl – its shape retention and movement, - are difficult to explain by the same reasons. Furthermore, such dust whirls are also moving on Mars, where there is no atmosphere - see. Fig. 2 [1]. Therefore, in the explanation of the dust whirls the main question is about the source of energy.
There are much more powerful phenomena related to dust whirls - Sandy tsunami - see Fig. 2a and Fig. 2c. The existing view that the causes of the movement of this colossus are a breeze and nonlinear medium seems unconvincing. It seems that this "device" has its own motor within, and the resistance of the medium is just a catalyst, a force that pushes the gas pedal.

Below we present a mathematical model of the dust whirl, which uses a system of Maxwell-like gravitational equations. It is shown that the energy source for the sand vortex is the energy of the gravitational field - see Appendix. In any case, it is hard to find any other source of energy on the planet Mars.
The model is based on the following assumptions. Sandy dust whirl is composed of material particles – sand grains. The movement of these particles is likened to mass currents. Mass currents in the gravitational field are described by Maxwell-like gravitational equations [2] (hereinafter - MLG-equations). The interaction between the moving masses is described by the Lorentz gravity-magnetic (the GL-force) similar to the Lorentz forces in electrodynamics acting between moving electrical charges.

Currents arising in the dust whirl are circulating (as shown) in the cross section of the vortex and along the vertical (up and down). The kinetic energy of such circulation is spent on the losses from collisions of sand grains. It comes from a gravitating body. Potential energy of the dust whirl is not changed, and therefore is not consumed. I.e. in this case there is no conversion of potential energy into kinetic energy and vice versa. However, gravitating body expends its energy on creating and maintaining a mass current - see Appendix.

Supporting dust whirl upright is explained as follows. From the analogy between the Maxwell equations and MLG it follows that there may be a flow $S$ of gravitational energy. Such flow can exist and not change over time. Together with the flow there is a gravitational momentum. If the body is in the flow of gravitational energy (and this flow does not change over time), then on the body acts the force $F = S \times c$ (where $c$ is the speed of light), directed opposite to the flow direction. It follows from the law of conservation of momentum. We emphasize once again that it is - a complete analogy between gravitational and electromagnetic field. For the electromagnetic field, these relationships are discussed in [3, 4].

In the body of the dust whirl together with constant mass currents exists (as shown below) a flow of gravitational energy, constant over time. It is directed downward. In accordance with the above, an upward force acts on the body of the dust whirl, thus holding it in an upright position.

2. Mathematical Model

MLG-equations for gravity-magnetic intensity $H$ and density of mass currents $J$ in stationary gravity-magnetic field are as follows:

$$\text{div}(H) = 0,$$  \hspace{1cm} (1)

$$\text{rot}(H) = J,$$  \hspace{1cm} (2)
In the simulation of dust whirl we shall use cylindrical coordinates $r$, $\varphi$, $z$. Then the MLG equations will be:

$$\frac{H_r}{r} + \frac{\partial H_r}{\partial r} + \frac{1}{r} \cdot \frac{\partial H_\varphi}{\partial \varphi} + \frac{\partial H_z}{\partial z} = 0,$$

(3)

$$\frac{1}{r} \cdot \frac{\partial H_z}{\partial \varphi} - \frac{\partial H_\varphi}{\partial z} = J_r,$$

(4)

$$\frac{\partial H_r}{\partial r} - \frac{\partial H_z}{\partial z} = J_\varphi,$$

(5)

$$\frac{H_\varphi}{r} + \frac{\partial H_\varphi}{\partial r} - \frac{1}{r} \cdot \frac{\partial H_r}{\partial \varphi} = J_z,$$

(6)

The model is based on the following facts:

