Bell’s Theorem
on Quantum Correlations is based on the Hopf Fibration \( \mathbb{RP}1 \rightarrow S1 \rightarrow S0 = \{-1,+1\} \).

Joy Christian has shown that it is more realistic
to base Quantum Correlations on the Hopf Fibrations
\( S1 \rightarrow S3 \rightarrow S2 = \mathbb{CP}1 \) and \( S3 \rightarrow S7 \rightarrow S4 = \mathbb{QP}1 \) and \( S7 \rightarrow S15 \rightarrow S8 = \mathbb{OP}1 \)
where \( R, C, Q, \) and \( O \) are Real, Complex, Quaternion, and Octonion Division Algebras.

In his book “Disproof of Bell’s Theorem” (BrownWalker Press, 2nd ed, 2014)
Joy Christian said:
“... Every quantum mechanical correlation can be understood as
a classical, local-realistic correlation among a set of points of a parallelized 7-sphere
... physical space ... respects the symmetries and topologies of a parallelized 7-sphere
... because 7-sphere ... is homeomorphic to the ...[Octonion]... division algebra ...
it is the property of division that ... is responsible for ... local causality in the world ...
To understand this reasoning better, recall that, just as a parallelized 3-sphere is a 2-
sphere worth of 1-spheres but with a twist in the manifold \( S3 (= S2 \times S1) \),
a parallelized 7-sphere is a 4-sphere worth of 3-spheres
but with a twist in the manifold \( S7 (= S4 \times S3) \)
... just as \( S3 \) is
a nontrivial fiber bundle over \( S2 \) with Clifford parallels \( S1 \) as its linked fibers,
\( S7 \) is also
a nontrivial fiber bundle ... over \( S4 \) ... with ... spheres \( S3 \) as its linked fibers.
...
it is the twist in the bundle \( S3 \) that forces one
to forgo the commutativity of complex numbers (corresponding to the circles \( S1 \))
in favor of the non-commutativity of quaternions.
In other words, a 3-sphere is not parallelizable by the commuting complex numbers but
only by the non-commuting quaternions. And it is this noncommutativity that gives rise
to the non-vanishing of the torsion in our physical space.

In a similar vein, the twist in the bundle \( S7 (= S4 \times S3) \) forces one to forgo the
associativity of quaternions (corresponding to the fibers ) in favor of the non-
associativity of octonions.
In other words, a 7-sphere is not parallelizable by the associative quaternions but
only by the non-associative octonions.
... it can be parallelized ... because its tangent bundle happens to be trivial:
Once parallelized by a set of unit octonions,
both the 7-sphere and each of its 3-spherical fibers remain closed under multiplication.
This, in turn, means that
the factorizability or locality condition of Bell is ... satisfied within a parallelized 7-sphere.
The lack of associativity of octonions, however, entails that, unlike the unit 3-sphere
[which is homeomorphic to the ... group SU(2)], a 7-sphere is not a group manifold ...".
The torsion within the 7-sphere ... varies from one point to another of the manifold.
It is this variability of the parallelizing torsion within that is ultimately responsible for the
diversity and non-linearity of the quantum correlations we observe in nature ..."

The 7-sphere S7 is the unit sphere in 8-dim space.
S7 is not a Lie algebra, but is a Malcev algebra
and is naturally embedded in the D4 Lie algebra Spin(8) which
is topologically composed of ( but \(\neq\) the simple product \(S7 \times S7 \times G2\) )
2 copies of S7 and 14-dim Lie Algebra G2 of the Octonion Automorphism Group.
The 8-dim quantum position - momentum unitary group is SU(8).

David Brown, in May 2012 comments on scottaaronson.com blog, said:
"... Where did Bell go wrong? Bell used quantum SU(1) states
whereas Christian correctly uses quantum SU(8) states ...[from]...
Christian’s parallelized 7-sphere model. ...
Every quantum mechanical Christian SU(8) correlation can be understood
as a realistic, non-local Christian SU(8) correlations among a set of points
of a parallelized 7-sphere ... More importantly, if Christian’s theory of local realism is true
then SU(8) should be the gauge group for physical reality ...".

Unimodular SL(8,R) is the non-compact Lie algebra corresponding to SU(8).
SL(8,R) effectively describes the 8-dim SpaceTime of E8 Physics
as a generalized checkerboard of SpaceTime HyperVolume Elements.
Anderson and Finkelstein in Am . J. Phys. 39 (1971) 901-904 said:
"... Unimodular relativity ... expresses the existence of a fundamental element of
spacetime hypervolume at every point. ...".
From the Real Clifford Algebra Cl(16) and 8-Periodicity
64-dim R+SL(8,R) appears from factoring Cl(16) = tensor product Cl(8)xCl(8)
as the tensor product of the 8-dim vector spaces 8v of each of the Cl(8) factors
so that 64-dim R+SL(8,R) = 8v x 8v
If you regard the two Cl(8) as Fourier duals then
one 8v describes 8-dim Spacetime Position and the other 8v describes its Momentum.

