Conjecture that states that numbers (4^n − 1)/3 where n is odd are divisible by Poulet numbers

Marius Coman
email: mariuscoman13@gmail.com

Abstract. In this paper I conjecture that any number of the form (4^n − 1)/3 where n is odd greater than 3 is divisible by a Poulet number (it is known that any number of this form is a Poulet number if n is prime greater than 3; such a number is called Cipolla pseudoprime to base 2, see the sequence A210454 in OEIS).

Conjecture:

Any number of the form a(n) = (4^n − 1)/3 where n is odd greater than 3 is divisible by a Poulet number.

Note: it is known that any number of this form is a Poulet number if n is prime greater than 3; such a number is called Cipolla pseudoprime to base 2, see the sequence A210454 in OEIS.

Verifying the conjecture:
(for the first twenty such numbers)

: a(5) = 341 which is a Poulet number;

: a(7) = 5461 which is a Poulet number;

: a(9) = 87381 which is divisible by 1387, a Poulet number;

: a(11) = 1398101 which is a Poulet number;

: a(13) = 22369621 which is a Poulet number;

: a(15) = 357913941 which is divisible by 341, 4681 and 49981, Poulet numbers;

: a(17) = 5726623061 which is a Poulet number;

: a(19) = 91625968981 which is a Poulet number;

: a(21) = 1466015503701 which is divisible by 5461, 14491, 42799, 233017, 688213, 1826203, Poulet numbers;

:   a(23) = 23456248059221 which is a Poulet number;

:   a(25) = 375299968947541 which is divisible by 341, 150851, 452051, 1082401, 2434651 and 7295851, Poulet numbers;

:   a(27) = 6004799503160661 which is divisible by 1387 and 22906579627, Poulet numbers;

:   a(29) = 96076792050570581 which is a Poulet number;

:   a(31) = 1537228672809129301 which is a Poulet number;

:   a(33) = 24595658764946068821 which is divisible at least by the Poulet number 2047;

:   a(35) = 393530540239137101141 which is divisible at least by the Poulet number 341;

:   a(37) = 6296488643826193618261 which is a Poulet number;

:   a(39) = 100743818301219097892181 which is divisible at least by the Poulet number 215749;

:   a(41) = 1611901092819505566274901 which is a Poulet number;

:   a(43) = 25790417485112089060398421 which is a Poulet number.