

# Non-Newtonian AI for solving NP-complete problems.

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## Abstract.

A new kind of dynamics for simulations based upon quantum-classical hybrid is discussed. The model is represented by a modified Madelung equation in which the quantum potential is replaced by different, specially chosen potentials. As a result, the dynamics attains both quantum and classical properties: it preserves superposition and entanglement of random solutions, while allowing one to measure its state variables using classical methods. Such an optimal combination of characteristics is a perfect match for quantum-inspired information processing. In this paper, the retrieval of stored items from an *exponentially large* unsorted database is performed by quantum-inspired resonance using *polynomial resources* due to quantum-like superposition effect.

## 1. Introduction.

Recent advances in quantum information theory have inspired an explosion of interest in new quantum algorithms for solving hard computational problems. They include a special class of so called NP-complete problems which are considered to be intractable by most of the theoretical computer scientists. One of the oldest (and still unsolved) problems of this class is a search problem: find one item in an unsorted database (for instance, find the name that matches a telephone number in a telephone book). As for any of NP-complete problems, here the algorithm for solution is very simple: try each item and compare with the sought one. Classically, this would require an average  $O(N)$  queries to the database if  $N$  is the number of items. Therefore, for exponentially large  $N$ , the problem becomes intractable. In terms of quantum computing, finding the item in the database corresponds to measuring the system and having it collapse to the state, which represents that item. Although the quantum algorithm requires only  $O(\sqrt{N})$  queries, its complexity is still exponential. Further progress in reducing the search complexity can be associated with incorporating a structure into databases, Zak, M., 2003. or using *simulations* instead of computations. However in the both cases, the corresponding quantum device must be implemented, and that is the main obstacle to practical application of these algorithms.

The basic idea of this paper is to create a new kind of dynamical system for simulations that would preserve superposition of random solutions, while allowing one to measure its state variables using classical methods. In other words, such a hybrid system would reinforce the advantages and minimize limitations of both quantum and classical aspects. The formal mathematical difference between quantum and classical mechanics is better pronounced in the Madelung (rather than the Schrödinger) equation. Two factors contribute to this difference: the scale of the system introduced through the Planck constant and the topology of the Madelung equations that includes the feedback (in the form of the quantum potential) from the Liouville equation to the Hamilton-Jacobi equation. Ignoring the scale factor as well as the concrete form of the feedback, we concentrated upon preserving the topology while varying the types of the feedbacks. A general approach to the choice of the feedback was introduced and discussed in [4]. More specific feedbacks linked to the behavioral models of Livings were presented in [5–8]. In this paper we are concerned only with computational capabilities of the proposed model disregarding possible physical interpretations. In particular, the problem of modeling associative memory of exponential capacity using only polynomial resources is addressed. The classical formulation of associative memory is: store a set of patterns in such a way that when presented with a new pattern, the network responds by producing whichever one of the stored patterns most closely resembles the stored one. The correct response follows training on a set of examples. The best solution provided by recurrent neural nets is the storage capacity proportional to  $N/\log N$  where  $N$  is the dimensionality of the corresponding neural net. The proposed associative memory is based upon quantum-like superposition of solutions to motor dynamics, and the resonance between motor and mental dynamics.

## 2. Starting with quantum mechanics.

The starting point of our approach is the Madelung equation that is a hydrodynamics version of the Schrödinger equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left( \frac{\rho}{m} \nabla S \right) = 0 \quad (1)$$

$$\frac{\partial S}{\partial t} + (\nabla S)^2 + F - \frac{\hbar^2 \nabla^2 \sqrt{\rho}}{2m\sqrt{\rho}} = 0 \quad (2)$$

Here  $\rho$  and  $S$  are the components of the wave function  $\psi = \sqrt{\rho} e^{iS/\hbar}$ , and  $\hbar$  is the Planck constant divided by  $2\pi$ . The last term in Eq. (2) is known as quantum potential. From the viewpoint of Newtonian mechanics, Eq. (1) expresses continuity of the flow of probability density, and Eq. (2) is the Hamilton-Jacobi equation for the action  $S$  of the particle. Actually the quantum potential in Eq. (2), as a feedback from Eq. (1) to Eq. (2), represents the difference between the Newtonian and quantum mechanics, and therefore, it is solely responsible for fundamental quantum properties.

The Madelung equations (1), and (2) can be converted to the Schrödinger equation using the ansatz

$$\sqrt{\rho} = \Psi \exp(-iS/h) \quad (3)$$

where  $\rho$  and  $S$  being real function.

