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Some Results on Total Mean Cordial Labeling of Graphs

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Abstract: A graph $G = (V, E)$ with p vertices and q edges is said to be a Total Mean Cordial graph if there exists a function $f : V(G) \to \{0, 1, 2\}$ such that for each edge xy assign the label $\left\lceil \frac{f(x)+f(y)}{2} \right\rceil$ where $x, y \in V(G)$, and the total number of 0, 1 and 2 are balanced. That is $|ev_f(i) - ev_f(j)| \leq 1$, $i, j \in \{0, 1, 2\}$ where $ev_f(x)$ denotes the total number of vertices and edges labeled with x $(x = 0, 1, 2)$. In this paper, we investigate the total mean cordial labeling behavior of $L_n \odot K_1$, $S(P_n \odot 2K_1)$, $S(W_n)$ and some more graphs.

Key Words: Smarandachely total mean cordial labeling, cycle, path, wheel, union, corona, ladder.

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§1. Introduction

Throughout this paper we considered finite, undirected and simple graphs. The symbols $V(G)$ and $E(G)$ will denote the vertex set and edge set of a graph G. A graph labeling is an assignment of integers to the vertices or edges, or both, subject to certain conditions. Labeled graphs serves as a useful mathematical model for a broad range of application such as coding theory, X-ray crystallography analysis, communication network addressing systems, astronomy, radar, circuit design and database management [1]. Ponraj, Ramasamy and Sathish Narayanan [3] introduced the concept of total mean cordial labeling of graphs and studied about the total mean cordial labeling behavior of Path, Cycle, Wheel and some more standard graphs. In [4,6], Ponraj and Sathish Narayanan proved that $K_n^c + 2K_2$ is total mean cordial if and only if $n = 1, 2, 4, 6, 8$ and they investigate the total mean cordial labeling behavior of prism, gear, helms. In [5], Ponraj, Ramasamy and Sathish Narayanan investigate the Total Mean Cordiality of Lotus inside a circle, bistar, flower graph, $K_{2,n}$, Olive tree, P_n^2, $S(P_n \odot K_1)$, $S(K_{1,n})$. In this paper we investigate $L_n \odot K_1$, $S(P_n \odot 2K_1)$, $S(W_n)$ and some more graphs. If x is any real number. Then the symbol $\lfloor x \rfloor$ stands for the largest integer less than or equal to x and $\lceil x \rceil$ stands for the smallest integer greater than or equal to x. For basic definitions that are not defined here are used in the sense of Harary [2].

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§2. Preliminaries

Definition 2.1 *A total mean cordial labeling of a graph $G = (V, E)$ is a function $f : V(G) \to \{0, 1, 2\}$ such that for each edge xy assign the label $\left\lceil \frac{f(x)+f(y)}{2} \right\rceil$ where $x, y \in V(G)$, and the total number of 0, 1 and 2 are balanced. That is $|ev_f(i) - ev_f(j)| \leq 1$, $i, j \in \{0, 1, 2\}$ where $ev_f(x)$ denotes the total number of vertices and edges labeled with x $(x = 0, 1, 2)$. If there exists a total mean cordial labeling on a graph G, we will call G is total mean cordial.*

Furthermore, let $H \leq G$ be a subgraph of G. If there is a function f from $V(G) \to \{0, 1, 2\}$ such that $f|_H$ is a total mean cordial labeling but $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ is a constant for all edges in $G \setminus H$, such a labeling and G are then respectively called Smarandachely total mean cordial labeling and Smarandachely total mean cordial labeling graph respect to H.

The following results are frequently used in the subsequent section.