1. The intensity of the gravitational field is directed along the axis of dust whirl,
2. It creates a vertical flow of sand grains - a mass current $J_z$.
3. Vertical mass current $J_z$ generates annular gravity-magnetic field $H_\varphi$ and radial gravity-magnetic field $H_r$ - see (6).
4. Gravity-magnetic field $H_\varphi$ deflects by GL-forces sand grains of vertical flow in the radial direction, creating a radial flow of sand grains - radial mass current $J_r$.
5. Gravity-magnetic field $H_\varphi$ deflects by GL-forces sand grains of radial flow perpendicular to the radius, creating a vertical mass current $J_z$.
6. Gravity-magnetic field $H_r$ deflects by GL-forces sand grains of vertical flow perpendicular to the radius, creating an annular mass current $J_\varphi$.
7. Gravity-magnetic field $H_r$ deflects the GL-forces sand grains of annular flow is perpendicular to the radius, creating a vertical mass current $J_z$.
8. The mass current $J_r$ generates a vertical gravity-magnetic field $H_z$ and annular gravity-magnetic field $H_\varphi$ - see (4).
9. The mass current $J_\varphi$ generates a vertical gravity-magnetic field $H_z$ and radial gravity-magnetic field $H_r$ - see (5).
10. The mass current $J_z$ generates an annular gravity-magnetic field $H_\varphi$ and radial gravity-magnetic field $H_r$ - see (6).
Thus, the main mass current $J_o$ creates additional mass currents $J_r$, $J_\phi$, $J_z$ and gravity-magnetic fields $H_r$, $H_\phi$, $H_z$. They must satisfy Maxwell equations (3-6). The currents must also satisfy the continuity condition

$$\operatorname{div}(J) = 0,$$

or, in cylindrical coordinates,

$$\frac{J_r}{r} + \frac{\partial J_r}{\partial r} + \frac{1}{r} \frac{\partial J_\phi}{\partial \phi} = 0. \quad (9)$$

Mass currents are

$$J = n \cdot m \cdot v,$$

and their kinetic energy -

$$W = n \cdot m \cdot v^2 / 2, \quad (11)$$

where $n$ - the number of sand grains in the flow, $m$ - the mass of one sand grain, $v$ - the speed of sand grains flow. Thus, equal mass currents may have different kinetic energy.

The solution of system (3-6, 9) has been found in [5] and has the following form:

$$H_r = \frac{\alpha}{2} h_\phi r \sin(\alpha \phi), \quad (14)$$

$$H_\phi = h_\phi r \cos(\alpha \phi), \quad (15)$$

$$H_z = -\frac{1}{2} j_\phi r^2 \sin(\alpha \phi), \quad (16)$$

$$J_r = -\frac{\alpha}{2} j_\phi r \cos(\alpha \phi), \quad (17)$$

$$J_\phi = j_\phi r \sin(\alpha \phi), \quad (18)$$

$$J_z = h_\phi \left( (1 - \alpha^2 / 2) \cos(\alpha \phi) - \alpha \sin(\alpha \phi) \right). \quad (19)$$

where $j_\phi$, $h_\phi$ - some constants, $\alpha$ - an integer constant.

Mass currents circulate along the cross sections of the dust whirl body and along the vertical. The structure of currents $J_r$ and $J_\phi$ is described in detail in [5]. Here we are looking at the trajectory of the mass on the assumption that the current created by this mass is proportional to the speed of this mass. Assume that the mass is kept at the same distance from the center. Then its trajectory will be described by the vector-function of the form

$$\overrightarrow{J} = \overrightarrow{J_r}(r, \phi) + \overrightarrow{J_\phi}(r, \phi) + \overrightarrow{J_z}(r, \phi),$$
where the vector summands are calculated by the formulas (17-19) for \( r = \text{const}, \, \varphi = \text{var} \). Fig. 3. Shows three trajectories for \( r = 1, \, r = 13, \, r = 29 \) and \( 0 \leq \varphi = \pi, \, \alpha = 10, \, j_{\varphi} = 1, \, h_{\varphi} = 1 \). Undoubtedly, the same sand grain cannot be located constantly on the same radius, which means that the sand grains substitute one another on every radius. But in average just these trajectories can be considered the trajectories of the jets in the flow of sand grains in the dust whirl.

Fig. 3.