Our approach is based upon a modification of the Madelung equation, and in particular, upon replacing the quantum potential with a different Liouville feedback, Fig.1

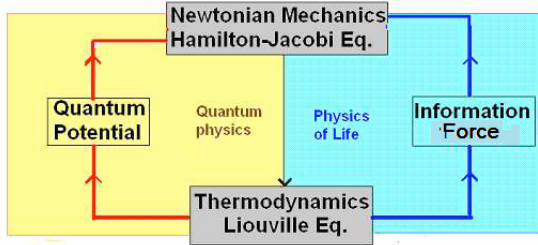


Figure 1. Classic Physics, Quantum Physics and Physics of Life.

In Newtonian physics, the concept of probability  $\rho$  is introduced via the Liouville equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{F}) = 0 \quad (4)$$

generated by the system of ODE

$$\frac{d\mathbf{v}}{dt} = \mathbf{F}[\mathbf{v}_1(t), \dots, \mathbf{v}_n(t), t] \quad (5)$$

where  $\mathbf{v}$  is velocity vector.

It describes the continuity of the probability density flow originated by the error distribution

$$\rho_0 = \rho(t=0) \quad (6)$$

in the initial condition of ODE (6).

Let us rewrite Eq. (2) in the following form

$$\frac{d\mathbf{v}}{dt} = \mathbf{F}[\rho(\mathbf{v})] \quad (7)$$

where  $\mathbf{v}$  is a velocity of a hypothetical particle.

This is a fundamental step in our approach: in Newtonian dynamics, the probability never explicitly enters the equation of motion. In addition to that, the Liouville equation generated by Eq. (7) could be nonlinear with respect to the probability density  $\rho$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \{\rho \mathbf{F}[\rho(\mathbf{V})]\} = 0 \quad (8)$$

and therefore, the system (7),(8) departs from Newtonian dynamics. However although it has the same topology as quantum mechanics (since now the equation of motion is coupled with the equation of continuity of probability density), it does not belong to it either. Indeed Eq. (7) is more general than the Hamilton-Jacoby equation (2): it is not necessarily conservative, and  $\mathbf{F}$  is not necessarily the quantum potential although further we will impose some restriction upon it that links  $\mathbf{F}$  to the concept of information. The relation of the system (7), (8) to Newtonian and quantum physics is illustrated in Fig.1.

*Remark.* Here and below we make distinction between the random variable  $v(t)$  and its values  $V$  in probability space.

### 3. Information force instead of quantum potential.

In this section we propose the structure of the force  $\mathbf{F}$  that plays the role of a feedback from the Liouville equation (8) to the equation of motion (7). Turning to one-dimensional case, let us specify this feedback as

$$F = -\frac{c_2}{\rho} \frac{\partial \rho}{\partial v} \quad (9)$$

Then Eq.(9) can be reduced to the following:

$$\dot{v} = -\frac{c_2}{\rho} \frac{\partial \rho}{\partial v} \quad (10)$$

and the corresponding Liouville equation will turn into the Fokker-Planck PDE

$$\frac{\partial \rho}{\partial t} = c_2 \frac{\partial^2 \rho}{\partial V^2} \quad (11)$$

that must satisfy the normalization constraint

$$\int_{-\infty}^{\infty} \rho dV = 1 \quad (12)$$

However as shown in [2], this constraint is satisfied: in physical space it expresses conservation of mass, and it can be easily scale-down to the constraint (12) in probability space.

The solution of Eq. (11) subject to the sharp initial condition

$$\rho = \frac{1}{2\sigma\sqrt{\pi}} \exp\left(-\frac{V^2}{4\sigma^2 t}\right) \quad (13)$$

describes diffusion of the probability density, and that is why the feedback (10) will be called a diffusion feedback.

Substituting this solution into Eq. (11) at  $V=v$  one arrives at the differential equation with respect to  $v(t)$

