Beyond Quantum Fields: A Classical Fields Approach to QED

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A classical field theory is introduced that is defined on a tower of dimensionally increasing spaces and is argued to be equivalent to QED. The domain of dependence is discussed to show how an equal times picture of the many coordinate space gives QED results as part of a well posed initial value formalism. Identical particle symmetries are not, a priori, required but when introduced are clearly propagated. This construction uses only classical fields to provide some explanation for why quantum fields and canonical commutation results have been successful. Some old and essential questions regarding causality of propagators are resolved. The problem of resummation, generally forbidden for conditionally convergent series, is discussed from the standpoint of particular truncations of the infinite tower of functions and a two step adiabatic turn on for scattering. As a result of this approach it is shown that the photon inherits its quantization $\hbar\omega$ from the free lagrangian of the Dirac electrons despite the fact that the free electromagnetic lagrangian has no $\hbar$ in it. This provides a possible explanation for the canonical commutation relations for quantum operators, $[\hat{P},\hat{Q}] = i\hbar$, without ever needing to invoke such a quantum postulate. The form of the equal times conservation laws in this many particle field theory suggests a simplification of the radiation reaction process for fields that allows QED to arise from a sum of path integrals in the various particle time coordinates. A novel method of unifying this theory with gravity, but that has no obvious quantum field theoretic computational scheme, is introduced.

1 Introduction

Quantum field theory, in some ways, marks the ultimate state of our understanding of physics. In its computational exactness, it can be thrilling yet its conceptual grounding is very unsatisfactory. Field theory has its origins in the 1920's and 1930's when attempts to include particle creation and the quantization of the photon necessitated a larger mathematical structure [13, 17]. Fock space seemed to have sufficient features to encompass the intrinsic quantum and particle number variable features. The ladder operators of the harmonic oscillator could be formally modified to give an algebra that allowed these various particle number spaces to interact. Different attempts to generate an equation of motion and find transition rates led to various formal procedures. Classical lagrangians were varied in a formal manner with "second quantized" operators in approaches by Schwinger and Tomanaga and systematic procedures to handle the divergent terms were introduced [15, 17]. Feynman gave a very intuitive approach using path-integrals that was put into a formal structure by Dyson. This approach has gained prominence due to its ease of organizing the terms of the expansion.

Quantum mechanics is the quantum theory of fixed particle number systems. Certain quasi-classical approaches made the treatment of radiative decay possible without QED at low energies. Nevertheless, even in this low energy domain, the theory had lingering conceptual problems. Measurement and the "collapse of the wavefunction" led to paradoxes that have spawned an enormous literature [7]. Decoherence is a popular "explanation" of these effects but these tend to rely on assumptions that are just pushed off to other parts of the analysis [16]. The Born interpretation, due to its simplicity and historical inertia, still dominates most treatments of classical-quantum interactions. Some may object that there are now ways to treat measurements independently of the Born interpretation to handle to new sorts of quantum nondemolition measurements [11] but these ultimately involve other ad hoc statistical assumptions. Quantum statistical mechanics has never found any solid conceptual footing despite the frequent success of its formalism in describing thermodynamic behavior and providing numerical results. This problem is often given a short comment in books on the subject and little progress has been made. Ultimately, an initial data formulation approach must resolve all of these issues in terms of the dynamical equations and evidence for the kinds of initial data that is physically relevant.

The quantum field theory approach to quantum mechanics is on a solid footing. Even though operators may change the particle number, it is always changed back at every order in the expansion. One may show [15] that this gives an exact isomorphism with the Schrödinger, Heisenberg and interaction picture versions of QM. This leads to the Feynman path integral approach to quantum mechanics which, while equivalent, generally gives absurdly difficult derivations of results compared to other means. In contrast, regularization of the path integral has never had a very solid mathematical foundation but applying the theory in a "standard" fashion gives correct results. The main uses of QFT is in relativistic physics, quasiparticle motions in condensed matter and in the "Wick rotated" form which converts temporal evolution to a high temperature expansion of the thermodynamic potentials. The correspondence of QFT in the case of quasiparticle evolution to that of Schrödinger evolution is itself challenging [2]. Fundamentally, one must give a description of the many-body wavefunction's excited states to give such a correspondence. This has led to the popularity of Green's function methods in condensed matter physics since it sidesteps this difficult work and leads directly to calculations. The validity of the derivation of the Kubo formula [8] has been extensively criticized [9] but it has, nevertheless, proved to be of great use over a broader range of phenomena than should be expected.

Given that no true classical-quantum correspondence of objects is known, it is unclear when one should impose classical structures (like hydrodynamics) on the system and when to extract certain properties (like viscosity) by quantum means. This is of particular interest in the study of ultracold gas dynamics [5] and superfluid Helium. There are popular and sometimes successful approaches for doing this but it is never clear that they must follow from the true many body dynamical theory or that we have simply made enough assumptions to stumble on to the tail of a correct derivation, the first, and correct part of which is a mystery to us. The general vagueness and nonspecificity of the subject allows theorists great freedom to generate calculations that then can be compared with experimental or Monte-Carlo data for affirmation of which ones to keep. This very freedom should undermine our reasons for faith in our theory and intuition. Instead it, together with professional publication demands, seems to create a selective pressure in favor of optimism and credulity on the part of practitioners and an air of mystical prophecy of our physics fathers and those who derive experimentally matching results.

In relativistic field theory, where particle creation is important, there are additional problems. Renormalization is necessary because of the local interactions of particles and fields. Classical physics certainly has such a problem and the radiation reaction problem of classical electrodynamics still has unanswered questions [14]. The series derived from QFT in the relativistic and quasiparticle cases tend to be asymptotic series and conditionally converging. Nonetheless, it seems very important to resum these series over subsets of diagrams to get desired approximations and Green's functions that are analytically continued to give the propagator pole structure corresponding to masses and lifetimes of resonances. The path integral itself has too large a measure to give a rigorous derivation. Regularization procedures, like putting the integrals on a Euclidean lattice for computation, length scale cutoffs, Wilson momentum cutoffs, dimensional regularization and others, are introduced to get finite results [13]. Of the conceptual problems facing quantum theory, renormalization will be shown to be a rather modest one. Justifying the use of resummation will be much more serious.

The Schrödinger approach to quantum mechanics has a special place. Questions of causality and geometric intuition are most naturally discussed in a real space picture. The diffusive nature of this equation is problematic but vanishes in the relativistic limit of the Dirac equation. Unfortunately, this is exactly where particle creation effects become important. In relativistic classical field theory, all causality questions are moot. The structure of the equations ensures that it is valid. Other advantages of classical fields are that they are deterministic, propagate constraints exactly, give clearly obeyed conservation laws and introduce a specificity that allows all philosophical questions and thought experiments to be resolved through examination of their own mathematically consistent structure. In some cases, like relativity, our intuition may need to be updated but how this is to be done is made clear through such examples. QFT clearly works at the level of computation for many problems. This makes one believe that maybe our precursory arguments and descriptions leading to those calculations are fine and merely need elaboration. Given the success of so many calculations, it comes as a great disappointment that almost any interacting field theory is inconsistent [6].

Beyond these problems, the use of one particle lagrangians and couplings that get promoted to many body interacting theory through canonical quantization or propagator methods lead to a kind of conceptual disconnect that makes the solid implications of classical field theory, e.g. Noether's Thm. and conservation laws, unclear. These conservation laws can be formally defined by a correspondence of operators and checked but are no longer strict implications of the symmetries of a lagrangian. The symmetries of one-particle systems themselves require a more explicit definition in the many body case where multiple coordinate labels of the wavefunction Ψ can describe independent motions but the current state of theory does not present a solid enough foundation to show how and when to make this manifest as an important symmetry. The meaning of a "propagator" in classical theory is simple yet it is often not appreciated that the full reality described by a Klein-Gordon field is not necessarily contained in the support of ϕ in a given constant time slice due to its second order nature. This is often lost in confusing discussions in terms of positive and negative energy components. This will be resolved for both KG and Dirac equations in the classical and quantum cases and clear up any apparently acausal effects without reference to commutation relations and formal measurement.

It is an emotionally identical state to feel that something is wrong but unclear, lacking sufficient specificity, or that we simply don't understand. The formal character of quantum field theory has produced a useful computational tool but left enough vague and ill-defined that there is plenty to improve. It is interesting that it has been proved that no interacting quantum field theory is consistent [6]. People typically shrug this off as with the other conceptual troubles in quantum theory. At some point people have to generate work or do something else but eventually formal approaches are destined to lose productivity. Beyond that is the lack of satisfaction that one really understands what one is doing. It is very common in physics to find clever solutions or long derivations that turn out to be flawed. Classical systems exist as well posed initial value problems so that they can be tackled from many angles: perturbation theory, conservation laws, idealized systems... A well posed such problem the describes field theory would doubtless open some new doors.