Definition 2.2 *The product graph $G_1 \times G_2$ is defined as follows: Consider any two vertices $u = (u_1, u_2)$ and $v = (v_1, v_2)$ in $V = V_1 \times V_2$. Then u and v are adjacent in $G_1 \times G_2$ whenever $[u_1 = v_1$ and u_2 adj $v_2]$ or $[u_2 = v_2$ and u_1 adj $v_1]$. Note that the graph $L_n = P_n \times P_2$ is called the ladder on n steps.*

Definition 2.3 *Let G_1 and G_2 be two graphs with vertex sets V_1 and V_2 and edge sets E_1 and E_2 respectively. Then their join $G_1 + G_2$ is the graph whose vertex set is $V_1 \cup V_2$ and edge set is $E_1 \cup E_2 \cup \{uv : u \in V_1$ and $v \in V_2\}$. Also the graph $W_n = C_n + K_1$ is called the wheel.*

Definition 2.4 *Let G_1, G_2 respectively be (p_1, q_1), (p_2, q_2) graphs. The corona of G_1 with G_2, $G_1 \odot G_2$ is the graph obtained by taking one copy of G_1 and p_1 copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2.*

Definition 2.5 *The union of two graphs G_1 and G_2 is the graph $G_1 \cup G_2$ with $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$.*

Definition 2.6 *The subdivision graph $S(G)$ of a graph G is obtained by replacing each edge uv of G by a path uwv.*

Theorem 2.7([7]) *Let G be a (p, q) Total Mean Cordial graph and $n \neq 3$ then $G \cup P_n$ is also total mean cordial.*

Main Results

Theorem 3.1 *$S(W_n)$ is total mean cordial.*

Proof Let $V(S(W_n)) = \{u, u_i, x_i, y_i : 1 \leq i \leq n\}$, $E(S(W_n)) = \{u_i y_i, y_i u_{i+1} : 1 \leq i \leq n-1\} \cup \{u_n y_n, y_n u_1\} \cup \{u x_i, x_i u_i : 1 \leq i \leq n\}$. Clearly $|V(S(W_n))| + |V(S(W_n))| = 7n + 1$.

Case 1. $n \equiv 0 \pmod{12}$.

Let $n = 12t$ and $t > 0$. Define $f : V(S(W_n)) \to \{0, 1, 2\}$ by $f(u) = 0$,

$$\begin{aligned} f(x_i) &= 0, \quad 1 \le i \le 12t \\ f(u_i) &= 0, \quad 1 \le i \le 2t \\ f(u_{2t+i}) &= 2, \quad 1 \le i \le 7t \\ f(u_{9t+i}) &= 1, \quad 1 \le i \le 3t \\ f(y_i) &= 1, \quad 1 \le i \le 2t-1 \\ f(y_{2t-1+i}) &= 2, \quad 1 \le i \le 7t \\ f(y_{9t-1+i}) &= 1, \quad 1 \le i \le 3t+1. \end{aligned}$$

Here $ev_f(0) = 28t + 1$, $ev_f(1) = ev_f(2) = 28t$.

Case 2. $n \equiv 1 \pmod{12}$.

Let $n = 12t + 1$ and $t > 0$. Define a map $f : V(S(W_n)) \to \{0, 1, 2\}$ by $f(u) = 0$,

$$\begin{aligned} f(x_i) &= 0, \quad 1 \le i \le 12t+1 \\ f(u_i) &= 0, \quad 1 \le i \le 2t \\ f(u_{2t+i}) &= 2, \quad 1 \le i \le 7t \\ f(u_{9t+i}) &= 1, \quad 1 \le i \le 3t+1 \\ f(y_i) &= 1, \quad 1 \le i \le 2t-1 \\ f(y_{2t-1+i}) &= 2, \quad 1 \le i \le 7t+1 \\ f(y_{9t+i}) &= 1, \quad 1 \le i \le 3t+1. \end{aligned}$$

Here $ev_f(0) = ev_f(1) = 28t + 3$, $ev_f(2) = 28t + 2$.

Case 3. $n \equiv 2 \pmod{12}$.