“Contractions of Exceptional Lie Algebras and SemiDirect Products”, showed that
SL(8,R) appears in the E8 Maximal Contraction = semi-direct product H92 x SL(8,R)
where
H92 is (8+28+56 +1+ 56+28+8)-dim Heisenberg Creation/Annihilation Algebra
so that $H^92 \times SL(8, R)$ has 7-graded structure:

grade -3 = Creation of 1 fermion (tree-level massless neutrino) with 8 SpaceTime Components for a total of 8 fermion component creators (related to SpaceTime by Triality)

grade -2 = Creation of $8+3+1 = 12$ Bosons for Standard Model and 16 Conformal $U(2,2)$ Bosons for MacDowell-Mansouri Gravity for a total of 28 Boson creators

grade -1 = Creation of 7 massive Dirac fermion each with 8 SpaceTime Components for a total of 56 fermion component creators

grade 0 = $1 + SL(8) = 1+63 = 64$-dim representing 8-dim SpaceTime of HyperVolume Elements

grade 1 = Annihilation of 7 massive Dirac fermions each with 8 SpaceTime Components for a total of 56 fermion component annihilators

grade 2 = Annihilation of $8+3+1 = 12$ Bosons for Standard Model and 16 Conformal $U(2,2)$ Bosons for MacDowell-Mansouri Gravity for a total of 28 Boson annihilators

grade 3 = Annihilation of 1 fermion (tree-level massless neutrino) with 8 SpaceTime Components for a total of 8 fermion component annihilators (related to SpaceTime by Triality)

The E8 expansion of $H^92 \times SL(8, R)$ has physical interpretation leading to a Local Classical Lagrangian with Base Manifold Spacetime, Gravity + Standard Model Gauge Boson terms, and Fermion terms for 8-dim spacetime and First-Generation Fermions (with 4+4 dim Kaluza-Klein and Second and Third Fermion Generations emerging with Octonionic Symmetry being broken to Quaternionic):

248-dim E8 = 120-dim D8 + 128-dim half-spinors of D8

In Symmetric Space terms:

$E8 / D8 = (64+64)$-dim $(OxO)P^2$ Octo-Octonionic Projective Plane

$64 = 8$ components of 8 fermion particles

$64 = 8$ components of 8 fermion antiparticles

$D8 / D4xD4 = 64$-dim = 8 position coordinates x 8 momentum coordinates

one $D4 = 28 = 12$ Standard Model Ghosts + 16 Conformal Gravity Gauge Bosons (4 of the 16 are not in the 240 E8 root vectors, but are in its 8-dim Cartan subalgebra)

other $D4 = 28 = 16$ ConformalGravity Ghosts + 12 Standard Model Gauge Bosons (4 of the 12 are not in the 240 E8 root vectors, but are in its 8-dim Cartan subalgebra)
How does Bell-Christian-Brown SU(8) Quantum Theory fit with the Bohm Quantum Potential of E8 Physics (http://vixra.org/pdf/1405.0030vG.pdf)?

Comparison of Bohm's Quantum Potential hidden variable "lambdas" with Bell's "lambdas" and Joy Christian's (arxiv 0904.4259)"lambdas":

Peter Holland, in his book "The Quantum Theory of Motion, an Account of the de Broglie - Bohm Causal Interpretation of Quantum Mechanics" (Cambridge 1993) said:

"... 11.5.1 Bell's Inequality ... In discussing the EPR spin experiment Bell supposed that the results of the two spin measurements are determined completely by a set of hidden variables lambda and made two assumptions which he claimed should be satisfied by a local hidden-variables theory:

(i) The result A of measuring sigma1 . a on particle 1 is determined solely by a and lambda, and the result B of measuring sigma2 . b on particle 2 is determined solely by b and lambda, where a and b are unit vectors with a . b = cos(delta).

Thus A = A( a , lambda ) = +/- 1 and B = B( b , lambda ) = +/- 1
Possibilities such as A = A( a , b , lambda ) and B = B( a , b , lambda ) are excluded.

(ii) The normalized probability distribution of the hidden variables depends only on lambda : rho = rho( lambda ).
Possibilities such as rho = rho( lambda , a , b ) are excluded.