The foregoing was to show that some new approach to the reality described by QFT is justified. In doing so, QFT's successes are the best guide to start. In the following we will seek a well-posed classical relativistic theory over a tower of spaces of increasing dimension that will have some loose correspondence with Fock space. This will not be guided by the computational convenience it affords but logical and mathematical consistency and specificity. Since we are taking the point of view that the fields are valid at all time (so implicitly have an "emergent measurement theory" at work) we don't need to think of "particles" as something more than a label for some axes in our higher dimensional space. It will turn out that we will need a larger encompassing structure than field theory on Fock space to describe the phenomenology of QFT adequately. From this we can derive QFT phenomenology in a suitable limit and use its rigid structure to answer conceptual questions in a more convincing fashion. Since this will strictly be a deterministic covering to QFT we consider for it a new name, *deterministic wave mechanics*, (DWM). It's purpose is to elucidate an explanation of why quantum field theory works and give a framework for modifications, like the inclusion of gravity, that may have a well posed structure but not exist in the frame work of QFT itself. In the following we will use QED as a particular case but the generalizations will be evident.

2 Overview

The goal here is to introduce set of many particle number spaces where energy, mass, charge, probability, stress... can travel between the spaces at two-body diagonals. This will necessitate we make sense of multiple time labels and have a well defined set of initial data and regions where interacting fields can consistently evolve in this high dimensional many-time structure. Because there will be no "field operators" there will be no need for a translationally invariant vacuum to build particles from. If we start with N electrons, the number of photons may increase and electron-positron pairs can appear but the net charge is the same in every space where nonzero amplitude exists. This eliminates the basis of Haag's theorem and its contraction.

Firstly, we will introduce separate equations of motion and particle labels for electrons and positrons. The amplitude of each of these will be positive locally and interactions will not change this. Negative norm states exist but are never utilized by the system. This is due to a symmetry of the dynamical equations not a constraint akin to the Gupta-Bueler formalism. The photon fields will be described by A and $\dot{A}$ labels so that, as spin labels bifurcate the number of fields, so do photon labels. An important distinction here with QFT is that there will be nonzero functions in the "tower" of fields that have zero norm. For example, in a one-electron zero-photon system, $\psi(x)$ has full norm while the function in the one-electron and Φ_{em} sector is nonzero. The norm of electromagnetic fields will not be a simple square of the function amplitude but a function of its amplitude and derivatives in such a way that only if there are imaginary parts will it contribute to the "norm." Thus our tower of functions will involve many nonzero ones that have no norm and the electromagnetic field can pick up some complex components. This suggests that our theory may have a larger configuration space than QFT. A explanation of QFT may arise from this by thinking of QFT tracking the flow of norm and other conserved quantities through the system while ignoring these higher nonzero functions and, in some gauges, treating them as constraints.

Once we have a suitable configuration space, equations of motion and reasonable sense of "future" we seek a mapping of QED into the space. The tools used to treat scattering in QFT involve "adiabatic turn on/off" of the interactions, regularization and renormalization. Typically we sum over special subsets of diagrams and adjust the "bare" parameters to get the right free behavior for these modifications. The regularization can be easily dealt with as in classical theory by assuming finite size effects. This is essential for the radiation reaction. It is still unclear how QFT can treat the radiation reaction adequately so this alone may introduce new physics. The sort of initial data with interactions already "on" requires we work with a truncated set of the total space on interactions. Implicit here is that the bare parameters be chosen to give the right momenta and other observable for the "free" particles (in the sense that they are ballistic not that interactions are turned off). The structure of the theory allows us to adjust couplings and interactions with far more freedom than QFT for perturbative purposes. Resummation has always been the most dubious aspect of QFT. Conditionally convergent series should not be rearranged so having a limiting method to make sense of this is an important improvement. In this paper we will not prove an isomorphism with QED, and, given the inconsistencies in the theory, this may be for the best. A foundation is laid with some arguments for its ability to generate QED results, but given the scope of the subject, much more work remains than can be done in this one paper.

Finally we will discuss a method of combining this with gravity by promoting the γ matrices themselves. This will require some extension of most fields to allow dual pairs so that the quadratic lagrangians become bilinear. Such a method is distinct from vierbein approaches and works on a flat background. Some important extensions of the notion of gauge freedom arise here and the "reality" of the particles can be shown to move causally yet not be definable in any obvious fashion in terms of the fields.

3 The Configuration Space

3.1 Dirac Fields

In the early days of the Dirac equation, interpretations have evolved from a proposed theory of electrons and protons to that of electrons and positrons with positrons as "holes" in an infinitely full electron"sea" to that of electrons with positrons as electrons moving "backwards in time." The first interpretation failed because the masses of the positive and negative parts are forced to be equal. The second was introduced out of fear that the negative energy solutions of the Dirac equations would allow a particle to fall to endlessly lower energies. The last was introduced as a computational tool. The negative mass solutions were to be reinterpreted as positive mass with negative charge. Necessary fixes to this idea are subtly introduced through the anticommutation relations and the algebraic properties of the vacuum ground state used in the field theory approach.* If we are going to seek a classical field theory approach to this problem we need another mechanism.

In a universe containing only electrons and positrons we require the fields Ψ_e, Ψ_p, Ψ_{ee}, $\Psi_{ep}\ldots$ where the number of spinor and coordinate labels is given by the number of particle type labels as in $\Psi_{ee} = \Psi_{ee}^{ab}(x^\mu, y^\nu)$. The lagrangian density must distinguish electrons and positron by their charge only. Since we have not included any photons yet and we have asserted that positive norm will be enforced on the initial data (and suggested it will be propagated even in the interacting case) these will have equations of motion that follow from the related one particle lagragians

$$\mathcal{L}_e = i\hbar\bar{\psi}_e\gamma^\mu\nabla_\mu\psi_e - m\bar{\psi}_e\psi_e \tag{1}$$
$$\mathcal{L}_p = i\hbar\bar{\psi}_p\gamma^\mu\nabla_\mu\psi_p + m\bar{\psi}_p\psi_p \tag{2}$$

The sign of the charge will be discussed when the electromagnetic field is added but, at this point, could be chosen either $\pm q$. We confine ourselves to the Dirac representation and the positron lagrangian is chosen so that its rest positive energy contribution is in the v component of the spinor $\binom{u}{v}$ unlike the the electron case. We will only be interested in initial data with positive energy. Later we will see that this is consistent with the kinds of creation and annihilation operator couplings in QED that allows positrons to have positive energy. We still need a lagrangian for our many particle wavefunctions. In this noninteracting case, we consider this to be built of a sum of the one particle ones so that the lagrangian of the two electron field $\Psi_{ab}(x^\mu, y^\nu)$ is

$$\mathcal{L}_{ee} = i\hbar\Psi^*_{af}\gamma^0_{ab}\gamma^\mu_{bc}\nabla_\mu\Psi_{cf} - m\Psi^*_{ab}\gamma^0_{ac}\Psi_{cb} \tag{3}$$
$$+\, i\hbar\Psi^*_{fa}\gamma^0_{ab}\gamma^\nu_{bc}\nabla_\nu\Psi_{fc} - m\Psi^*_{ab}\gamma^0_{ac}\Psi_{cb} \tag{4}$$

where we have explicitly written out the indices associated with spinor labels and coordinates and the summation convention is assumed for all repeated indices. The action is to be computed by integrating over a region in the 2-fold Lorentz space $\mathbb{R}^4\times\mathbb{R}^4$. Variation of the function can be done holding it constant along y and x respectively leading to the usual equations of motion along the separate time coordinates t^x, t^y for a product function $\Psi = \psi_1(x^\mu)\psi_2(y^\nu)$.

From a dynamical point of view, we are mostly interested in the cases where the fields are all evaluated at equal times. However we should ask what it even means to evaluate a function at two different times. When is this even meaningful? If we specify $\Psi(x_1^\mu, x_2^\nu)$ at $t_1 = t_2$ we desire to know into what region of this many-time future we should expect a solution. Further explanation of the equal time evolution is discussed in Sec. 3.4.