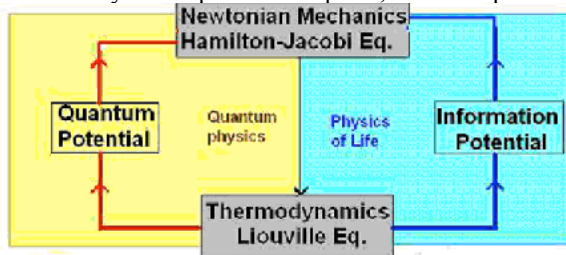
$$\dot{v} = \frac{v}{2t} \quad (14)$$

and therefore,

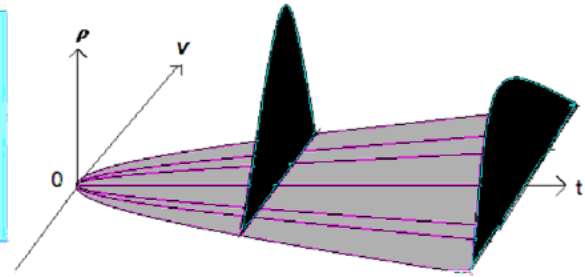
$$v = C\sqrt{t} \quad (15)$$

where  $C$  is an arbitrary constant. Since  $v=0$  at  $t=0$  for any value of  $C$ , the solution (15) is consistent with the sharp initial condition for the solution (13) of the corresponding Liouville equation (12). The solution (15) describes the simplest irreversible motion: it is characterized by the “beginning of time” where all the trajectories intersect (that results from the violation of Lipschitz condition at  $t=0$ , Fig.2), while the backward motion obtained by replacement of  $t$  with  $(-t)$  leads to imaginary values of velocities. One can notice that the probability density (13) possesses the same properties

It is easily verifiable that the solution (15) has the same structure as the solution of the Madelung equation [7], although the dynamical system (11), (12) is not quantum! The explanation of such a “coincidence” is very simple: the system (11), (12) has the same dynamical topology as that of the Madelung equation where the equation of conservation of the probability is coupled with the equation of conservation of the momentum. As will be shown below, the system (11), (12) neither quantum nor Newtonian, and we will call such systems quantum-inspired, or self-supervised.



**Figure 1 . Classical Physics, Quantum physics, and Physics of Life**



**Figure 2 Stochastic process and probability density**

Further analysis of the solution (15) demonstrates that the solution (15) is *unstable* since

$$\frac{dv}{dv} = \frac{1}{2t} > 0 \quad (16)$$

and therefore, an initial error always grows generating *randomness*. Initially, at  $t=0$ , this growth is of infinite rate since the Lipschitz condition at this point is violated

$$\frac{dv}{dv} \rightarrow \infty \text{ at } t \rightarrow 0 \quad (17)$$

This type of instability has been introduced and analyzed in [8]. The unstable equilibrium point ( $v = 0$ ) has been called a terminal attractor, and the instability triggered by the violation of the Lipschitz condition – a non-Lipschitz instability. The basic property of the non-Lipschitz instability is the following: if the initial condition is infinitely close to the repeller, the transient solution will escape the repeller during a *bounded* time while for a regular repeller the time would be *unbounded*. Indeed, an escape from the simplest regular repeller can be described by the exponent  $v = v_0 e^t$ . Obviously  $v \rightarrow 0$  if  $v_0 \rightarrow 0$ , unless the time period is unbounded. On the contrary, the period of escape from the terminal attractor (15) is bounded (and even infinitesimal) if the initial condition is infinitely small, (see Eq. (17)).

Considering first Eq. (15) at fixed  $C$  as a sample of the underlying stochastic process (13), and then varying  $C$ , one arrives at the whole ensemble characterizing that process, (see Fig. 2). One can verify that, as follows from Eq. (13), [9], the expectation and the variance of this process are, respectively

$$\bar{v} = 0, \quad \tilde{v} = 2\sigma^2 t \quad (18)$$

The same results follow from the ensemble (15) at  $-\infty \leq C \leq \infty$ . Indeed, the first equality in (18) results from symmetry of the ensemble with respect to  $v=0$ ; the second one follows from the fact that

$$\tilde{\nu} \propto \nu^2 \propto t \quad (19)$$

It is interesting to notice that the stochastic process (15) is an alternative to the following Langevin equation, [9]

$$\dot{\nu} = \Gamma(t), \quad \bar{\Gamma} = 0, \quad \tilde{\Gamma} = \sigma \quad (20)$$

that corresponds to the *same* Fokker-Planck equation (12). Here  $\Gamma(t)$  is the Langevin (random) force with zero mean and constant variance  $\sigma$ .

Thus, the emergence of self-generated stochasticity is the first basic non-Newtonian property of the dynamics with the Liouville feedback.