Fig. 4.
Fig. 4 shows the value of $J_z$ on the section plane for $\alpha = 10$, $h_\varphi = 1$, $R = 50$, where $R$ is the dust whirl radius. Here it is important to note that the vertical currents circulate so that the sum of the currents on each section is equal to zero - see. (19). Thus, the dust particles move along a closed path and gravity does not perform work on this trajectory. Nevertheless, some work is done to overcome the frictional forces between dust particles when they are moved by GL-forces. This work is performed by the energy of the gravitational field - see Appendix.

We shall assume that the work of friction force between the sand grains

$$P = \rho |J|^2,$$

(20)

where $\rho$ - the resistivity of the mass current, independent of its magnitude and direction (similar to electrical resistance). Then, the entire work can be defined in the same manner as in [5]. It is equal to

$$P = \pi R^2 L \rho \left( j_\varphi^2 R^2 \left( 1/4 + \alpha^2/16 \right) + h_\varphi^2 \left( 1 + \alpha^4/4 \right) \right),$$

(21)

where $R$, $L$ - radius and height of the dust whirl accordingly. These formulas are similar to the formulas for a length of wire with a constant current. Assuming that for mass currents (as well as for electric currents), the principle of minimum thermal losses is observed, it is possible to find the ratio of [5]

$$j_\varphi = h_\varphi \eta / R.$$

(22)

where

$$\eta = \sqrt{\left( 4 + \alpha^4 \right) / \left( 1 + \alpha^2 / 4 \right)}.$$

(23)

Then

$$P = \pi R^2 L \rho h_\varphi^2 \left( 1/4 + \alpha^4/16 \right).$$

(24)

That is the power that must come from the gravitational field for the existence of dust whirl.

### 3. The Energy Flows

By analogy with electrodynamics let us write the connection between the mass current and the gravity-electrical intensity in the form of

$$E = \rho J.$$

(25)

Also by analogy with electrodynamics let us determine the density of gravitation energy flows in the form

$$S = E \times H.$$

(26)
Then we can find
\[ S = \rho (J \times H). \]  \hspace{1cm} (27)

The vector product \((J \times H)\) in cylindrical coordinates looks as follows:

\[
\begin{bmatrix}
S_r \\
S_{\varphi} \\
S_z
\end{bmatrix} = \frac{J \times H}{\rho} =
\begin{bmatrix}
J_{\varphi} H_z - J_z H_{\varphi} \\
J_z H_r - J_r H_z \\
J_r H_{\varphi} - J_{\varphi} H_r
\end{bmatrix}
\]  \hspace{1cm} (28)

Energy flows and form stability were used in similar mathematical models [6, 7]. By analogy, one could argue that there is no power flow out of the body of the dust whirl. Inside the body it is directed
- along the radius from periphery to center \(- S_r \);  
- circumferentially \(- S_{\varphi} \);  
- vertical down \(- S_z \).

These energy flows provide
- preservation of the dust whirl form, for the change of its form requires external energy inflow [7],  
- vertical stability,  
- the dust whirl motion.

### 4. Vertical Stability

The body of the dust whirl is permeated by flows of gravitational energy that are created by mass currents. A formulaic relationship between the currents and energy flows is discussed in [5] for direct current. The same dependencies can be used in this case. In particular, in the body of a vortex there is a flow of energy directed vertically, with a density
\[ S_z = - j_{\varphi} h_{\varphi} r^2 \frac{\alpha}{2}. \]  \hspace{1cm} (30)

In the Introduction it was shown that a flow with a given density permeating a body creates a pressure force acting on the body with a density (pressure)
\[ F_z = \frac{S_z}{c}, \]  \hspace{1cm} (31)

In a direction opposite to the flow. Let us find the full force of the pressure exerted in each section of the body of the dust whirl of radius \(R\),
\[ F_{z_0} = -\frac{1}{c} \int_{0}^{R} S_r 2\pi r \, dr = \frac{1}{c} j_\varphi h_\varphi \pi \alpha \int_{0}^{R} r^3 \, dr = \frac{j_\varphi h_\varphi \pi \alpha R^4}{4c}. \]  

(32)

As the flow of energy (30) is directed downwards, the force of opposite (32) is directed upwards and supports the dust whirl in an upright position. The gravity counteracts to the above force and balances it.