Considering free propagators we can evolve the data from (x_1, x_2) in the t_1 direction indefinitely and similarly for t_2. The domain of dependence is then the union of the two backwards light-cones $|x_1' - x_1| < c(t_{1'} - t_1)$ and $|x_2' - x_2| < c(t_{2'} - t_2)$. Interactions will allow free evolution for such a function except on 2-body diagonals $x^\mu = y^\nu$. When these cones intersect these regions sources and sinks with other particle number functions will arise. When these produce a net change in amplitude versus simply a potential force remains to be seen. Furthermore, it is still unclear that we can derive the static electromagnetic force effects from such a restricted local interaction. This will be explained later but first we investigate the case of free photons.

3.2 Photons

The classical electromagnetic field is a real vector field A^μ. For our many body generalization as $\Psi^\mu_a \sim \psi_a(x)A^\mu(y)$ we will have, generally nonseparable, combinations of electromagnetic and electron fields so making the assignment of which is "real" is ambiguous. We will find that phase differences between these fields on the many body diagonals give sources and sinks of amplitude from one particle number space to another. Firstly, let us consider the classical electromagnetic field which we can, loosely, think of as a single particle field.† The lagrangian of the electromagnetic field is

$$\mathcal{L}_A = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} \tag{5}$$

where $F_{\mu\nu} = \partial_{[\mu}A_{\nu]} = \partial_\mu A_\nu - \partial_\nu A_\mu$. For now consider only the "classical" field theory case where we have one field of each type on $\mathbb{R}^4$. The complex Klein-Gordon field has a norm conservation law induced by the global phase change $\phi \to \phi e^{i\gamma}$. In this case of a noninteracting electromagnetic field we have equations of motion $\Box A^\mu = 0$ and, allowing complex values, we have four independent global phase changes allowed in

*It is interesting to note that it is precisely the properties of this ground state that lead to the inconsistencies show by Haag's Thm.

†Generally classical em fields are considered as combinations of photon fields of all photon number.

addition to the usual $A_\mu \to A_\mu + \nabla\xi$ gauge freedom. We will revisit this shortly and reveal how photon quantization arises naturally from the lagrangian once coupling is introduced.

One important distinction of the electromagnetic fields versus the Dirac fields is that the equations are second order. These can be rendered into first order equations by introducing an auxiliary field $C^\mu = \dot{A}^\mu$ so that the equations of motion become

$$\partial_t A^\mu = C^\mu \tag{6}$$
$$\partial_t C^\mu = \partial_i \partial^i A^\mu \tag{7}$$

The extension to the many particle case leads to a proliferation of functions akin to the rapid number of increasing spin states for multiple Dirac fields. In each time direction of a two photon state $A^{\mu\nu}(x^\alpha, y^\beta)$ we need first and second order time derivatives. A complete set of first order initial data is then A, $C_x = \partial_{t^x} A$, $C_y = \partial_{t^y} A$, and $C_{x,y} = \partial_{t^x, t^y} A$ with equations of motion

$$\partial_{t^x} A^{\mu\nu} = C_x^{\mu\nu} \tag{8}$$
$$\partial_{t^y} A^{\mu\nu} = C_y^{\mu\nu} \tag{9}$$
$$\partial_{t^x} C_x^{\mu\nu} = \partial_i \partial^i A^{\mu\nu} \tag{10}$$
$$\partial_{t^y} C_y^{\mu\nu} = \partial_j \partial^j A^{\mu\nu} \tag{11}$$
$$\partial_{t^x} C_y^{\mu\nu} = \partial_{t^y} C_x^{\mu\nu} = C_{x,y}^{\mu\nu} = C_{y,x}^{\mu\nu} \tag{12}$$
$$\partial_{t^x} C_{x,y}^{\mu\nu} = \partial_i \partial^i C_y^{\mu\nu} \tag{13}$$
$$\partial_{t^y} C_{x,y}^{\mu\nu} = \partial_j \partial^j C_x^{\mu\nu} \tag{14}$$
$$\tag{15}$$

where the roman indices are spatial indices related to the corresponding spacetime indices as $(t^x, x^i) = x^\mu, (t^y, y^j) = y^\nu$, etc. We can see that the number of first order fields for a source free N-photon system is $4 \cdot 2^N$ analogous to the number of spin subspaces for an N-electron system. A convenient notation for this is (P, Q) where P, Q can be 0 or 1 and the pair indicates how many derivatives of A with respect to x and y are taken. This notation gives (suppressing spacetime indices)

$$A = C_{00} \tag{16}$$
$$C_x = C_{10} \tag{17}$$
$$C_y = C_{01} \tag{18}$$
$$C_{x,y} = C_{11} \tag{19}$$
$$\tag{20}$$

which will be convenient for later generalization

3.3 Interactions

The presence of interactions is what makes dynamics interesting. The mixing of gauge freedom means that any notion of "reality" of an electron now involves a photon field as is illustrated through the A-B effect. This is seen in the definition of a gauge invariant electron current in its explicit use of A. In the many body case we need a set of interaction terms tailored for our, now distinct, equations of motion for electrons and positrons. It also radically constrains our domain of dependence in this many time coordinate space.

Let us begin with the classical or "one body" case. The interaction terms tailored for electrons and positrons are respectively:

$$\Lambda_{eA} = -q\bar{\psi}_a^{(e)} \gamma_{ab}^\mu A_\mu \psi_b^{(e)} \tag{21}$$
$$\Lambda_{pA} = -q\bar{\psi}_a^{(p)} \gamma_{ab}^\mu A_\mu \psi_b^{(p)} \tag{22}$$

The free Dirac equation does not require such extra terms but we will include them from now on to make the interaction terms nicer. The sign stays the same here because of the sign flip in the charge induced by the γ^0 factor in the Dirac representation where we assume the amplitude for the resting positron is chosen in the "v" component of the spinor $\psi = \binom{u}{v}$. We previously changed the sign of the mass term in $\mathcal{L}_p$ so that the energy of this field is positive.

Including the interaction term $\mathcal{L}_e$, variation of the action yields the equations of motion

$$\frac{\partial F_{\mu\nu}}{\partial x^\nu} = q j_\mu = q\bar{\psi}\gamma\psi \tag{23}$$
$$i\hbar\gamma\psi + qA^\mu \gamma_\mu \psi - m\psi = 0 \tag{24}$$

These are not all dynamic. Since the first is a second order equation of motion, the equations of motion must have two time derivatives. In this case we have the constraint $\nabla \cdot E = q\rho = q j_0$ which is propagated by the equations of motion. This is induced by the conservation law we derive from the sources, $\partial_\mu j^\mu = 0$ which shows that only three of these equations are now dynamical. We can rewrite this as a set of first order equations by the the definition $C_\mu = \partial_t A_\mu$. Choosing the Lorentz gauge, $\partial_\mu A^\mu = -C_t + \partial_i A^i = 0$, we obtain $\Box A^\mu = q j^\mu$ in a form that automatically generates compatibility with the conservation of charge and is propagated for all time.

Interactions for the many body case, QED, involves two ways of coupling electrons and positrons to the electromagnetic field: a lone electron can couple to a lone electron and a photon or a photon can couple to an electron and a positron. We are not interested in any of the common "backwards in time" mnemonics or procedures here since this is an initial value approach. Firstly we should give a picture of the "tower" of states that need to be coupled.

$$\begin{aligned}
&\alpha \\
&\Psi^\mu_{(A),Q}(x), \Psi^{\mu\nu}_{(AA),QR}(x,y)\ldots \\
&\Psi_{(e),a}(x), \Psi^\mu_{(eA),aQ}(x,y), \Psi^{\mu\nu}_{(eAA),aQR}(x,y,z)\ldots \\
&\Psi_{(p),a}(x), \Psi^\mu_{(pA),aQ}(x,y), \Psi^{\mu\nu}_{(pAA),aQR}(x,y,z)\ldots \\
&\Psi_{(ep),ab}(x,y), \Psi^\mu_{(epA),abQ}(x,y,z), \Psi^{\mu\nu}_{(epAA),abQR}(x,y,z,w)\ldots \\
&\ldots
\end{aligned} \tag{25}$$