Let $n = 12t + 2$ and $t > 0$. Define a function $f : V(S(W_n)) \to \{0, 1, 2\}$ by $f(u) = 0$,

$$\begin{aligned} f(x_i) &= 0, \quad 1 \le i \le 12t+2 \\ f(u_i) &= 0, \quad 1 \le i \le 2t \\ f(u_{2t+i}) &= 2, \quad 1 \le i \le 7t+1 \\ f(u_{9t+1+i}) &= 1, \quad 1 \le i \le 3t+1 \\ f(y_i) &= 1, \quad 1 \le i \le 2t \\ f(y_{2t+i}) &= 2, \quad 1 \le i \le 7t+1 \\ f(y_{9t+1+i}) &= 1, \quad 1 \le i \le 3t+1. \end{aligned}$$

In this case $ev_f(0) = ev_f(1) = ev_f(2) = 28t + 5$.

Case 4. $n \equiv 3 \pmod{12}$.

Let $n = 12t - 9$ and $t > 0$. For $n = 3$, the Figure 1 shows that $S(W_3)$ is total mean cordial.

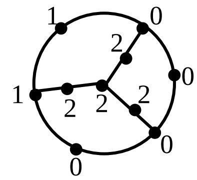

Figure 1

Now assume $t \geq 2$. Define a map $f : V(S(W_n)) \to \{0, 1, 2\}$ by $f(u) = 0$,

$$\begin{aligned} f(x_i) &= 0, \quad 1 \leq i \leq 12t-9 \\ f(u_i) &= 0, \quad 1 \leq i \leq 2t-2 \\ f(u_{2t-2+i}) &= 2, \quad 1 \leq i \leq 7t-5 \\ f(u_{9t-7+i}) &= 1, \quad 1 \leq i \leq 3t-2 \\ f(y_i) &= 1, \quad 1 \leq i \leq 2t-3 \\ f(y_{2t-3+i}) &= 2, \quad 1 \leq i \leq 7t-5 \\ f(y_{9t-8+i}) &= 1, \quad 1 \leq i \leq 3t-1. \end{aligned}$$

In this case $ev_f(0) = ev_f(1) = 28t - 21$, $ev_f(2) = 28t - 20$.

Case 5. $n \equiv 4 \pmod{12}$.

Let $n = 12t - 8$ and $t > 0$. The following Figure 2 shows that $S(W_4)$ is total mean cordial.

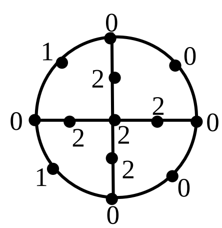

Figure 2

Now assume $t \geq 2$. Define $f : V(S(W_n)) \to \{0, 1, 2\}$ by $f(u) = 0$,

$$\begin{aligned} f(x_i) &= 0, \quad 1 \leq i \leq 12t-8 \\ f(u_i) &= 0, \quad 1 \leq i \leq 2t-2 \\ f(u_{2t-2+i}) &= 2, \quad 1 \leq i \leq 7t-5 \\ f(u_{9t-7+i}) &= 1, \quad 1 \leq i \leq 3t-1 \\ f(y_i) &= 1, \quad 1 \leq i \leq 2t-3 \\ f(y_{2t-3+i}) &= 2, \quad 1 \leq i \leq 7t-4 \\ f(y_{9t-7+i}) &= 1, \quad 1 \leq i \leq 3t-1. \end{aligned}$$

In this case $ev_f(0) = 28t - 19$, $ev_f(1) = ev_f(2) = 28t - 18$.

Case 6. $n \equiv 5 \pmod{12}$.

Let $n = 12t - 7$ and $t > 0$. Define a function $f : V(S(W_n)) \to \{0, 1, 2\}$ by $f(u) = 0$,

$$\begin{aligned} f(x_i) &= 0, \quad 1 \le i \le 12t-7 \\ f(u_i) &= 1, \quad 1 \le i \le 4t-3 \\ f(u_{4t-3+i}) &= 2, \quad 1 \le i \le 7t-4 \\ f(u_{11t-7+i}) &= 1, \quad 1 \le i \le t \\ f(y_i) &= 0, \quad 1 \le i \le 4t-3 \\ f(y_{4t-3+i}) &= 2, \quad 1 \le i \le 7t-4 \\ f(y_{11t-7+i}) &= 1, \quad 1 \le i \le t. \end{aligned}$$

In this case $ev_f(0) = ev_f(1) = ev_f(2) = 28t - 16$.

