Constant quality of the Riemann zeta's non-trivial zeros

Petr E. Pushkarev
Kaliningrad, Russia
petr@petr.space

March 13, 2017

Abstract

In this article we are closely examining Riemann zeta function's non-trivial zeros. Especially, we examine real part of non-trivial zeros. Real part of Riemann zeta function's non-trivial zeros is considered in the light of constant quality of such zeros. We propose and prove a theorem of this quality. We also uncover a definition phenomenons of zeta and Riemann xi functions. In conclusion and as an conclusion we observe Riemann hypothesis in perspective of our researches.

1 Introduction

In this research we consider constant quality of the Riemann zetas non-trivial zeros. To uncover the subject indicated in the title, suppose a theorem and prove it.

Theorem 1.1. *If zeta function defined on the set of its non-trivial zeros (recurrently), all non-trivial zeros of defined zeta function have equal and constant real part.*

$$\forall itr \in \mathbb{N} \Longleftrightarrow \zeta_{itr} := \sum_{n=1}^{\infty} \frac{1}{n^{s_{itr}}}, where \tag{1}$$

$$\begin{cases} s_{itr} \in const_{itr-1} + i\,\alpha_{itr-1} \\ \alpha_{itr-1} = \xi(t_{itr-1}) \\ \xi(t_{itr-1}) = \frac{1}{2}\,s_{itr-1}\,(s_{itr-1} - 1)\,\pi^{-s_{itr-1}/2}\,\Gamma(\frac{s_{itr-1}}{2})\,\zeta_{itr-1}(s_{itr-1}), where \\ s_{itr-1} = \frac{1}{2} + i\,t_{itr-1} \end{cases} \tag{2}$$

$$\Rightarrow \zeta_{itr}(s_{itr}) = 0,\ where\, s_{itr} = const_{itr} + i\,\alpha_{itr} \tag{3}$$

Step of each zeta function's definition, we denote as itr from word "*iteration*". Respectively, defined zeta function is denoted as ζ_{itr}.

Equal and constant real part of non-trivial zeros, we defined as $const_{itr}$.

Now, let us prove it.

2 Definition on the Set

For a start, let us consider possibility to define Zeta function on the set of its non-trivial zeros (iterated zeta) itself. Could be such definition justified as part of mathematical research?

Let us examine such questions: could be justified such mathematical theory where exist a Zeta function which is defined on a set of its non-trivial zeros? Will such Zeta's definition lead to consequences which possible only in case of such definition and which is absurd by itself?

It is quite obvious that the set of non-trivial zeros must be somehow equal to the set of complex numbers. In this and only in this case, the result of iterated zeta's definition may somehow correlate with a definition of Zeta function itself.

It is clearly from Hardy's proof [2, Chapter X. The Zeroes on Critical Line. 10.2] that the zeta function has an infinite number of non-trivial zeros on the critical line.

But what about a density of the sets? Is the density of non-trivial zeros' set equal to the density of $\mathbb{C}$?

It is possible to note in the particular definition of 1.1 that we consider the Zeta function irrespectively to any particularity of any sets. As the number of Zeta's iterations is infinite, the behavior of each iterated zeta must be equivalent to any other. Otherwise, the number of iterations would be countable (countable infinity is absurd for us) or theorem has to be contradictory (what is has to be researched).

In this way, we can define an relation between two elements of the non-trivial zeros' set by following axiomatic relationship:

Axiom 1. The difference between two sequential elements of the set of non-trivial zeros ρ_n equivalent to the difference between their sequence numbers n.

$$\zeta(\rho_n - \rho_{n-1}) \sim \xi(n - (n - 1)) \tag{4}$$

, where ρ — the element of non-trivial zeros set, n — number of that element (we assume $\forall n \in \mathbb{N}$).

The relationship 4 obviously follows from the general Zeta function's definition and remains true in every Zeta's particularity ($\forall itr \in \mathbb{N}$) of theorem 1.1.

3 Conformity Object

To complete the picture, we find an indirect proof of theorem 1.1 first and direct after.

But before that it is necessary to observe question about the subject of equivalence and its place in our of research. When we say that $a = b$ and $b = c$ so $c = a$, the equality between c and a directly follow from subject of equality. But how happed such following for equivalence? When we say that $a \sim b$ and $b \sim c$ so $c \sim a$, it is necessary to explain fundamental reason that equivalence between c and a actually follow.

We can leave explanation of this reason behind the scope of this research because we directly define elements of each equivalence. In other words, fundamental reason that equivalence between c and a actually follows for us lies in our fundamental definitions of particular a, b, c. Thus, by referring to the definition of equivalence' elements, we can assume consistency of mathematical theory.

3.1 Absence of Object

Let there is such s_{itr}, that $s_{itr} = \neg const_{itr} + i\,\alpha_{itr}$ and $s_{itr} \in const_{itr-1} + i\,\alpha_{itr-1}$. Define it as $\neg s_{itr}$. Since the relation 4 is an axiom, then the relationship between elements of 1.1 should also comply with such relation.