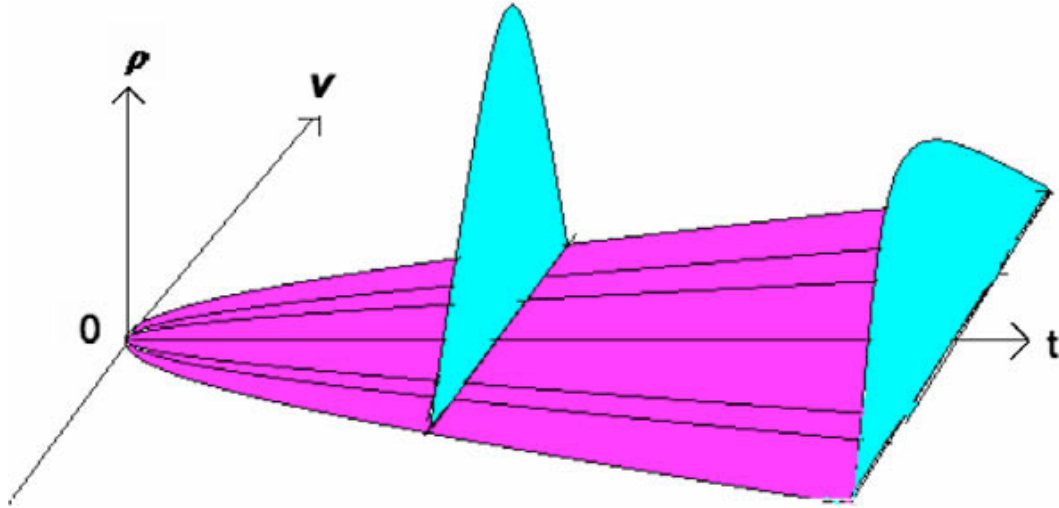
#### 4. Proposed dynamics for simulations.

Let us introduce the following inhomogeneous dynamical system with the diffusion feedback

$$\dot{\nu} = -\frac{1}{\rho} \left[ a^2 \frac{\partial \rho}{\partial \nu} - e^{-\omega t} \sum_{k=1}^{k=m} \frac{l}{2\pi k} \sin \frac{2\pi k}{l} \nu \right], \quad 0 \leq \nu \leq l, \quad t > 0, \quad (21)$$

Then the corresponding Liouville equation takes the form of an inhomogeneous parabolic equation subject to an aperiodic force

$$\frac{\partial \rho}{\partial t} - a^2 \frac{\partial^2 \rho}{\partial \nu^2} = e^{-\omega t} \sum_{k=1}^{k=m} \cos \frac{2\pi k}{l} \nu \quad (22)$$



**Fig. 3. Stochastic process and probability density.**

It should be noticed that the sums in Eqs. (1) and (2) are finite, and they do not represent even truncated Fourier expansions, while all the harmonic terms are equally powerful.

We will solve this equation subject to the following initial and boundary conditions

$$\rho(\nu, 0) = \delta(\nu - 0.5l), \quad \frac{\partial \rho}{\partial \nu}(0, t) = 0, \quad \frac{\partial \rho}{\partial \nu}(l, t) = 0 \quad (23)$$

and the normalization constraint

$$\int_0^l \rho(\xi, t) d\xi = 1 \quad (24)$$

Before writing down the solution, we will verify satisfaction of the constraint (4). For that purpose, let us integrate Eq. (2) with respect to  $v$

$$\frac{\partial}{\partial t} \int_0^l \rho d\xi - a^2 \int_0^l \frac{\partial^2 \rho}{\partial V^2} = e^{-\omega t} \sum_{k=1}^m \int_0^l \cos \frac{2\pi k}{l} \xi d\xi \quad (25)$$

As follows from the boundary conditions in (3),

$$\frac{\partial \rho}{\partial V} \Big|_{V=0} = \frac{\partial \rho}{\partial V} \Big|_{V=l} = 0, \text{ and therefore, } \int_0^l \frac{\partial^2 \rho}{\partial V^2} d\xi = 0; \text{ obviously, } \int_0^l \cos \frac{2\pi k}{l} \xi d\xi = 0 \text{ as well.}$$

Hence,  $\frac{\partial}{\partial t} \int_0^l \rho d\xi = 0$ . But, according to the initial condition in (3)  $\int_0^l \rho d\xi = \int_0^l \delta(\xi = 0.5l) d\xi = 1$  at  $t=0$ .

Therefore, the normalization constraint will be satisfied for all  $t \geq 0$

Exploiting the superposition principle for the linear equation (2), we will represent the solution as a sum of free and forced components. These components are, respectfully

$$\rho_1 = \frac{2}{l} \sum_{j=1}^{\infty} e^{-\left(\frac{\pi j}{l}\right)^2 a^2 t} \cos \frac{\pi j}{l} V \cdot \cos \frac{\pi j}{2l} + \frac{1}{l} \quad (26)$$

$$\rho_2 = \sum_{k=1}^m \left[ \int_0^t e^{-\left(\frac{\pi k}{l}\right)^2 a^2 (t-\tau) - \omega \tau} d\tau \right] \cos \frac{2\pi k}{l} V \quad \text{if} \quad \omega \neq \left(\frac{\pi k}{l}\right)^2 a^2 \quad (27)$$

$$\rho_2^* = t e^{-\omega t} \cos \frac{2\pi k}{l} V \quad \text{if} \quad \omega = \left(\frac{\pi k}{l}\right)^2 a^2 \quad (28)$$