Case 7. $n \equiv 6 \pmod{12}$.

Let $n = 12t - 6$ and $t > 0$. Define a function $f : V(S(W_n)) \to \{0, 1, 2\}$ by $f(u) = 0$,

$$\begin{aligned} f(x_i) &= 0, \quad 1 \le i \le 12t-6 \\ f(u_i) &= 0, \quad 1 \le i \le 2t-1 \\ f(u_{2t-1+i}) &= 2, \quad 1 \le i \le 7t-4 \\ f(u_{9t-5+i}) &= 1, \quad 1 \le i \le 3t-1 \\ f(y_i) &= 1, \quad 1 \le i \le 2t-2 \\ f(y_{2t-2+i}) &= 2, \quad 1 \le i \le 7t-3 \\ f(y_{9t-5+i}) &= 1, \quad 1 \le i \le 3t-1. \end{aligned}$$

In this case $ev_f(0) = ev_f(1) = 28t - 7$, $ev_f(2) = 28t - 6$.

Case 8. $n \equiv 7 \pmod{12}$.

Let $n = 12t - 5$ and $t > 0$. Define a function $f : V(S(W_n)) \to \{0, 1, 2\}$ by $f(u) = 0$,

$$\begin{aligned} f(x_i) &= 0, \quad 1 \le i \le 12t-5 \\ f(u_i) &= 0, \quad 1 \le i \le 2t-1 \\ f(u_{2t-1+i}) &= 2, \quad 1 \le i \le 7t-3 \\ f(u_{9t-4+i}) &= 1, \quad 1 \le i \le 3t-1 \\ f(y_i) &= 1, \quad 1 \le i \le 2t-2 \\ f(y_{2t-2+i}) &= 2, \quad 1 \le i \le 7t-3 \\ f(y_{9t-5+i}) &= 1, \quad 1 \le i \le 3t. \end{aligned}$$

Here $ev_f(0) = ev_f(1) = 28t - 11$, $ev_f(2) = 28t - 12$.

Case 9. $n \equiv 8 \pmod{12}$.

Let $n = 12t - 4$ and $t > 0$. Define a function $f : V(S(W_n)) \to \{0, 1, 2\}$ by $f(u) = 0$,

$$\begin{aligned}
f(x_i) &= 0, \quad 1 \le i \le 12t - 4 \\
f(u_i) &= 0, \quad 1 \le i \le 2t - 1 \\
f(u_{2t-1+i}) &= 2, \quad 1 \le i \le 7t - 2 \\
f(u_{9t-3+i}) &= 1, \quad 1 \le i \le 3t - 1 \\
f(y_i) &= 1, \quad 1 \le i \le 2t - 1 \\
f(y_{2t-1+i}) &= 2, \quad 1 \le i \le 7t - 3 \\
f(y_{9t-4+i}) &= 1, \quad 1 \le i \le 3t.
\end{aligned}$$

In this case $ev_f(0) = ev_f(1) = ev_f(2) = 28t - 9$.

Case 10. $n \equiv 9 \pmod{12}$.

Let $n = 12t - 3$ and $t > 0$. Define a function $f : V(S(W_n)) \to \{0, 1, 2\}$ by $f(u) = 0$,

$$\begin{aligned}
f(x_i) &= 0, \quad 1 \le i \le 12t - 3 \\
f(u_i) &= 0, \quad 1 \le i \le 2t - 1 \\
f(u_{2t-1+i}) &= 2, \quad 1 \le i \le 7t - 2 \\
f(u_{9t-3+i}) &= 1, \quad 1 \le i \le 3t \\
f(y_i) &= 1, \quad 1 \le i \le 2t - 2 \\
f(y_{2t-2+i}) &= 2, \quad 1 \le i \le 7t - 1 \\
f(y_{9t-3+i}) &= 1, \quad 1 \le i \le 3t.
\end{aligned}$$

In this case $ev_f(0) = ev_f(1) = 28t - 7$, $ev_f(2) = 28t - 6$.