The first line holds a complex value α that indicates occupancy of the "vacuum" state. The next line gives the pure photon states. The N photon state has $4 \cdot 2^N$ degrees of freedom (dof) in the free case if we have not imposed any gauge constraints. Below this are the one electron states with the 1,2... photon states to the right. Below is are the one positron states with the various photon number states then the electron and positron states with corresponding photon number cases. The action to describe these as free fields is given by a collection of independent actions

$$S_{(e)} = \int i\hbar\Psi^*_a\gamma^\mu_{ab}\partial_\mu\Psi_b - m\Psi^*_a\Psi_a dx \tag{26}$$

$$S_{(ee),1} = \iint i\hbar\Psi^*_{ba}\gamma^0_{bc}\gamma^\mu_{cd}\partial_\mu\Psi_{da} - m\Psi^*_{ba}\gamma^0_{bc}\Psi_{ca}dxdy \tag{27}$$

$$S_{(ee),2} = \iint i\hbar\Psi^*_{ab}\gamma^0_{bc}\gamma^\nu_{cd}\partial_\nu\Psi_{ad} - m\Psi^*_{ab}\gamma^0_{bc}\Psi_{ac}dxdy \tag{28}$$

$$\ldots \tag{29}$$

The action for a single particle photon field is

$$S_{(A)} = -\int \frac{1}{4}F^{\mu\nu}F_{\mu\nu}dx \tag{30}$$

$$= -\frac{1}{4}\int(\partial^{[\mu}\Psi^{*\nu]}_{(A)})(\partial_{[\mu}\Psi^{(A)}_{\nu]})dx \tag{31}$$

$$\tag{32}$$

where we have included a complex conjugation. This seems unnecessary since we generally consider the electromagnetic field to be real. When we consider the functions that correlate electron and photon fields we will see that we cannot neglect it. The two photon actions are*

$$S_{(AA),1} = -\frac{1}{4}\int\left(\partial^{[\mu}_{(x)}\Psi^{*\nu]\alpha}_{(AA)}(x^{\cdot},y^{\cdot})\right)\left(\partial^{(x)}_{[\mu}\Psi^{(AA)}_{\nu]\alpha}(x^{\cdot},y^{\cdot})\right)dxdy \tag{33}$$

$$S_{(AA),2} = -\frac{1}{4}\int\left(\partial^{[\mu}_{(y)}\Psi^{*\nu\alpha]}_{(AA)}(x^{\cdot},y^{\cdot})\right)\left(\partial^{(y)}_{[\mu}\Psi^{(AA)}_{\nu\alpha]}(x^{\cdot},y^{\cdot})\right)dxdy \tag{34}$$

$$S_{(AC),1} = -\frac{1}{4}\int\left(\partial^{[\mu}_{(x)}\Psi^{*\nu]\alpha}_{(AC)}(x^{\cdot},y^{\cdot})\right)\left(\partial^{(x)}_{[\mu}\Psi^{(AC)}_{\nu]\alpha}(x^{\cdot},y^{\cdot})\right)dxdy \tag{35}$$

$$S_{(CA),2} = -\frac{1}{4}\int\left(\partial^{[\mu}_{(y)}\Psi^{*\nu\alpha]}_{(CA)}(x^{\cdot},y^{\cdot})\right)\left(\partial^{(y)}_{[\mu}\Psi^{(CA)}_{\nu\alpha]}(x^{\cdot},y^{\cdot})\right)dxdy \tag{36}$$

where the $1,2\ldots$ subscripts on the actions indicate the respective coordinate label $x^{\cdot}, y^{\cdot}\ldots$ where the derivatives are being taken. The previous notation we used to distinguish coordinate order for the Dirac fields is not available here because of the more complicated index structure and we replace A and C as field labels with $\Psi_{(A)}$ and $\Psi_{(C)}$ for the sake of a uniform notation when both electrons and photons are present. Here we explicitly include the coordinates and label the first coordinate, $x^{\cdot}$, in the derivative operator $\partial^\mu_{(x)}$ and order the indices in $\Psi^{\mu\nu}$ to correspond to $x^{\cdot}$ and $y^{\cdot}$ respectively. The square backets, [ ], indicate antisymmetry over the two indices immediately to their open sides. The first order time derivative data from the "inactive" coordinates, those not being dynamically evolved by the particular lagrangian, are included with the C labels to get a full set of first order initial data. Variation of these lagrangians, through a combination of explicit and implicit expressions, gives the four functions $\Psi^{\mu\nu}_{(C_{PQ})}$ and eight linear EoM for each function in each of the two time directions t^x, t^y.

The (noninteracting) mixed one-electron one-photon actions on $\Psi(x^{\cdot}, y^{\cdot})$ to generate EoM in each time label are

$$S_{(eA),1} = \int i\hbar\Psi^{*,\nu}_{(eA),a}\gamma^\mu_{ab}\partial^{(x)}_\mu\Psi_{(eA),b\nu} - m\Psi^{*\nu}_{(eA),a}\Psi^{(eA)}_{a\nu}dx \tag{37}$$

$$S_{(eA),2} = -\frac{1}{4}\int\left(\partial^{[\mu}_{(y)}\Psi^{*\nu]}_{(eA),a}\right)\left(\partial^{(y)}_{[\mu}\Psi^{(eA)}_{a\nu]}\right)dx \tag{38}$$

Generalizations to higher particle numbers from here are evident but rapidly becomes onerous. Symmetries among identical particle types are not required by these actions but it is not hard to see that imposing them as initial data lets them be propagated.

To give an interesting theory there must be interactions. The vacuum u is strictly formal and does not couple to anything. We know that electrons and positrons can annihilate and electrons/positrons can scatter and produce a photon. Taking our cues from QED, coupling terms must be of the form The couplings must be "local" in some sense that we enforce, with inspiration from QED, as

$$S_{(e)} = \int i\hbar\bar{\Psi}^*_a\gamma^\mu_{ab}\partial_\mu\Psi_b - m\bar{\Psi}^*_a\Psi_a dx + \Lambda_{(e-eA)} \tag{39}$$

$$S_{(eA),1} = \int i\hbar\bar{\Psi}^{*,\nu}_{(eA),a}\gamma^\mu_{ab}\partial^{(x)}_\mu\Psi_{(eA),b\nu} - m\bar{\Psi}^{*\nu}_{(eA),a}\Psi^{(eA)}_{a\nu}dx + \Lambda_{(eA-eAA)} \tag{40}$$

$$S_{(eA),2} = -\frac{1}{4}\int\left(\partial^{[\mu}_{(y)}\Psi^{*\nu]}_{(eA),a}\right)\left(\partial^{(y)}_{[\mu}\Psi^{(eA)}_{a\nu]}\right)dx + \Lambda_{(e-eA)} + \Lambda_{(eep-eA)} \tag{41}$$

$$\ldots \tag{42}$$

where the "bar" action over the Ψ is hiding a γ^0 considered to be contracted on the active spinor indices. Here we see that the one-electron field $\psi = \Psi_{(e)}$ feels the electromagnetic field from $\Psi_{(eA)}$ as we evolve in its time coordinate direction $t^{(e)}$. The notion of locality for this interaction is chosen so that ψ feels the field of $\Psi_{(eA)}$ when all three spacetime coordinates agree. In this case, this gives only a self energy contribution but will give the usual two body static interaction for two

*The "upper" or "lower" state of the particle type labels (AA), (AC), (eA) etc. have no meaning but are chosen to make the expression as uncluttered as possible. Summation conventions are in effect for spacetime and spinor indices.

charges. Conversely, the field $\Psi_{(eA)}$ feels the influence of ψ as a source where all three coordinates agree when we evolve in the time direction $t_2^{(eA)}$, the second time label corresponding to A. For an electron-positron pair production or annihilation amplitude we give a similar definition of locality.

Explicitly, the couplings are

$$\Lambda_{e-eA} = -q \int \bar{\Psi}_a^{(e)}(x^{\cdot})\gamma^{\mu}_{ab}\Psi^{(eA)}_{\mu,b}(y^{\cdot}, z^{\cdot})\delta(x^{\cdot} - y^{\cdot}) \tag{43}$$

$$\delta(x^{\cdot} - z^{\cdot})dxdydz \tag{44}$$

$$\Lambda_{p-pA} = -q \int \bar{\Psi}_a^{(p)}(x^{\cdot})\gamma^{\mu}_{ab}\Psi^{(pA)}_{\mu,b}(y^{\cdot}, z^{\cdot})\delta(x^{\cdot} - y^{\cdot}) \tag{45}$$

$$\delta(x^{\cdot} - z^{\cdot})dxdydz \tag{46}$$

$$\Lambda_{ep-A} = \pm q \int \Psi^{(ep)*}_{ab}(x^{\cdot}, y^{\cdot})\gamma^0_{ac}\gamma^{\mu}_{cb}\Psi^{(A)}_{\mu}(z^{\cdot})\delta(x^{\cdot} - y^{\cdot}) \tag{47}$$

