Conjectures on (q+2)]c[n]c[q and (q-4)]c[n]c[q where n is equal to 1]c[2]c[...]c[p and p, q are primes

Abstract. In this paper I make the following two conjectures: (I) let n be a number obtained concatenating the positive integers from 1 to p, where p prime of the form 6*k – 1 (e.g. n = 12345 for p = 5); there exist an infinity of primes q of the form 6*h + 1 such that the number r obtained concatenating q + 2 with n then with q is prime (e.g. for n = 12345 there exist q = 19 such that r = 211234519 is prime); (II) let n be a number obtained concatenating the positive integers from 1 to p, where p prime of the form 6*k – 1; there exist an infinity of primes q of the form 6*h + 1 such that the number r obtained concatenating q - 4 with n then with q is prime (e.g. for n = 12345 there exist q = 37 such that r = 331234537 is prime). I use the operator "]c[" with the meaning "concatenated to".

Conjecture I:

Let n be a number obtained concatenating the positive integers from 1 to p, where p prime of the form 6*k – 1 (e.g. n = 12345 for p = 5); there exist an infinity of primes q of the form 6*h + 1 such that the number r obtained concatenating q + 2 with n then with q is prime (e.g. for n = 12345 there exist q = 19 such that r = 211234519 is prime).

The sequence of primes r for n = 12345:

: 211234519, 691234567, 991234597, 21312345211, 27912345277, 41112345409, 43512345433, 44112345439, 46512345463, 63312345631, 67512345673, 71112345709, 90912345907, 96912345967, 99312345991 (...)

obtained for q = 19, 67, 97, 211, 277, 409, 433, 439, 463, 631, 673, 709, 907, 969, 991 (...)

The sequence of primes r for n = 1234567891011:

: 912345678910117, 1411234567891011139, 3151234567891011313, 3811234567891011379, 5491234567891011547, 6091234567891011607, (...)

obtained for q = 7, 139, 313, 379, 547, 607 (...)

The least r for n = 1234567891011121314151617:

: 1051234567891011121314151617103.

Conjecture II:

Let n be a number obtained concatenating the positive integers from 1 to p, where p prime of the form 6*k – 1; there exist an infinity of primes q of the form 6*h + 1 such that the number r obtained concatenating q - 4 with n then with q is prime (e.g. for n = 12345 there exist q = 37 such that r = 331234537 is prime).

The sequence of primes r for n = 12345:

: 331234537, 931234597, 17712345181, 19512345199, 30312345307, 36312345367, 36912345373, 40512345409, 41712345421, 45312345457, 57312345577, 82512345829, 84912345853, 87312345877, 87912345883 (...)

obtained for q = 37, 97, 181, 199, 307, 367, 373, 409, 421, 457, 577, 829, 853, 877, 883 (...)

The sequence of primes r for n = 1234567891011:

: 1231234567891011127, 1471234567891011151, 1951234567891011199, 3271234567891011331, 4171234567891011421, 6571234567891011661, 8191234567891011823, 9631234567891011967 (...)

obtained for q = 127, 151, 199, 331, 421, 661, 823, 967 (...)

Note:

A wider statement would not require for the number q to be prime but number of the form 6*k + 1; in this case, the sequences from Conjecture 1 would contain also the numbers:

24912345247, 25512345253, 45312345451, 48312345481, 90312345901, 90912345907, 93123456789101191, 5131234567891011511, 5851234567891011583, 6811234567891011679, 8431234567891011841, 8731234567891011871 (...)

and the sequences from Conjecture 2 would contain also the numbers:

871234591, 18312345187, 25512345259, 47712345481, 63312345637, 90912345913, 1831234567891011187, 2551234567891011259, 3991234567891011403, 6991234567891011703, 8431234567891011847, 9391234567891011943 (...).

Observation:

Note the remarkable symmetry between the sequences from the two conjectures: up to q = 1000, for n = 12345, 15 primes q in the sequence from the first conjecture, 15 primes q in the sequence from the second conjecture; for n = 1234567891011, 6 primes q in the sequence from the first conjecture, 8 primes q in the sequence from the second conjecture; for non-primes satisfying the statements, for n = 12345, 6 in the sequence from the first conjecture, 6 in the sequence from the second conjecture; for n = 123456789101112, 6 in the sequence from the first conjecture, 6 in the sequence from the second conjecture.