Case 11. $n \equiv 10 \pmod{12}$.

Let $n = 12t - 2$ and $t > 0$. Define a function $f : V(S(W_n)) \to \{0, 1, 2\}$ by $f(u) = 0$,

$$\begin{aligned}
f(x_i) &= 0, \quad 1 \le i \le 12t - 2 \\
f(u_i) &= 0, \quad 1 \le i \le 2t - 1 \\
f(u_{2t-1+i}) &= 2, \quad 1 \le i \le 7t - 1 \\
f(u_{9t-2+i}) &= 1, \quad 1 \le i \le 3t \\
f(y_i) &= 1, \quad 1 \le i \le 2t - 2 \\
f(y_{2t-2+i}) &= 2, \quad 1 \le i \le 7t - 1 \\
f(y_{9t-3+i}) &= 1, \quad 1 \le i \le 3t + 1.
\end{aligned}$$

In this case $ev_f(0) = 28t - 5$, $ev_f(1) = ev_f(2) = 28t - 4$.

Case 12. $n \equiv 11 \pmod{12}$.

Let $n = 12t - 1$ and $t > 0$. Define a function $f : V(S(W_n)) \to \{0, 1, 2\}$ by $f(u) = 0$,

$$
\begin{aligned}
f(x_i) &= 0, \quad 1 \le i \le 12t - 1 \\
f(u_i) &= 1, \quad 1 \le i \le 4t - 1 \\
f(u_{4t-1+i}) &= 2, \quad 1 \le i \le 7t \\
f(u_{11t-1+i}) &= 1, \quad 1 \le i \le t \\
f(y_i) &= 0, \quad 1 \le i \le 4t - 1 \\
f(y_{4t-1+i}) &= 2, \quad 1 \le i \le 7t - 1 \\
f(y_{11t-2+i}) &= 1, \quad 1 \le i \le t + 1.
\end{aligned}
$$

In this case $ev_f(0) = ev_f(1) = ev_f(2) = 28t - 6$.

Hence $S(W_n)$ is total mean cordial. □

Theorem 3.2 *$S(P_n \odot 2K_1)$ is total mean cordial.*

Proof Let $V(S(P_n \odot 2K_1)) = \{u_i, v_i, w_i, x_i, y_i : 1 \le i \le n\} \cup \{u_i' : 1 \le i \le n-1\}$ and $E(S(P_n \odot 2K_1)) = \{u_iu_i', u_i'u_{i+1} : 1 \le i \le n-1\} \cup \{u_iv_i, u_iw_i, v_ix_i, w_iy_i : 1 \le i \le n\}$. Clearly $|V(S(P_n \odot 2K_1))| + |V(S(W_n \odot 2K_1))| = 12n - 3$. Now we define a map $f : V(S(P_n \odot 2K_1)) \to \{0, 1, 2\}$ by $f(v_1) = 0$, $f(w_1) = 1$, $f(u_n) = 0$,

$$
\begin{aligned}
f(u_i) &= f(u_i') &= 0, \quad 1 \le i \le n - 1 \\
f(v_i) &= f(w_i) &= 1, \quad 2 \le i \le n \\
f(x_i) &= f(y_i) &= 2, \quad 1 \le i \le n.
\end{aligned}
$$

In this case $ev_f(0) = ev_f(1) = ev_f(2) = 4n - 1$.