$$\delta(x^{\cdot} - z^{\cdot})dxdydz \tag{48}$$

The sign of the pair production term is not clearly constrained here and neither is our choice of where to place the complex conjugations. Comparison with QED suggests that the sign be chosen negative and these be the correct choices of conjugation and contraction with γ^0 factors. The evolution of the free equations ensures conservation of the stress-energy, charge and particle number. These coupling terms can introduce relative phase differences at these many-body diagonals so can act as source and sink terms for amplitude. The complexity of the quantum version of the photon is important in generating these sources and in creating a norm conservation law that governs the flow of "norm-flux" between these spaces. Interestingly the conservation of charge and norm arise from the same global phase symmetry. The electron-positron field has no net charge yet will have a well defined norm from the phase symmetry $\Psi_{ab} \to e^{i\theta}\Psi_{ab}$ in the free lagrangians

$$S_{(ep),1} = \iint i\hbar\Psi^*_{ba}\gamma^0_{bc}\gamma^{\mu}_{cd}\partial^{(x)}_{\mu}\Psi_{da} - m\Psi^*_{ba}\gamma^0_{bc}\Psi_{ca}dxdy \tag{49}$$

$$S_{(ep),2} = \iint i\hbar\Psi^*_{ab}\gamma^0_{bc}\gamma^{\nu}_{cd}\partial^{(y)}_{\nu}\Psi_{ad} + m\Psi^*_{ab}\gamma^0_{bc}\Psi_{ac}dxdy \tag{50}$$

$$\tag{51}$$

There is an obvious extension of these interactions to the tower of fields. We need to discuss why the equal times slice of the evolution* here is most related to what we see and experience. Before we do this let us consider the electrostatic interaction between two electrons. It has always seemed a little ad hoc that we impose the two point interaction $\frac{q}{4\pi}|x_1^2 - x_2^2|^{-1}$ for a function $\Psi(x, y)$ in quantum mechanics. Certainly we can write down a one body wavefunction $\psi(x)$ and vector potential A^{μ} and impose a classical 4D lagrangian. We find an electrostatically driven self spreading distribution where the density of the norm gives the charge density. This is not at all what we see for the two charge quantum system. No such self-force is manifested beyond the usual quantum pressure.

Given the fields $\Psi^{(ee)}_{ab}(x^{\cdot}, y^{\cdot})$ and $\Psi^{(eeA),\mu}_{ab}(x^{\cdot}, y^{\cdot}, z^{\cdot})$, we see that when we impose the Coulomb gauge that

$$\Psi^{(eeA),t}_{ab} = \Psi^{(ee)}_{ab}(x, y)\,\frac{q}{4\pi}|\vec{x}^{\,2} - \vec{z}^{\,2}|^{-1}\,\delta(\vec{y} - \vec{z}). \tag{52}$$

The nature of the self-energy for such a theory seems more opaque than in the classical case where we can consider it in terms of finitely sized objects [14]. Locality and causality here are not so forgiving with such a construction and a constituent based approach would likely require an infinite number of fields of vanishing mass and charge that bind to a state of finite extent with the center-of-mass coordinates appearing as the x^{μ}, y^{ν} coordinates in our $\Psi^{(ee)}_{ab}$. We will not discuss this point further but should be aware of the complication in managing self field contributions that affect both the energy and momenta of particles. Shortly we will see that even though $\Psi^{(eeA),t}_{ab}$ is nonzero it contains zero norm and that there is an infinite tower of such nonzero fields above it. This is not so evident in QFT which we may think of as tracking the nonzero norm of the fields through the tower. Now would be a good time to emphasize that these are all classical fields in a tower of spaces of growing dimensionality. There are no Grassmann variables, q-numbers or field operators and their associated commutation relations. These have always been conceptually dubious or ad hoc constructions on which field theory is built and the goal of our construction is to show why (and when) they work.

3.4 Diagonal Time Evolution

The relationship between the quantum and classical worlds is an enduring problem. It is not just explaining quantum measurement that is troublesome. Encoding the classical world in a quantum description is a challenge to do correctly. Naive approaches have led to such useful results as band theory and the Kubo relations but ultimately lead to inconsistencies. One approach is to assume the classical world is a very restricted subset of localized many body wavefunctions that are sparsely distributed in the total Fock space. The usual quantum statistics then follow trivially along with an arrow of time [1, 3]. The new problem is justifying such initial data. In this many time description we have the further challenge of justifying why we, as observers, seem to observe the universe of "equal times" and not the vast regions of unequal space and time locations where the many body quality of the description is more evident.

Possible explanations for this is that interactions occur at many body diagonals. Since our observations require interactions this is the part of the universe we see. In general, many body wavefunctions do not act in a form similar to discrete state machines which seem to underlie our notions of memory

*Specifically, for any many body point of any function of the tower, we choose all the times corresponding to the spatial coordinates equal: $t^1 = t^2 = t^3 \ldots$.

and consciousness. The special cases that do seem to define our classical world. We will show that the equal times evolution defines the motion everywhere so all the other regions are defined by them and so give no other possible observations of the world.

As an example, consider the evolution of the two photon field $A^{\mu\nu}(x^\alpha, y^\beta)$ along the $t^x = t^y$ axis with respect to $t = t^x + t^y$

$$\partial_t A^{\mu\nu} = C_x^{\mu\nu} + C_y^{\mu\nu} \tag{53}$$

$$\partial_t C_x^{\mu\nu} = \partial_i \partial^i A^{\mu\nu} + C_{x,y}^{\mu\nu} \tag{54}$$

$$\partial_t C_y^{\mu\nu} = \partial_j \partial^j A^{\mu\nu} + C_{x,y}^{\mu\nu} \tag{55}$$

$$\partial_t C_{x,y}^{\mu\nu} = \partial_i \partial^i C_y^{\mu\nu} + \partial_j \partial^j C_x^{\mu\nu} \tag{56}$$

$$\tag{57}$$

It is unclear if this is particularly useful but it does illustrate how the evolution along the equal times axis is locally determined in the equal time coordinate t. However, we still need to evolve spatially in a neighborhood of this diagonal so the many body and many time propagator approach seems hard to avoid.

3.5 Quantization of the Photon

Here we show that the quantization of the photon inherits its norm from the purely electron part of the lagrangians. This is the photon analog to the way that the "reality" of the Schrödinger electron picks up a contribution from A in the current $j^k = \frac{\hbar}{m}\nabla^k\phi - eA^k$.* This explain how the photon quantization condition can be a function of $\hbar$ despite having no such factor in its own lagrangian. It is quantized in the sense that if all the amplitude (normalized to 1) is initially in the lepton fields then it is all converted to a photon then the factor $\hbar\omega$ gives the magnitude of the photon norm. Up to this point we have been using units where $c = \mu_0 = \epsilon_0 = 1$ but left $\hbar$ general to make the connection more clear with standard formulas. In this section, we revert to full SI units to emphasize this connection more clearly.

In the free field cases, the usual definitions of momentum, energy... follow from the stress tensors for the classical Dirac and electromagnetic fields regardless of whether they are real or complex. The one additional conserved quantity that Dirac fields have is "norm" associated with the complex global phase freedom. The fields in the tower posses a U(1) symmetry in the sense that $\Psi \to \Psi e^{i\theta}$ and similar transformations for every function in the tower leaves the set of lagrangians invariant. When a fermion and photon field interact the coupling terms act as complex source terms resulting in, for example, a complex $\Psi^{(eA)}$ functions as the amplitude of $\Psi^{(e)}$ decreases. Since this is not generally a separable function, we cannot say whether the photon or electron part is complex individually but can predict the phase difference between the function pair and derive a many body conserved norm.

*We have neglected the "spin current" fraction here for simplicity.

Firstly, we can modify the photon lagragian to allow complex fields as

$$\mathcal{L}_A = \frac{1}{4\mu_0}(\partial_\mu A_\nu^* \partial^\mu A^\nu + \partial_\mu A_\nu \partial^\mu A^{\nu^*}) \tag{58}$$

This is essentially the massless Klein-Gordon field. The conserved current is

$$j_\mu = \frac{i}{4\mu_0}\left(\partial_\mu A_\nu \cdot A^{*\nu} - A_\nu \cdot \partial_\mu A^{*\nu}\right) \tag{59}$$

Consider the case of a complex plane wave solution $A_y(x,t) = \mathcal{A}e^{i(kx-\omega t)}$. If this was a real (classical) field there would be no current and norm would equal zero. For the complex case, $\rho = j_0 = \mathcal{A}^2\omega/2\mu_0$ and $j_x = -\mathcal{A}^2 k/2\mu_0$. In computing the norm for $\Psi^{(eA)}$ we need to use this $j^0_{(A)}$ and evaluate

$$\hat{\mathcal{N}}(\Psi_{(eA)}) = \frac{i}{2\mu_0}\iint\left(\partial_t^{(A)}\Psi_{a\nu}^{(eA)}\Psi_{(eA),a}^{*\nu} - \Psi_{a\nu}^{(eA)}\partial_t^{(A)}\Psi_{(eA),a}^{*\nu}\right)dx^3dy^3 \tag{60}$$

$$= \frac{i}{2\mu_0}\iint\left(\Psi_{a\nu}^{(eC)}\Psi_{(eA),a}^{*\nu} - \Psi_{a\nu}^{(eA)}\Psi_{(eC),a}^{*\nu}\right)dx^3dy^3 \tag{61}$$

where $\hat{\mathcal{N}}$ is the norm operator defined by j^0 for the argument function. A Dirac field gives a conserved $\int\psi^*\psi$ so this clearly gives the correct electron-photon conserved current in the noninteracting case so this is the quantity that is conserved along the equal times diagonal. Let the volume of the space be $V = 1$. Now let us investigate the implications of simultaneous conservation of energy and norm in a radiative decay process.