In this paper, we will be interested only in the case (8) that represents a resonance between two aperiodic terms, namely: exponentially decaying force and exponentially decaying free motion. Indeed, the solution (8) has a well-pronounced maximum at

$$t^* = 1/\omega \quad \text{if} \quad \omega = \left(\frac{\pi k}{l}\right)^2 a^2 \quad (29)$$

while the solutions (6) and (7) are monotonously decay.

Let us now reaffirm the scenario of transition from deterministic to random state described by Eqs. (13)- (15). For that purpose, rewrite Eq. (6) in a different, but an equivalent form (based upon reflections from the boundaries)

$$\rho_1 = \frac{1}{2a\sqrt{\pi t}} \sum_{n=-\infty}^{\infty} \left\{ e^{-\frac{(V-\xi+2nl)^2}{4a^2 t}} + e^{-\frac{(V+\xi-2nl)^2}{4a^2 t}} \right\} \quad (30)$$

It can be verified that for vanishingly small times

$$\rho_1 \rightarrow \frac{1}{2a\sqrt{\pi t}} \exp\left(-\frac{V^2}{4a^2 t}\right), \quad \text{and} \quad \rho_2^* \rightarrow 0 \quad \text{at} \quad t \rightarrow 0 \quad (31)$$

and therefore, the transition scenario remains the same.

It should be noticed that prior to running Eq. (1), the analytical solution of Eq. (2) in the form of the sum of Eqs. (6), (7), and (25) is to be substituted for  $\rho$ .

Before moving to  $n$ -dimensional case, we will discuss the basic properties of the solution to Eqs. 13)-(15). Although there are many similarities to those of quantum systems, we will concentrate again on superposition since it will be essential for the described approach.

In quantum mechanics, any observable quantity corresponds to an eigenstate of a Hermitian linear operator. The linear combination of two or more eigenstates results in quantum superposition of two or more values of the quantity. If the quantity is measured, the projection postulate states that the state will be randomly collapsed onto one of the values in the superposition (with a probability proportional to the square of the amplitude of that eigenstate in the linear combination). Let us compare the behavior of the model of the proposed system from that viewpoint. As follows from Eq. (15), all the particular solutions intersect at the same point  $v=0$  at  $t=0$ , and that leads to non-uniqueness of the solution due to violation of the Lipschitz condition. Therefore, the same initial condition  $v=0$  at  $t=0$  yields infinite number of different solutions forming a family (15); each solution of this family appears with a certain probability guided by the corresponding Fokker-Planck equation, Fig. 4.

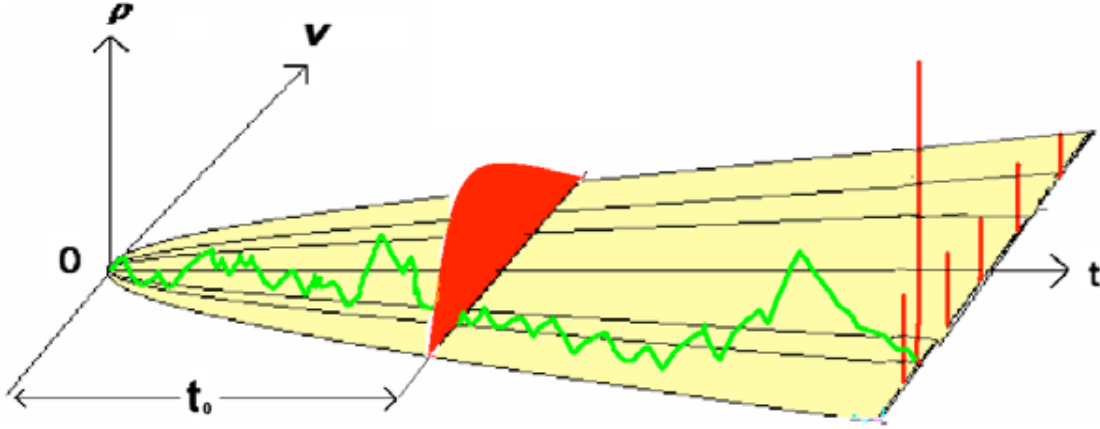


Figure 4. Resonance in the probability space

Turning to  $n$ -dimensional case we have

$$\dot{v}_i = -\frac{1}{\rho} \left[ a^2 \frac{\partial \rho}{\partial v_i} - e^{-\omega t} \sum_{k=1}^{k=m} \frac{l}{2\pi k} \sin \frac{2\pi k}{l_i} v_i \right], \quad 0 \leq v_i \leq l_i, \quad (32)$$