Hence $S(P_n \odot 2K_1)$ is total mean cordial. □

Theorem 3.3 *$L_n \odot K_1$ is total mean cordial.*

Proof Let $V(L_n \odot K_1) = \{u_i, v_i, x_i, y_i : 1 \le i \le n\}$ and $E(L_n \odot K_1) = \{x_iu_i, u_iv_i, v_iy_i : 1 \le i \le n\} \cup \{u_iu_{i+1}, v_iv_{i+1} : 1 \le i \le n-1\}$. Here $|V(L_n \odot K_1)| + |E(L_n \odot K_1)| = 9n - 2$. Define a map $f : V(L_n \odot K_1) \to \{0, 1, 2\}$ by

$$
\begin{aligned}
f(u_i) &= 0, \quad 1 \le i \le n \\
f(x_i) &= 0, \quad 1 \le i \le \left\lceil \tfrac{n}{2} \right\rceil \\
f(y_i) &= 1, \quad 1 \le i \le n \\
f(x_{\lceil \frac{n}{2} \rceil + i}) &= 1, \quad 1 \le i \le \left\lfloor \tfrac{n}{2} \right\rfloor \\
f(v_i) &= 2, \quad 1 \le i \le n.
\end{aligned}
$$

The following Table 1 shows that f is a total mean cordial labeling of $L_n \odot K_1$.

Nature of n	$ev_f(0)$	$ev_f(1)$	$ev_f(2)$
$n \equiv 0 \pmod 2$	$\left\lfloor \frac{9n-2}{3} \right\rfloor$	$\left\lceil \frac{9n-2}{3} \right\rceil$	$\left\lfloor \frac{9n-2}{3} \right\rfloor$
$n \equiv 1 \pmod 2$	$\left\lceil \frac{9n-2}{3} \right\rceil$	$\left\lfloor \frac{9n-2}{3} \right\rfloor$	$\left\lfloor \frac{9n-2}{3} \right\rfloor$

Hence $L_n \odot K_1$ is Total Mean Cordial. □

Theorem 3.4 *The graph $P_1 \cup P_2 \cup \ldots \cup P_n$ is total mean cordial.*

Proof We prove this theorem by induction on n. For $n = 1, 2, 3$ the result is true, see Figure 3.

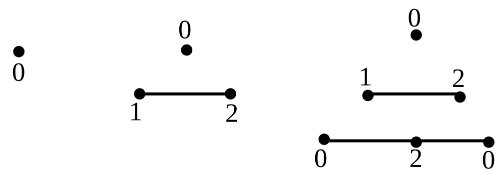

Figure 3

Assume the result is true for $P_1 \cup P_2 \cup \ldots \cup P_{n-1}$. Then by Theorem 2.7, $(P_1 \cup P_2 \cup \ldots \cup P_{n-1}) \cup P_n$ is total mean cordial. □

Theorem 3.5 *Let C_n be the cycle $u_1u_2 \ldots u_nu_1$. Let GC_n be a graph with $V(GC_n) = V(C_n) \cup \{v_i : 1 \leq i \leq n\}$ and $E(GC_n) = E(C_n) \cup \{u_iv_i, u_{i+1}v_i : 1 \leq i \leq n-1\} \cup \{u_nv_n, u_1v_n\}$. Then GC_n is total mean cordial.*

Proof Clearly, $|V(GC_n)| + |E(GC_n)| = 5n$.

Case 1. $n \equiv 0 \pmod 3$.

Let $n = 3t$ and $t > 0$. Define $f : V(GC_n) \to \{0, 1, 2\}$ by

$$\begin{aligned} f(u_i) &= f(v_i) &&= 0, \quad 1 \leq i \leq t \\ f(u_{t+i}) &= f(v_{t+i}) &&= 2, \quad 1 \leq i \leq t \\ f(u_{2t+i}) &= f(v_{2t+i}) &&= 1, \quad 1 \leq i \leq t-1 \end{aligned}$$

$f(u_{3t}) = 1$ and $f(v_{3t}) = 0$. In this case $ev_f(0) = ev_f(1) = ev_f(2) = 5t$.

Case 2. $n \equiv 1 \pmod 3$.