Suppose we start with an excited positronium state $\Psi^*_{(ep)}$ that radiates with frequency ω into the state $\{\Psi_{(epA)}, \Psi_{(epC)}\}^\dagger$ and possibly higher photon number ones. The resulting photon must have the same frequency ω since this is the frequency at which the source term oscillates. The initial norm for the states is $\hat{\mathcal{N}}\Psi_{(ep)} = 1$ and $\hat{\mathcal{N}}\Psi_{(epA)} = 0$. Our goal is to find the resulting norms after the transfer is completed, in these units. This will tell us the ratio of energy to norm transferred, which we construe as the meaning of photon quantization.

Assume the resulting function is $\Psi_{(epx)} = \Psi'_{(ep)}\mathcal{A}e^{i(kx-\omega t)}$ where $\hat{\mathcal{N}}(\Psi'_{(ep)}) = 1$. Since these lagrangians are coupled the coefficients define a relative size for them which are respectively $\hbar$ at $t = 0$ (from the factor in the kinetic term in the electron and positron lagrangians) and

$$\hat{\mathcal{N}}\Psi_{(epA)} = \hat{\mathcal{N}}\Psi_{(A)} = \mathcal{A}^2\omega/2\mu_0 \tag{62}$$

at $t = t_f$. Since these must be equal we obtain the amplitude of the wave as $\mathcal{A} = (2\mu_0\hbar/\omega)^{1/2}$. The final energy of the

†Note that the notation {, } does not denote anticommutation here. These are functions and the braces here just indicate a set.

system must be the same with the electron and positron in a new state with $\Delta E_{(ep)} = \Delta E_{(epA)}$. The photon contribution is given by $E_{(A)} = \int \frac{1}{2\mu_0} C^2 dx = \frac{1}{2\mu_0} \mathcal{A}^2 \omega^2 = \hbar\omega$. This shows that to radiate any more energy an additional photon would need to be generated.

In quantum mechanics and quantum field theory this is one of the assumptions that is hidden in the formalism. Since we are constructing an explicit classical field theory we do not have such a liberty. It was not, a priori, necessary that a transfer of energy, $\hbar\omega$, from a decay between two eigenstates give a unit norm transfer. We might have had a partial occupancy of the $\Psi_{(epA)}$ state and not completely emptied the $\Psi_{(ep)}$ one or had to resort to higher $\Psi_{(epAA...)}$ states to contain all the norm that was generated by the event. This is the first actual derivation of the "quantization" of the photon. In this model, the statement of photon quantization is more precisely stated that the ratio of energy flux to norm flux between different photon number states is $j_E/j_n = \hbar\omega$, at least for the case where the frequency of the radiation is monochromatic. It is interesting that the photon "norm" depends on $\hbar$ even though the only lagrangian with such a factor is that of the fermions. The coupling has done several things. It introduces a constraint on one of the components of the electromagnetic field from the current conservation of the charges. It mixes the "reality" of the A and ψ fields to give the electron current. Here we see that it also induces the proportionality constant in the norm flux of the photon between different particle number spaces. This relationship between norm and energy flux may be what underlies the success of the formal commutation relations for field operators $[\hat{P}, \hat{Q}] = i\hbar$ [4].

4 Dynamics

We have not firmly established an isomorphism with QED for a precise subset of initial data. Ideally, imposing the usual particle symmetries on such data and evolving will match the usual scattering amplitudes. We have several barriers to doing this. Firstly is renormalization and the singularity of the coupling terms. The dimensionality of the space is so enormous and the number of nonzero yet norm free subspaces is infinitely large so finding an economical and compact manner to even start the problem is unclear if possible at all. Even finding the suitable "dressed" particles to scatter is not yet accomplished. The largest hurdle to overcome is probably the fact that no interacting field theory is well defined by Haag's theorem. This has been solved here so it might be unfair to even ask for an isomorphism between the theories. However, QED has a record of impressive calculations and the most reasonable notion of "isomorphism" may be to reproduce these. The foundational aspects of QED were designed after the fact on the tail of a process of refining procedures to obtain useful calculations so the inconsistency of these foundations may not be so important. Let us begin with a process of restricting the subspaces in a fashion that gives observable particles with enough of the interactions necessary for good approximations. Given the expanse of QED we cannot do all the work necessary to make a convincing case for this theory in a single paper. Some of this section is meant to be suggestive of more essential work ahead not an exhaustive argument or thorough calculation to this end.

4.1 Scattering and Adiabatic Coupling Changes

One of the most frustrating aspects of QFT is that the interim state of the system is clouded in the language of "virtual particles" and it seems to be not well defined at every time. Our measurements are confined to in and out states once the interactions are over. The is a formulaic extension to bound states where the interaction persists but this does not solve this problem. The current formulation shows that there is a well defined state at every time. Ironically, the in and out state picture has more problems at $t = \pm\infty$! This is because the interactions have been "turned off" here so the "virtual cloud" of many particle states that must always accompany a particle are no longer there. By adjusting the bare mass parameter slowly we can make an association with such states of the same net mass and momentum.

This is already formally discussed in many books. Here we will make some small changes that don't affect the results but make the process a bit more logical. Firstly, notice that the equations of motion above have been selected to give the usual propagators in the single time coordinate functions and the couplings to model those of QED. The role of the many photon coordinate spaces has been suppressed by the QED formalism and we see that there are many more spaces to consider than in the usual treatment. Once we impose the Coulomb gauge, we see that many of the constraints described by the "longitudinal photons" are just nonzero zero-norm functions in the tower.

If we consider the case of scattering of two particles, say and electron and a positron, we should properly "dress" them first. This suggests we partition our tower into a set of higher photon and electron-positron pair spaces that only couple to these particles separately. By turning on the interaction parameters slowly enough we can force the net mass and momentum of these waves to be the same without inducing any unwanted reflection. Since we typically work with plane waves of infinite extent instead of planes waves, we don't have a natural way to let spatial separation of packets prevent them from interacting but we can now use a second adiabatic turn on that lets these towers now interact and couple to the set of higher photon and electron-positron pair spaces that include both of these in more interesting ways. The more flowery aspects of QED such as "the positron is an electron moving backwards in time" is removed by our positive mass independent equation for the positron and superluminal virtual particles are now to be understood as a feature of evolving propagators in separate time spaces to arrive at the equal times result. We will now show that the apparent superluminal contributions to the Feynman propagator is actually a constraint on consistent initial data not faster light effects that are cancelled by a measurement ansatz.

4.2 Causality Considerations

The divergences we see in field theory with interactions are directly related to the singular nature of the δ-function coupling in the lagrangian. This is usually phrased in the loose semi-classical language of quantum theory as the "particles are point-like." We already expressed that our opinion was that finer nonsingular structure existed at a level we cannot yet probe. The oldest method of handling such a situation is with "cutoffs." Naively done, these are intrinsically nonrelativistic for reasons beyond some small nonlocality. We can make them as mild a problem as possible by choosing them in the local frame defined by the two body currents at the interaction diagonal. Specifically, it is here we need to couple two fields such as $\Psi_{(e)}$ and $\Psi_{(eA)}$ so that the electron field of $\Psi_{(e)}$ generates the em field in $\Psi_{(eA)}$ as a source at the $x_e = x_A$ diagonal. The current $j^\mu(\Psi_{(e)})$ defines a velocity $v = j/\rho$. This specifies a local frame to construct a spherical region of radius r_0. We can then modify the electromagnetic source interaction term as $\bar{\Psi}_{(e)}\gamma^\mu\Psi_{(eA);a\mu}\delta(x_{(e),1} - x_{(e),2})\delta(x_{(e)} - x_{(A)}) \to \bar{\Psi}_{(e)}\gamma^\mu\Psi_{(eA);a\mu}\delta(x_{(e),1} - x_{(e),2}) f(v, x_{(e)}, \Theta(r_0 - |x_{(e)} - x_{(A)}|))$ where f gives a boost distortion to the r_0 sphere in the rest frame defined by the current. As long as the oscillations we consider are much longer than r_0 this has little contribution to nonlocal and nonrelativistic errors for a long time. It does create a recursive (hence nonlinear) definition. We only expect cutoffs to be useful when the details of the cutoff are not important in the result. It is expected that this extension of the usual cutoff procedure will give new radiation reaction contributions not present in QED although it is possible that other regularization procedures to cut off integrals may effectively do this implicitly. The small range of the boost dependent shape of the cutoff has effects only for field gradients that can probe it, however, this is exactly the case in the radiation reaction problem. There is considerable belief that the radiation reaction force and rate of particle creation is not captured by standard QED and that all such approaches are plagued with the pre-acceleration problems standard in the classical case [14] but some useful limits have been derived [10].

