A New Conjecture in Number Theory

Dao Thanh Oai

August 8, 2015

Abstract

I propose a conjecture of generalization of the Fermat last theorem and the Lander, Parkin, and Selfridge conjecture.

Conjecture 1. *Let n, m be two positive integers and $m \neq n$, let a be an integer and $a \neq 0$. Let $g(x)$ be a given polynomial, let $f(x) = g(x) + a.x^k$, then exist k_0 be the positive integer such that when $k \geq k_0$ then no $(n+m)$ positive integers x_1, x_2, x_n, y_1, y_2,..., y_m greater than 1 can satisfy an equation as follows:*

$$f(x_1) + f(x_2) + ... + f(x_n) = f(y_1) + f(y_2) + ... + f(y_m) \quad (1)$$

Example 1: Let $g(x) \equiv 0$, exist k_0 such that when $k \geq k_0$ then no three positive integers x, y, z can satisfy an equations:

$$x^k + y^k = z^k$$

By the Fermat Last Theorem, in **example 1** we get $k_0 = 3$

Example 2: Let n, m be two positive integers and $m \neq n$, let $g(x) \equiv 0$, exist k_0 such that when $k \geq k_0$ then no $(n+m)$ positive integers x_1, x_2,...,x_n, y_1, y_2,..., y_m can satisfy an equation:

$$x_1^k + x_2^k + ... + x_n^k = y_1^k + y_2^k + ... + y_m^k$$

By the Lander, Parkin, and Selfridge conjecture in **example 2**, $k_0 = n + m + 1$

Example 3: Let $g(x) = 4x^2 + 5$, exist k_0 such that when $k \geq k_0$ then no three integers x, y, z can satisfy an equation:

$$3x^k + 4x^2 + 5 + 3y^k + 4y^2 + 5 = 3z^k + 4z^2 + 5$$

Example 4: Let $g(x) = x$, exist k_0 such that when $k \geq k_0$ then no three integers x, y, z can satisfy an equation:

$$x^k + x + y^k + y = z^k + z$$

References

[1] L. J. Lander, T. R. Parkin, J. L. Selfridge; Parkin; Selfridge (1967). *A Survey of Equal Sums of Like Powers.* Mathematics of Computation 21 (99): 446459. doi:10.1090/S0025-5718-1967-0222008-0.

[2] Frits Beukers (January 20, 2006). *The generalized Fermat equation.* Staff.science.uu.nl. Retrieved 2014-03-06, available at

http://www.staff.science.uu.nl/ beuke106/Fermatlectures.pdf

Dao Thanh Oai: *Cao Mai Doai-Quang Trung-Kien Xuong-Thai Binh-Viet Nam*
E-mail address: *daothanhoai@hotmail.com*