Thus, from first theorem and relationship 4 follows that iterated alpha equivalent to alpha on previous iterated step, step which determines the set for definitions. We can say that this following happens by definition.

$$1.1 \overset{def}{\Rightarrow} \alpha_{itr} \sim \alpha_{itr-1} \Rightarrow \tag{5}$$

In order to relationship 4 and our assumption of $\neg s_{itr}$ presence has been correct, we have to also assume that iterated alpha of the set on which we define zeta are equivalent to $\neg const_{itr}$. From the definition of Xi, respectively, it is also follow such equivalence to t_{itr-1}.

$$\alpha_{itr-1} \sim \neg const_{itr} \Rightarrow \tag{6}$$

$$t_{itr-1} \sim \neg const_{itr} \Rightarrow \tag{7}$$

However, since the 4 is axiom, and 8 have to be true by definition, we can see that our assumption of $\neg s_{itr}$ leads to absurdity. Q.E.D.

$$t_{itr-1} \sim \neg const_{itr} \;\wedge\; t_{itr-1} \sim const_{itr} \;\wedge\; s_{itr-1} = \frac{1}{2} + i\,t_{itr-1} \tag{8}$$

$$\Leftrightarrow \frac{1}{2} \neq \frac{1}{2} \tag{9}$$

$$\square \tag{10}$$

Intuitively, 10 is possible to imagine as a such fact that context of something can be formed only from context of something which not contradict to that something.

3.2 Consistency of Object

Now we consider a direct proof of Theorem 1.1 From relationship 4 such relation directly follow for iterated zetas and xis on the same step of iteration. Furthermore, from Zeta and Xi definition equivalence also follow between iterated zetas and xis on different steps of iteration.

Respectively, all iterated zetas of 1.1 have to be equivalent to each other by Zeta's definition which take a part in relation 4.

$$4 \Rightarrow \tag{11}$$

$$\zeta_{itr}(\rho_n - \rho_{n-1}) \sim \xi_{itr}(n - (n-1)) \Rightarrow \tag{12}$$

$$\zeta_{itr}(\rho_n - \rho_{n-1}) \sim \xi_{itr-n}(n - (n-1)) \Rightarrow \tag{13}$$

$$\zeta_{itr} \underset{\xi}{\sim} \zeta_{itr-1} \underset{\xi}{\sim} \zeta_{itr-n} \Rightarrow \tag{14}$$

$$itr \underset{\zeta}{\sim} itr - 1 \underset{\zeta}{\sim} itr - n \Rightarrow \tag{15}$$

In this way and due a fact of zeta's definition on set of zeros, we can conclude that all iterated xis have to be equal as much as all zetas equivalent to each other. Intuitively, it can be represented as specific property of Xi from 4 by which Xi and all its iterated derivatives substantiate Zeta's zeros and fact of equivalence between iterated zetas.

$$\Rightarrow \xi(t_{itr}) = \xi(t_{itr-1}) = \xi(t_{itr-n}) \tag{16}$$

From definition of s obviously that such equality is possible if and only if all iterated s_{itr} equal by Xi. This implies that all real part on each iterated step is constant because such equality follows from relation 4. Q.E.D.

$$\Leftrightarrow s_{itr} \underset{\xi}{=} s_{itr-n} \longrightarrow const_{itr} = const\ \forall itr \in \mathbb{N} \tag{17}$$

$$\blacksquare \tag{18}$$

It may be curious to research Zeta's function in particularity of different *const*. However, such research is beyond the scope of this work where our aim was constant quality by itself. As a conclusion, we examine the Riemann hypothesis as special case of iterated zeta with *const* equal to $\frac{1}{2}$.

4 As an Conclusion

In conclusion, we can consider the Riemann hypothesis in the light of our discoveries.

$$(1.1) \longrightarrow \zeta(s_{itr}) = 0, where \tag{19}$$

$$s_{itr} = const_{itr} + i\,\alpha_{itr}\; and\; const_{itr} = \frac{1}{2} \tag{20}$$

Description of this famous hypothesis and overview of problems associated with the proof of this hypothesis may be found in the article "Problems of the millennium: The Riemann hypothesis" by Enrico Bombieri [1].

A proof of the Riemann hypothesis implies indirectly from 1.1 as 19. Respectively, we cannot say that work for finding proof of the Riemann hypothesis done until we do not consider its direct proof or an opportunity for that.

It's clearly, that any kind of direct proof should be somehow correlated with the theorem 1. Based on this fact, we can assume, that the direct proof suggest enumeration of **all** non-trivial zeros with real part equal to $\frac{1}{2}$ (as a $\forall itr \in \mathbb{N}$ from 1.1). Regardless of 1 and 19, an direct proof must plainly allow enumeration of an infinity. That contradict to the infinity's definition and remove for us possibility of an direct proofs for the Riemann's hypothesis.

Semantically, our research refutes the hypothesis, arguing that it is "critical line" beneath the non-trivial zeros, and not vice versa. At the same time, formally, the research make this hypothesis actually confirm.

References

[1] E Bombieri. Problems of the millennium: The riemann hypothesis, 2000.

[2] Edward Charles Titchmarsh and David Rodney Heath-Brown. *The theory of the Riemann zeta-function*. Oxford University Press, 1986.