The perturbative schemes generally built on the interaction representation yields a time order exponential [13, 17] of terms ordered by the number of discrete interactions in the terms. The details of this construction allow S_F to be pieced together from forwards and backwards propagators in a spacelike slice. This results in a propagator that lives outside the light cone. Usual arguments [13] tell us that the vanishing of the commutator of the field operators outside the light cone is sufficient for causality, an explanation that sounds excessively hopeful and reaching but all to familiar to students of QFT. For our initial data formalism there is no such analog. Firstly let us argue that this unconfined behavior of $S(x-y)$ at $t^x = t^y = 0$ is not an expression of acausal behavior just a statement that the "reality" the initial data has not been localized to start with. How can this be? We could start with a classical delta function source and evolve with this and arrive at a true solution that evolves past the light cone. The usual answer to this is obscured by the usual cloudy use of positive and negative energy states in QFT. Here we have distinct equations of motion or electrons and positrons so the "negative energy" components are a reality to contend with and not be "reinterpreted" through some measurement ansatz.

To address this consider the case of the classical (massive) KG equation

$$\nabla^2\phi - \partial_t^2\phi = \frac{m^2}{\hbar^2}\phi \tag{63}$$

where the propagator has the same problem. Here the initial data is ϕ and $\dot{\phi}$. Localizing ϕ as a delta function gives

$$\phi = \sum e^{i(px-\omega t)} \tag{64}$$

$$\dot{\phi} = -i\sum E_p e^{i(px-\omega t)} \tag{65}$$

$$\tag{66}$$

where $E_p = \omega(p) = \sqrt{p^2 + m^2/\hbar^2)}$. This shows that whatever reality is associated with the KG field ϕ is not localized even though ϕ itself is. Interestingly, if we force localization of $\dot{\phi}$ then $\phi = i(2\pi)^3 \sum E_p^{-1} e^{i(px-\omega t)} = i(2\pi)^3 G_p(x)$ so it embodies the delocalized initial data we complain about in the propagator. We can produce a localization of ϕ and $\dot{\phi}$ by setting $\phi(x) = \delta(x)$ and $\dot{\phi} = 0$ as the particular linear combination

$$\phi(x, t=0) = \frac{1}{2\pi}\int_0^\infty dk(ae^{ikx+i\omega(k)t} + be^{-ikx-i\omega(k)t})|_{t=0} \tag{67}$$

with $a + b = 1$ and $a - b = 0$ so $a = b = \frac{1}{4}$ but this will turn out not to be the interesting solution for coupling of KG to a positive energy Dirac field.

Our inability to constrain the total reality (charge, energy, mass...) of the particle to a point indicates that we have a constraint on our physical initial data not a measure of the incompleteness of our basis or a causality problem with our propagators. It should now not be surprising that a similar situation arises for the Dirac fields. For a spin up, positive energy state, localization of all components is inconsistent with the equations of motion. In coupling the Dirac field to the KG (or electromagnetic) field we cannot couple a delocalized Dirac packet to a localized one and the use of the propagator G_p to build the interaction now is more reasonable that the solution given by eqn. 67 since it follows directly from the Fourier transforms of the couplings Λ_{e-eA}, Λ_{ee-eeA}, etc.

4.3 Subspace Restrictions and Resummation

The problems of finding initial data and evolving in an infinite tower of spaces is daunting. The perturbative solutions embodied in the path integral approach are a way of working around this without stating it in these terms. The problems of field theory are often such that a finite perturbative approach is inadequate. Superconductivity is a canonical example of this where this "nonperturbative" behavior delayed an explanation for half a century. Summing over the same diagrammatic sequence such as with "ladder diagrams" lets us capture some small slice of the infinite character of the space and derive new effective propagators where effective mass terms arise. The number of terms in the total perturbative expansion grow exponentially so it is unclear if such a sum actually has any meaning to which we are attempting convergence. We now know that such series are generally asymptotic so that there is no meaning to them in this limit. However, these particularly abbreviated series have been very valuable and are often capturing essential parts of the physics.

In this article, we are seeking a higher standard of conceptual justification for such sums. Even though we cannot hope to complete this task in a single article, let us seek a foundation for such calculations based on the data set and coupling provided. The self energies have been addressed through a relativistically valid, if slightly nonlocal, approach through cutoffs. Consider a single particle of mass parameter m and momentum p. This should be thought of as including $\Psi_{(e)}, \Psi_{(eA)}, \Psi_{(eAA)} \ldots$ (and associated C_{PQ} fields) with all amplitude in the bottom state but constraints holding in the upper level functions but no other space couplings. This can be exactly and easily solved with the Coulomb gauge imposed at each level. Turning up the other interactions through the pair creation states $\Psi_{(eep)}, \Psi_{(eepA)} \ldots$ can be done independently since the couplings between all function pairs, labelled by q, can be controlled separately. These states acquire little contribution in dressing a lone charge because they add so much energy to the system although the effects can be larger during deep scattering events with other charges.

In order to evolve such a system with a gradually changing interaction term while preserving the net norm, mass, and charge (observed from a distance) we can control the m and q parameters and an overall multiplicative constant, β, of the system. The final observed mass is the net energy of the system in the rest frame. We assert that the observed charge is determined by the electric flux that we can observe through large spheres in the A-coordinate subspaces in $\Psi_{(eA)}, \Psi_{(epA)}, \Psi_{(eAA)} \ldots$. When a large "classical" body interacts with such a particle we assume it is broadly and uniformly distributed through a large variation in photon number spaces. This may seem ad hoc but for such a body to affect a lone dressed charge it must act in all the photon number spaces available or it leads to spectroscopic filtering of charge subspace components as they move in its field. Since this is not observed and we don't have a clear understanding of how classical bodies are represented with a quantum description, this seems like a reasonable supposition. These ideas lead to a prescription to modify the m, q, and β as we turn up the interaction. We need to be careful here as we now implicitly have multiple $q's$! This has been obscured by our choice of labeling them the same in our tower of interactions. There is the value q_{eA} that gives the self energy coupling in the towers of strictly photon number increasing states e.g. $\Psi_{(eA)}, \Psi_{(eAA)}, \Psi_{(eAAA)} \ldots$ and the value q_{eep} that gives the couplings to the towers of electron positron pair increase $\Psi_{(e)}$, $\Psi_{(eep)}$, $\Psi_{(eeepp)} \ldots$ Ultimately we want these parameters to be both the same. This seems to be a nontrivial process and it is somewhat impressive that the usual QED adiabatic turn on gets this to work by starting with a completely undressed charge and a single parameter.

Once we have dressed up lone charges on a subset of the towers deemed to be sufficiently rich to describe the dynamics of the process of interest. The interactions between them must be turned up. Given the states Ψ_{e_1A} and Ψ_{e_2A} we expect an antisymmetrized product of the two to give a first approximation to $\Psi_{e_1e_2A}$ and evolve these new "crossing" interaction parameters $q_{1,2}$ gradually and then hold it steady for a much longer period of time followed by a turn off of the interactions. If these adiabatic processes can be done in a way that leaves momenta of scattered waves unchanged then we can infer the actual scattering rates and angles for dressed particles. To this author, this is the simplest possible way to arrive at the scattering results from a well-posed initial value formalism. Ultimately, we must try other less restrictive subspace restrictions to show that our assumption that they made a small contribution was valid. There is reason to believe this actually works and gives the usual QED results and will be a subject of a followup work.