$$\frac{\partial \rho}{\partial t} - a^2 \sum_{i=1}^n \frac{\partial^2 \rho}{\partial V_i^2} = e^{-\omega t} \sum_{i=1}^n \sum_{k=1}^m \cos \frac{2\pi k}{l_i} V_i, \quad (33)$$

$$i = 1, 2, \dots, n, \quad k = 1, 2, \dots, m.$$

Eq. (13) has the following eigen-values of decay

$$\omega_{j_{k_1}, \dots, j_{k_n}} = (a^2 \pi^2 \sum_{i=1}^n \frac{j_{k_i}^2}{l_i^2}), \quad j = 1, 2, \dots, n, \quad i = 1, 2, \dots, n, \quad k = 1, 2, \dots, m \quad (34)$$

If the excitation  $\omega$  in Eqs. (12) and (13) are selected as following

$$\omega = \omega_{j_{k_1} \dots j_{k_n}} = (a^2 \pi^2 \sum_{i=1}^{i=n} \frac{j_{k_i}^2}{l_i^2}), j = 1, 2, \dots, n, i = 1, 2, \dots, n, k = 1, 2, \dots, m \quad (35)$$

it will generate resonance with the eigen-value (14), and the corresponding “decay” will dominate over the rest of decays; in terms of Eq. (13) this means that the probability density  $\rho$  will tend to its maximum at

$$t^* = 1 / \omega_{j_{k_1} \dots j_{k_n}} = (a^2 \pi^2 \sum_{i=1}^{i=n} \frac{j_{k_i}^2}{l_i^2})^{-1}, \quad (36)$$

along that trajectory which is the “winning” solution of the system (12), Fig. 25. The value of this maximum is irrelevant, but its location is important: it is given by the following values of the coordinates

$$v_i^* = v_i(t^*), i = 1, 2, \dots, n. \quad (37)$$

**4. Resonance-based hetero-associated memory.** The classical formulation of associative memory is: store a set of patterns in such a way that when presented with a new pattern, the network responds by producing whichever one of the stored patterns most closely resembles the stored one. The correct response follows training on a set of example. The algorithm for hetero-associated memory proposed in this paper is the following. Consider an unsorted data-base consisting of  $n^n$  items labeled with a string of numbers  $j_1, j_2, \dots, j_n$  as shown in Eq.(14) for  $n=m$ . Obviously a label includes permutations of these numbers. Turning to Eqs. (12), notice that each solution to this system can be labeled similarly if the winning solution in Eqs. (13) has its maximum at a point with the coordinates  $v_1^*, v_2^*, \dots, v_n^*$  defined by Eqs (16). Then one can introduce the forced excitation defined by Eq. (15) that provides the resonant solution of Eq. (13), and as a result, the coordinates  $v_1^*, v_2^*, \dots, v_n^*$  of this maximum will represent the address of the item in question. According to Eq. (15), the number of possible values of forced excitations  $\omega$  providing required resonances is equal to  $n^n$ , and that is exactly the number of the items to be retrieved. Therefore, each item can be retrieved by the corresponding resonance with the forced excitation (having the values from the set (15)) with the probability that dominates over the probabilities for wrong addresses to occur. Strictly speaking, a non-resonance solution has a smaller, but non-zero probability to occur; than by a few number of Bernoulli trials, the most probable solution can be found.

Let us now briefly review the procedure of the retrieval. Assume that the label of the item to be found is  $\omega_{j_{k_1} \dots j_{k_n}}$ . The first step is to write down the analytical solution to Eq. (13) that consists of free and forced motions as in the one-dimensional case:

$$\rho = \rho_1 + \rho_2^* \quad (38)$$

Here

$$\rho_1 = \sum_{j=1}^{\infty} C_j e^{-\left(\frac{\pi j}{l}\right)^2 a^2 t}, \quad (39)$$