Let $n = 3t+1$ and $t > 0$. Define $f : V(GC_n) \to \{0, 1, 2\}$ by

$$\begin{aligned} f(u_i) &= f(v_i) &&= 0, \quad 1 \leq i \leq t \\ f(u_{t+1+i}) &= f(v_{t+i}) &&= 2, \quad 1 \leq i \leq t \\ f(u_{2t+1+i}) &= f(v_{2t+1+i}) &&= 1, \quad 1 \leq i \leq t \end{aligned}$$

$f(u_{t+1}) = 0$, $f(v_{2t+1}) = 2$. In this case $ev_f(0) = 5t+1$, $ev_f(1) = ev_f(2) = 5t+2$.

Case 3. $n \equiv 2 \pmod 3$.

Let $n = 3t + 2$ and $t > 0$. Construct a vertex labeling $f : V(GC_n) \to \{0, 1, 2\}$ by

$$\begin{aligned} f(u_i) &= f(v_i) &&= 0, \quad 1 \le i \le t+1 \\ f(u_{t+2+i}) &= f(v_{t+1+i}) &&= 2, \quad 1 \le i \le t \\ f(u_{2t+2+i}) &= f(v_{2t+2+i}) &&= 1, \quad 1 \le i \le t \end{aligned}$$

$f(u_{t+1}) = 1$, $f(v_{2t+2}) = 2$. In this case $ev_f(0) = ev_f(1) = 5t + 3$, $ev_f(2) = 5t + 4$. Hence GC_n is total mean cordial. □

Example 3.6 A total mean cordial labeling of GC_8 is given in Figure 4.

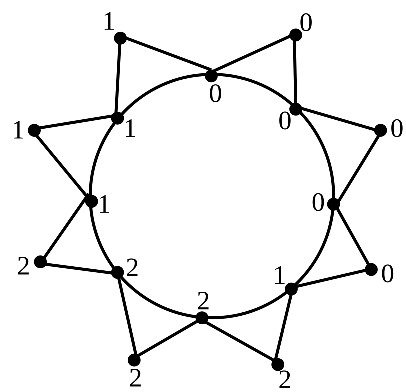

Figure 4

Theorem 3.6 *Let $St(L_n)$ be a graph obtained from a ladder L_n by subdividing each step exactly once. Then $St(L_n)$ is total mean cordial.*

Proof Let $V(St(L_n)) = \{u_i, v_i, w_i : 1 \le i \le n\}$ and $E(St(L_n)) = \{u_iw_i, w_iv_i : 1 \le i \le n\} \cup \{u_iu_{i+1}, v_iv_{i+1} : 1 \le i \le n-1\}$. It is clear that $|V(St(L_n))| + |E(St(L_n))| = 7n - 2$.

Case 1. $n \equiv 0 \pmod 6$.

Let $n = 6t$. Define a map $f : V(St(L_n)) \to \{0, 1, 2\}$ as follows.

$$\begin{aligned} f(u_i) &= 0, \quad 1 \le i \le 6t \\ f(w_i) &= 0, \quad 1 \le i \le t \\ f(w_{t+i}) &= 1, \quad 1 \le i \le 5t \\ f(v_i) &= 2, \quad 1 \le i \le 5t \\ f(v_{5t+i}) &= 1, \quad 1 \le i \le t. \end{aligned}$$

In this case $ev_f(0) = ev_f(1) = 14t - 1$, $ev_f(2) = 14t$.

Case 2. $n \equiv 1 \pmod 6$.

Let $n = 6t + 1$ and $t \geq 1$. Define a function $f : V(St(L_n)) \rightarrow \{0, 1, 2\}$ by

$$\begin{aligned} f(u_i) &= 0, \quad 1 \leq i \leq 6t+1 \\ f(w_i) &= 0, \quad 1 \leq i \leq t \\ f(w_{t+i}) &= 2, \quad 1 \leq i \leq 5t+1 \\ f(v_i) &= 1, \quad 1 \leq i \leq 4t+1 \\ f(v_{4t+1+i}) &= 2, \quad 1 \leq i \leq 2t. \end{aligned}$$

Here $ev_f(0) = 14t + 1$, $ev_f(1) = ev_f(2) = 14t + 2$.

