An unusual conjecture on primes involving concatenation and repunits

Abstract. In this paper I make the following conjecture: for any k positive integer there exist an infinity of primes p such that the number q, obtained concatenating (p – k) with p then, repeatedly k times, with the digit 1, is prime. Examples: for k = 1, there exist p = 19 such that q = 18191 is prime; for k = 2, there exist p = 5 such that q = 3511 is prime; for k = 3, there exist p = 7 such that q = 47111 is prime; for k = 4, there exist p = 37 such that q = 33371111 is prime; for k = 5, there exist p = 11 such that q = 61111111 is prime; for k = 6, there exist p = 17 such that q = 1117111111 is prime.

Conjecture :

For any k positive integer there exist an infinity of primes p such that the number q, obtained concatenating (p – k) with p then, repeatedly k times, with the digit 1, is prime. Examples: for k = 1, there exist p = 19 such that q = 18191 is prime; for k = 2, there exist p = 5 such that q = 3511 is prime; for k = 3, there exist p = 7 such that q = 47111 is prime; for k = 4, there exist p = 37 such that q = 33371111 is prime; for k = 5, there exist p = 11 such that q = 61111111 is prime; for k = 6, there exist p = 17 such that q = 1117111111 is prime.

The sequence of primes q for k = 1:

: 10111, 18191, 46471, 60611, 78791 (...)
obtained for p = 11, 19, 47, 61, 79 (...)

The sequence of primes q for k = 2:

: 3511, 5711, 272911, 353711, 414311, 454711, 515311, 697111, 777911, 10510711, 11111311, 14915111, 16516711, 17717911, 17918111, 18919111, 19719911 (...)
obtained for p = 5, 7, 29, 37, 43, 47, 53, 71, 79, 107, 113, 151, 167, 179, 181, 189, 199 (...)

The sequence of primes q for k = 3:

: 25111, 47111, 3841111, 4043111, 5659111, 8083111, 8689111, 100103111, 104107111, 106109111, 176179111, 178181111, 190193111 (...)
obtained for p = 5, 7, 41, 43, 59, 83, 89, 103, 107, 109, 179, 181, 193 (...)

The sequence of primes q for k = 4:

: 33371111, 39431111, 57611111 (...)
obtained for p = 37, 43, 61 (...)

The sequence of primes q for k = 5:

: 61111111, 485311111, 66711111, 747911111, 9610111111, 10811311111, 13213711111, 17618111111, 19419911111 (...)
obtained for p = 11, 53, 67, 79, 101, 113, 137, 181, 199 (...)

The sequence of primes q for k = 6:

: 1117111111, 2329111111, 101107111111, 133139111111 (...)
obtained for p = 17, 29, 107, 139 (...)

The sequence of primes q for k = 7:

: 64711111111, 90971111111, 1021091111111, 1241311111111, 1841911111111, 1901971111111 (...)
obtained for p = 71, 97, 109, 131, 191, 197 (...)

The sequence of primes q for k = 8:

: 31111111111, 233111111111, 455311111111, 818911111111, 9510311111111, 12313111111111, 14915711111111, 16517311111111, 18519311111111, 19119911111111 (...)
obtained for p = 11, 31, 53, 89, 103, 131, 157, 173, 193, 199 (...)

The sequence of primes q for k = 9:

: 2231111111111, 6473111111111, 92101111111111, 158167111111111 (...)
obtained for p = 31, 73, 101, 167(...)

The sequence of primes q for k = 10:

: 9191111111111, 21311111111111, 49591111111111, 911011111111111, 1471571111111111, 1831931111111111, 1871971111111111, 1891991111111111 (...)
obtained for p = 19, 31, 59, 101, 157, 193, 197, 199 (...)

Note: all the possible primes q are listed above, for k up to 10 and p up to 199.