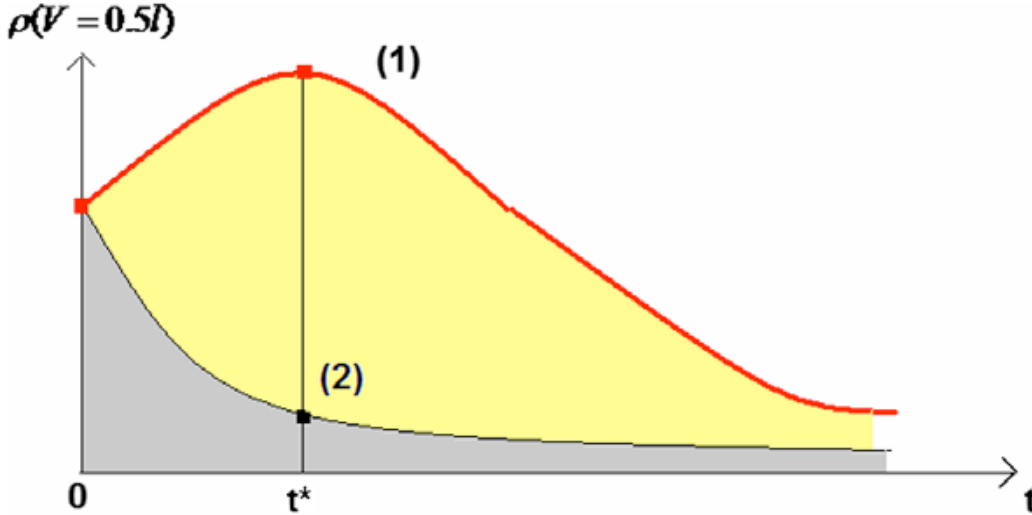
$$\rho_2^* = t e^{-\omega^* t} \sum_{i=1}^n \sum_{k=1}^m \cos \frac{2\pi k}{l_i} V_i \quad (40)$$

where  $C_j$  are constants to be found from the initial conditions, and

$$\omega^* = \omega_{j_{k_1} \dots j_{k_n}} = (a^2 \pi^2 \sum_{i=1}^{i=n} \frac{j_{k_i}^2}{l_i^2}), j = 1, 2, \dots, n, i = 1, 2, \dots, n, k = 1, 2, \dots, m \quad (41)$$



The second step is to substitute the solution (1) into Eq. (2.12). The third step is to run the system (2.12), measure the values of  $v_i$  at  $t = 1/\omega^*$  and obtain the address of the item in the form of a string of coordinates  $v_1^*, v_2^*, \dots, v_n^*$ , Fig. 5.