Case 3. $n \equiv 2 \pmod 6$.

Let $n = 6t + 2$ and $t \geq 0$. The Figure 5 shows that $St(L_2)$ is total mean cordial.

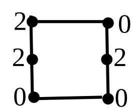

Figure 5

Consider the case for $t \geq 1$. Define $f : V(St(L_n)) \rightarrow \{0, 1, 2\}$ as follows.

$$\begin{aligned} f(u_i) &= 0, \quad 1 \leq i \leq 6t+2 \\ f(w_i) &= 0, \quad 1 \leq i \leq t \\ f(w_{t+i}) &= 1, \quad 1 \leq i \leq 5t+1 \\ f(v_i) &= 2, \quad 1 \leq i \leq 5t+1 \\ f(v_{5t+1+i}) &= 1, \quad 1 \leq i \leq t. \end{aligned}$$

and $f(w_{6t+2}) = 2$, $f(v_{6t+2}) = 0$. Here $ev_f(0) = ev_f(1) = ev_f(2) = 14t + 4$.

Case 4. $n \equiv 3 \pmod 6$.

Let $n = 6t - 3$ and $t \geq 1$. Define a function $f : V(St(L_n)) \rightarrow \{0, 1, 2\}$ by

$$\begin{aligned} f(u_i) &= f(w_i) &&= f(v_i) &&= 0, \quad 1 \leq i \leq 2t-2 \\ f(u_{2t-1+i}) &= f(w_{2t-1+i}) &&= f(v_{2t+i}) &&= 1, \quad 1 \leq i \leq 2t-2 \\ f(u_{4t-2+i}) &= f(w_{4t-1+i}) &&= f(v_{4t-2+i}) &&= 2, \quad 1 \leq i \leq 2t-2 \end{aligned}$$

$f(u_{2t-1}) = f(w_{2t-1}) = 0$, $f(u_{4t-2}) = f(w_{4t-2}) = f(w_{4t-1}) = 1$ and $f(u_{6t-3}) = f(v_{6t-3}) = 2$. In this case $ev_f(0) = 14t - 7$, $ev_f(1) = ev_f(2) = 14t - 8$.

Case 5. $n \equiv 4 \pmod 6$.

Let $n = 6t - 2$ and $t > 0$. Define $f : V(St(L_n)) \to \{0, 1, 2\}$ by

$$\begin{aligned} f(u_i) &= 0, \quad 1 \le i \le 6t-2 \\ f(w_i) &= 0, \quad 1 \le i \le t \\ f(w_{t+i}) &= 1, \quad 1 \le i \le 5t-2 \\ f(v_i) &= 2, \quad 1 \le i \le 5t-2 \\ f(v_{5t-2+i}) &= 1, \quad 1 \le i \le t. \end{aligned}$$

In this case $ev_f(0) = ev_f(1) = 14t - 5$, $ev_f(2) = 14t - 6$.

Case 6. $n \equiv 5 \pmod 6$.

Let $n = 6t - 1$ and $t > 0$. Define a function $f : V(St(L_n)) \to \{0, 1, 2\}$ by

$$\begin{aligned} f(u_i) &= 0, \quad 1 \le i \le 6t-1 \\ f(w_i) &= 0, \quad 1 \le i \le t \\ f(w_{t+i}) &= 1, \quad 1 \le i \le 5t-1 \\ f(v_i) &= 2, \quad 1 \le i \le 5t-1 \\ f(v_{5t-1+i}) &= 1, \quad 1 \le i \le t. \end{aligned}$$

Here $ev_f(0) = ev_f(1) = ev_f(2) = 14t - 3$.

Hence $St(L_n)$ is total mean cordial. □

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