### 5. The Motion of the Dust whirl

The trajectory of the dust whirl is poorly predictable. We can say that the dust whirl makes chaotic movement. In order to show that the motion of the dust whirl is accomplished by the internal energy (and not by the force of the wind) let us again turn to the consideration of the internal flow of electromagnetic energy. In [5] it is shown that in the body of the dust whirl there is a flow of energy directed radially with density

\[ S_r = \frac{1}{2\sqrt{2}} \left( h_\varphi^2 \left( 2 - \alpha^2 \right) \cdot r - j_\varphi^2 r^3 \right). \]  

(33)

As for the vertical energy flow, a force with the density

\[ F_r = \frac{S_r}{c}. \]  

(34)

also corresponds to this flow.

Let us find the total force acting in the dust whirl's body along the radius:

\[ F_{z_0} = \frac{1}{c} \int_{0}^{R} S_r r \, dr. \]  

(35)

For a symmetrical distribution of the radial flow total force (35) is zero. If the axial symmetry of the vortex is broken, then there appears an uncompensated force. Let \( \xi < 1 \) - be a coefficient characterizing the symmetry breaking. Then uncompensated force can be found from the formula

\[ F_{z_0} = \frac{1}{c} \left( \int_{0}^{R/2} S_r r \, dr - \xi \int_{R/2}^{R} S_r r \, dr \right). \]  

(36)

or

\[ F_{z_0} = \frac{1 - \xi}{c} \int_{R/2}^{R} S_r r \, dr. \]  

(37)

or, in view of (33),
\[ F_{zo} = \frac{(1-\xi)}{2\sqrt{2c}} \int_{R/2}^{R} \left( h_{\varphi}^{2}(2-\alpha^{2}) \cdot r^{2} - f_{\varphi}^{2} r^{4} \right) dr = \]
\[ = \frac{(1-\xi)}{2\sqrt{2c}} \left( h_{\varphi}^{2}(2-\alpha^{2}) \cdot \frac{R^{3}}{3} - f_{\varphi}^{2} \frac{R^{5}}{5} \right) \quad (38) \]

This force results of the motion of the dust whirl as a whole. It can be shown that the reason for this distortion is air resistance and sand grains inertia (but that is another topic).

**Appendix**

Conservative forces (by definition) do not perform work on a closed trajectory. The force of gravity is a conservative force (which is proved mathematically). Hence the conclusion is reached that

1) there does not exist a motor using only conservative forces (specifically, the force of gravity) to perform work.

Next an unproven conclusion is made that

2) there does not exist a motor using the energy of conservative forces source (including the gravity forces), for performing the work.

Coulomb forces are also conservative. From this by analogy one can make the conclusion 1). However, the conclusion 2) is easily refuted: there exists, for example, a DC motor with self-excitation. Its energy source is a constant voltage source, i.e., a source of Coulomb forces. Therefore, in the general case, the assertion 2) is incorrect, and the true statement is as follows:

3) **There can exist** a motor using the energy of conservative forces source for performing work.

Nevertheless, the existence of the motor that uses energy of the electrical conservative forces source (SECF) does not mean that there is a motor that uses the energy source of the gravitational conservative forces (SGCF).

Electrical forces create the charges motion along a closed trajectory – electric current which forms a magnetic field. Due to this the energy of SECF turns into magnetic energy. It occurs even if the energy is not expended for the motion of the charges on the closed path. Thus, the energy of SECF exceeds the energy of the mechanical motion of the charges. This is the reason for the existence of a motor using the energy SECF.

Gravity forces also can create a mass motion on a closed trajectory – mass current. Let us assume that mass current also forms a gravity magnetic
field (it is shown in [2]). Then by analogy with the previous we may assume that

4) there can exist a motor using the energy of the source of gravity conservative forces for performing work.

This does not contradict the law of conservation of energy: it is the energy of SGCF that is converted into work, and SGCF power source loses some of its energy (it cannot be said that the energy of SGCF may be used only for the movement of the masses).

References