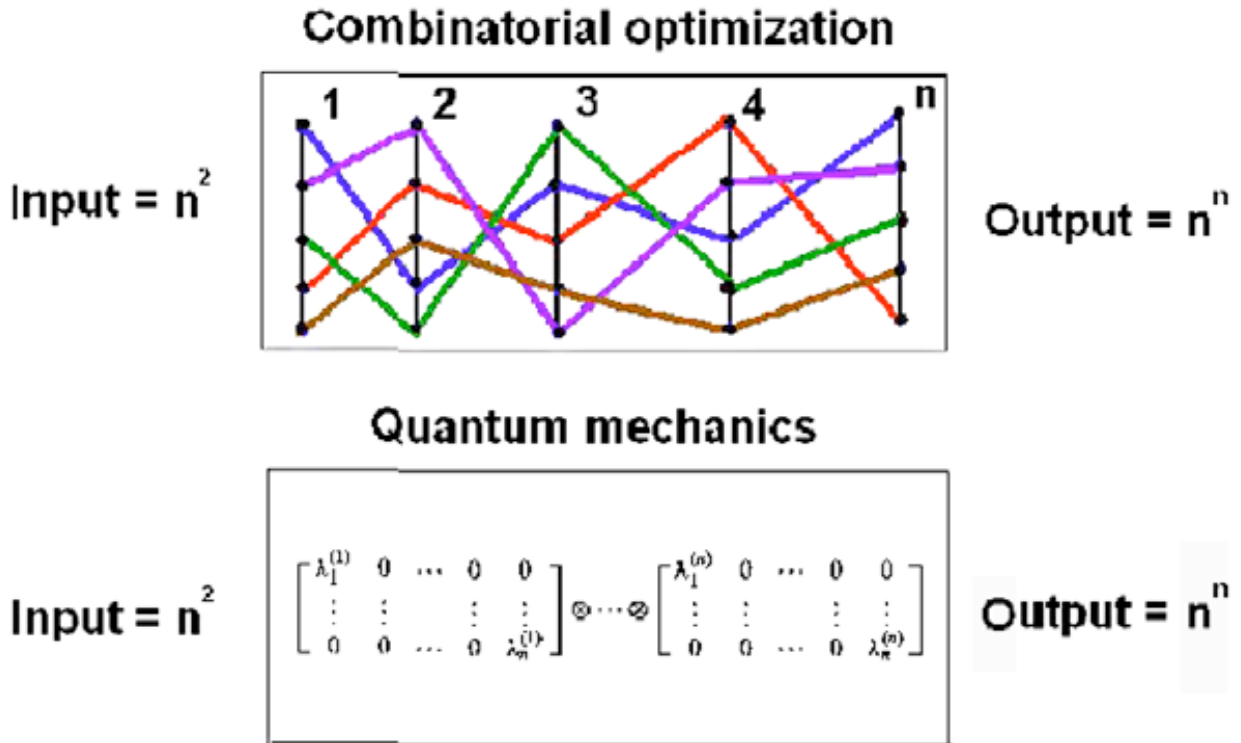


**Figure 5. Maximum probability, selected (1) and not selected (2) items.**

It should be noticed that the capacity of the proposed hetero-associated memory is of order  $O(n^n)$  i.e. exponential with respect to its dimensionality  $n$ , while all the resources providing its implementation are of order  $O(n)$ , i.e. polynomial since the number of equations in the system (2.12) is  $n$ , and the number of terms in the analytical solution to Eq.(2.13) (to be substituted into Eqs. (2.12)) are of the order  $O(n)$  as well. Indeed, the infinite sum in Eq. (2) converges very fast to equal distribution of the probability density, and practically, only the forced component of the solution represented by Eq. (3) is important, and this component contains  $O(n^2)$  number of terms.

## 5. Conclusion.

The basic idea of this paper is to create a dynamical system that would preserve superposition of random solutions, while allowing one to measure its state variables using classical methods. In other words, such a hybrid system would reinforce the advantages and minimize limitations of both quantum and classical aspects. The basic properties of these systems have been analyzed in [8]. It has been shown there that along with preservation of superposition, such an important property of quantum systems as direct-product-decomposability here is *lost*. Let us recall that the main advantage of this property in terms of quantum information is in blowing up an input of a polynomial complexity into an output of exponential complexity, with no additional resources required. This property was successfully exploited in [3] for solving an NP-complete problem with quantum resonance, Fig.6.



**Figure 6, Mapping combinatorial optimization to quantum mechanics.**

The greatest challenge of our approach in this paper was in finding a “replacement” for the fundamental property of the Schrödinger equation that is direct-product decomposability of the unitary operator. It turns out that eigen-values of linear parabolic PDE possess similar property. Indeed, consider a linear  $n$ -dimensional parabolic PDE subject to boundary conditions. Then the eigen-values corresponding to each variable form a sequence of monotonously increasing positive numbers  $\lambda_1^{(1)}, \dots, \lambda_n^{(n)}$ . However, each linear combination of these eigen-values represents another eigen-value of the solution, and that is the same “combinatorial explosion” that is illustrated in [3]. Due to that property, for each  $n$ -string-number label, one can find an excitation force that activates the corresponding eigen-value. The second challenge was to satisfy a global (normalization) constraint imposed upon the probability density (in addition to boundary conditions). That was achieved via a special form of the excitation force.

Thus, a retrieval of stored items from an exponentially large unsorted database is performed by quantum-inspired resonance using polynomial resources due to quantum-like superposition effect.

There is another interesting aspect of the dynamical system introduced above.

The proposed model illuminates the “border line” between living and non-living systems. The model introduces an intelligent particle that, in addition to Newtonian properties, possesses the ability to process information. The probability density can be associated with the *self-image* of the intelligent particle as a member of the class to which this particle belongs, while its ability to convert the density into the information force - with the *self-awareness* (both these concepts are adopted from psychology). Continuing this line of associations, the equation of motion (such as Eq (10)) can be identified with a motor dynamics, while the evolution of density (see Eq. (11)) –with a mental dynamics. Actually the mental dynamics plays the role of the Maxwell sorting demon: it rearranges the probability distribution by creating the information potential and converting it into a force that is applied to the particle. One should notice that mental dynamics describes evolution of the whole class of state variables (differed from each other only by initial conditions), and that can be associated with the ability to generalize that is a privilege of intelligent

systems. The association of these systems with livings implies that human could be more effective in solving NP problems than “rational” tools of AI.

Thus, a new kind of dynamics for simulations based upon quantum-classical hybrid is discussed. The model is represented by a modified Madelung equation in which the quantum potential is replaced by different, specially chosen potentials. As a result, the dynamics attains both quantum and classical properties: it preserves superposition and entanglement of random solutions, while allowing one to measure its state variables using classical methods. Such an optimal combination of characteristics is a perfect match for quantum-inspired information processing. The retrieval of stored items from an exponentially large unsorted database is performed by quantum-inspired resonance using polynomial resources due to quantum-like superposition effect.

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