... We now consider to what extent assumptions (i) and (ii) are valid in the causal [Bohm Potential] interpretation ... The hidden variables are then the particle positions x1 , x2 (the internal orientation spin vectors s1 , s2 along the trajectories are determined by the positions and the wavefunction ...) ... the eventual results ... for each of sz1 and sz2 is determined by the initial positions of both particles and by delta, i.e., A = A( x1 , x2 , a . b ) , B = B( x1 , x2 , a . b ) Thus assumption (i) is not valid ...
Neither is assumption (ii) satisfied. ... In reproducing ... the quantum mechanical correlation function ...

Ppsi( a , b ) = ... = - cos( delta ) ... the causal [Bohm Potential] interpretation disobeys both of Bell's basic assumptions. ...".

So, Bell's "lambdas" obey (i) and (ii) and so obey Bell's inequality and Bohm's "lambdas" violate (i) and (ii) and so violate Bell's Inequality but obey the quantum experimentally observed correlation function.
Joy Christian (see arxiv 0904.4259) explicitly violates (i) by replacing
  \[ A = A( a, \lambda ) = \pm 1 \] \text{and} \[ B = B( b, \lambda ) = \pm 1 \]
with
  \[ A = A( a, \lambda ) \text{ in } S^2 \] \text{and} \[ B = B( b, \lambda ) \text{ in } S^2. \]
However, Joy does not violate (ii). Joy says: "... once the state \( \lambda \) is
specified and the two particles have separated, measurements of \( A \) can
depend only on \( \lambda \) and \( a \), but not \( b \), and likewise measurements of \( B \)
can depend only on \( \lambda \) and \( b \), but not \( a \) ...[ compare the (ii)-violation by
Bohm's \( \lambda \)tas stated above ]... Assuming ... that the distribution
\( \rho( \lambda ) \) is normalized on the space \( \Lambda \), we finally arrive at the
inequalities ... exactly what is predicted by quantum mechanics ... we have
been able to derive these results without specifying what the complete state
\( \lambda \) is or the distribution \( \rho( \lambda ) \) is, and without employing any
averaging procedure ... the correlations [ for the examples of 0904.4259 ] ...
are simply the local, realistic, and deterministic correlations among certain
points of ... \( S^3 \) and \( S^7 \) ... This implies that the violations of Bell inequalities
... have nothing to do with quantum mechanics per se ...".

So, even though Joy's \( \lambda \)s do not violate (ii), when Joy "... derive[s] ... the exact quantum mechanical expectation value ... - a . b " Joy's result is
consistent with that of Bohm's "\( \lambda \)tas".

Joy's "\( \lambda \)tas" are classical and local (in Joy's sense).

Bohm's "\( \lambda \)tas" are quantum and, since Joy does not change Bell's (ii),
nonlocal (in Joy's sense).

Joy's "\( \lambda \)tas" and Bohm's "\( \lambda \)tas" are consistent with each other
with respect to their calculated quantum expectation values.
Could Joy's "lambdas" be considered as a Classical Limit of Bohm's "lambdas"?

Consider again Peter Holland's book in which he says:
"... 6.9 Remarks on the path integral approach ... Feynman[s] ... route to quantum mechanics ... rests on the trajectory concept and so may be expected to have some connection with the causal [Bohm Potential] formulation. ... Feynman provides a technique for computing ... the transition amplitude (Green function or propagator) ... from the classical Lagrangian ... One considers all the paths ... and associates with each an amplitude ... These tracks are ... called 'classical paths' ... one sums (integrates) over all the paths ... the solution .. is given by ... Huygens' principle ... of all the paths ... one of them will be the actual trajectory pursued by the quantum particle according to the [Bohm Potential] guidance formula ... We shall refer to ... it ... as the 'quantum path' ... For an infinitesimal time interval ... the propagator is just the classical wavefunction ... a finite path may be decomposed into many such infinitesimal steps, the net propagator being obtained by successive applications of Huygens' construction ... We may view the Feynman procedure as a method of obtaining the quantum action from the set of all classical actions. ...".

If Joy Christian's classical "lambdas" are identified with Feynman path Lagrangian / Green function propagators, and if their Huygens' sums can be seen to produce the Bohm "lambdas", then Joy's work will show a nice smooth classical limit (as opposed to Bell's discordant classical limit) for the Bohm Quantum Potential.

If the Bohm Quantum Potential can then be used as a basis for a construction of a realistic AQFT (Algebraic Quantum Field Theory) then maybe Joy Christian's work will help show a useful connection (and philosohical reconciliation) between the Classical Lagrangian physics so useful in detailed understanding of the Standard Model and of AQFT along the lines of generalization of the Hyperfinite II1 von Neumann factor algebra.