5 Conclusions

The need for establishing a well-defined space and set of dynamical equations for the reality described by QED, and QFT in general, has been discussed and presented in the form of a tower of spaces of continuum functions. Subsets of the dimensional labels of these spaces give meaning to the notion of "particle" and symmetries in the couplings and initial data define "identicality" of them. There have been a number of subtle issues to confront. Not the least of these is how to give meaning the the many time labels that arise in such a construction and why we, as observers built from the fields, should observe only one time. Such a construction has a number of advantages. It removes the ad hoc character of the construction and the need for the notion of "quantum fields." The inconsistencies described by Haag's theorem are resolved by a partitioning of the tower space into subsets of fixed lepton number that never couple to the ground state. Most importantly we have given an explanation for the quantization of the photon and an indication of the origin of the quantization conditions for quantum operators and the appearance of $\hbar$ in them.

The biggest downside of this construction is that of computability. QED was built from computations and arose out of many ad hoc attempts to make sense of observed dynamics on the part of many stellar physicists. The actual foundations of the subject are almost a necessary afterthought. Of course, no class is taught this way and the foundations must come first regardless of how flimsly they are. A cynic might worry that field theory courses are filtering students based on their levels of credulity or lack of concern with consistency, a possible advantage in a field driven by extreme publication pressures.

The work here is still hardly complete and it is still to be shown that such construction can validate the successful results of QED for scattering. The subject of bound state corrections has been untouched here and an important topic that needs attention. There is good reason to believe that, ultimately, this theory will have corrections that are not found in QED and therefore be inequivalent at some level of accuracy. The subject of the radiation reaction and QED is still disputed. Given that the classical radiation reaction is resolved by keeping track of the self fields that traverse the extent of a finite body, one might worry that the renormalization procedure to handle self energy might be too simplistic and miss the asymmetric forces that must arise to give the back reaction. A primary motivation for this construction is the incorporation of gravity in a consistent fashion with the quantum world and other fundamental forces. A recent construction by the author in a classical direction relies on a greatly expanded gauge group and a flat background construction. Here couplings mock up the "geometric" effects of general relativity to observers and provides a new avenue for this problem as discussed briefly in the appendix.

A Gravity

Recently the author has presented a treatment of classical GR, electromagnetism and the Dirac field on a flat background that retains the apparently geometric features of GR and yet puts the fields on a similar footing [3]. The motivation for this is in promoting the Dirac γ^μ matrices to dynamic fields without imposing the vierbein approach. This has a number of consistency challenges to work out that will not be reproduced here. One of the essential features is that the γ^0 that is hidden in the $\bar{\Psi}$ has to go. We must replace all the $\bar{\Psi}$'s with new independent fields Φ's that implicitly do the work of them. The quadratic nature of the equations then become bilinear and, while the fields may not evolve causally, it can be shown that the gauge invariant reality of them do. Promoting the γ^μ_{ab} matrices to dynamical fields necessitates that we reinterpret them as vectors in the μ index and scalars in the a, b indices. This seems at odds with the usual SU(2) representation theory. This can be resolved by keeping track of the gauge invariant quantities and allowing new rules to actively boost fields in the space. The various details surrounding this are discussed in Chafin [1].

The metric and its inverse can be defined in terms of these fields as

$$\begin{aligned} g^{\mu\nu} &= -4^{-1}\mathrm{Tr}_{ac}\gamma^{(\mu}_{ab}\gamma^{\nu)}_{bc} \\ g_{\mu\nu} &= \mathrm{Inv}(-4^{-1}\mathrm{Tr}_{ac}\gamma^{(\mu}_{ab}\gamma^{\nu)}_{bc}), \end{aligned} \tag{68}$$

however the complexity of the inverse definition makes it more convenient to define an auxiliary field λ^μ and define the γ matrix with its index down.

$$\begin{aligned} g^{\mu\nu}\delta_{ac} &= -2^{-1}\{\lambda^\mu, \lambda^\nu\} = -\lambda^{(\mu}, \lambda^{\nu)} \\ g_{\mu\nu}\delta_{ac} &= -2^{-1}\{\gamma_\mu, \gamma_\nu\} = -\gamma_{(\mu}, \gamma_{\nu)} \end{aligned} \tag{69}$$

Some dynamic interaction terms will then lead to these forcing of the inverse matrix relation for the trace of these at low enough energy e.g. through the "Higgs-ish" coupling in the action

$$S_c = M|g_{\mu\nu}(\gamma)g^{\nu\rho}(\lambda) - \delta^\rho_\mu|^2 \tag{70}$$

for a large "mass" parameter M.

In our many body tower of functions we need to ask how the couplings with such a gravity field γ^μ_{ab} would work. Modeling it on the electromagnetic field by introducing γ and λ labels to Ψ as in $\Psi^\nu_{(eA\gamma),\mu,abc}(x, y, z)$ has some appeal in thinking of gravitons as correlated with other particles but is problematic in the details. When we look at the modified Dirac lagrangian we find that there is always an extra μ index to accomodate:

$$\mathcal{L} = i(\phi_a\gamma^\mu_{ab}\partial_\mu\psi_b - \partial_\mu\phi_a\gamma^\mu_{ab}\psi_b) - 2m\phi_a\psi_a \tag{71}$$

Furthermore the γ function will need to span the full coordinate set of the function it is evolving. For example, when we wish to evolve $\Psi_{(eA)}(x, y)$ in the t^e direction we must multiply by a function $\gamma^\mu_{ab}(x, y)$ so that the $x^{(A)} = y$ coordinate must still be present even if it is only in a passive role. For these reasons it seems important to include not just a dual field $\Phi_{(eA)}$ to go with $\Psi_{(eA)}$ but an independent $\gamma^\mu_{(eA)}(x, y)$ field to contract with the derivative operator $\partial^{(e)}_x$. Note that we have labeled the gravity function γ^μ_{ab} with the electron and photon coordinate labels not some new graviton coordinate and it has only one μ and two a, b indices. This will persist regardless of how many coordinate functions are embedded in it. Thus the tower of functions of electron, positron and photon fields (and their Φ associated fields) there is an associated tower

$$\begin{aligned} &\gamma^\mu_{(A),Q}(x), \gamma^\mu_{(AA),abQR}(x, y)\ldots \\ &\gamma^\mu_{(e),ab}(x), \gamma^\mu_{(eA),abQ}(x, y), \gamma^\mu_{(eAA),abQR}(x, y, z)\ldots \\ &\gamma^\mu_{(p),ab}(x), \gamma^\mu_{(pA),abQ}(x, y), \gamma^\mu_{(pAA),abQR}(x, y, z)\ldots \\ &\gamma^\mu_{(ep),ab}(x, y), \gamma^\mu_{(epA),abQ}(x, y, z), \gamma^\mu_{(epAA),abQR}(x, y, z, w)\ldots \\ &\ldots \end{aligned} \tag{72}$$

This allows these functions to be straightforwardly coupled into the electron, positron and photon lagrangians using the mapping $g^{\mu\nu} = -8^{-1}\mathrm{Tr}\{\lambda^\mu, \lambda^\nu\}$.

The problem now is reduced to giving an evolution equation for these various γ^{μ}_{ab} functions in each of the implicit time directions. The Einstein-Hilbert action $S_{EH} = \int R\sqrt{g_{..}}$ suggests a start. The measure can be extracted from $g_{\mu\nu} = -8^{-1}\mathrm{Tr}\{\gamma^{\mu}, \gamma^{\nu}\}$. The geometric meaning of these terms is not clear but it is not necessarily required. We know that we want GR to arise in some, probably uncorrelated classical limit of particles over the energy scales we currently observe but beyond that we only require that we have a well defined set of evolution equations. Define the Riemann operator $\hat{R}_{(e)_i}$ to be the Riemann function of the connections $\Gamma(\lambda, \gamma)$ in terms of the two associated gravity fields where all the derivatives are taken with respect to the $x^{(e)_i}$ coordinate label, ith electron label, in the $\gamma_{(eee...ppp...AAA...)}$ function. The interactions are provided by the remaining classical lagrangians that now needs no delta function to localize the interaction.

The global gauge freedom we associate with norm $\Psi \to \Psi e^{i\theta}$ and $\Phi \to \Phi e^{-i\theta}$ does not involve the γ functions so it seems to not acquire or lose amplitude in the fashion of particle creation so exists as a new kind of field entity that makes gravity seem fundamentally different than the other fields even though the geometric nature of the theory is subverted in favor of a flat background formalism. It seems that any generalization of this theory needs three fields (with various particle label sets). It would be interesting to see if there is some high energy unification which treats them in a more symmetric fashion.

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