Fractal Emergence Matches Marginal Economics and Offers Quantum Mechanics Insights

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Abstract

As a fractal structure emerges – repeating a simple rule – it appears to share direct properties familiar to classical economics, including production, consumption, and equilibrium. Fractal geometry is found universally and is said to be one of the best descriptions of our reality – from clouds and trees to market price behaviour. This paper investigated whether the mathematical principles behind 'the market' – marginalism – is an aspect or manifestation of a fractal geometry or attractor. Total and marginal areas (assumed to stand for utility) and the cost of production were graphed as the fractal grew and compared to a classical interpretation of diminishing marginal utility theory and the market supply and demand. PED and PES were also calculated and analysed with respect to (iteration) time and decay. It was found that the fractal attractor demonstrates properties and best models classical economic theory, and from this, it was deduced that the market is a fractal attractor phenomenon where all properties are inextricably linked. The fractal, at equilibrium, appears to be a convergent – zeta function – series, able to be described by Fourier analysis and involves Pi, *i, e*, 0, and 1 (of Euler's identity) in one model. It also demonstrated growth, development, evolution and Say's Law – production before consumption. Insights from the fractal on knowledge and knowing are also revealed, with implications on what exactly 'science' is and what 'art' is. A connection between reality and quantum mechanics was identified. It was concluded marginal; classical economics is an aspect of fractal geometry.

Keywords

Marginal, Fractal, Elasticity, Utility, Cost, Production, Price, Growth and Development, Say's Law

1 INTRODUCTION

The marginal approach of economics is the foundation of classical economics: it forms the 'supply/demand' framework fundamental to our understanding of 'markets' – of all types and scales (macro and micro). It is described and presented in all elementary economics textbooks. It focuses on the extra of a select variable from another output unit. It deals with economic 'problems' surrounding the production and consumption of things and defines the value and prices of these 'things' through the equilibrium of the two.

As convincing as marginal economics is – and its ramifications – it often does not sit well with general society as a 'solid' explanation of reality and is almost always met with resistance where it tests 'our' morality. For this reason, it is rejected by the so-called 'natural sciences' and those on what is termed the extreme left, who see it as human creation and something to be controlled. Market lead 'capitalism' is assumed wrong, so markets and marginalism must be wrong; governments of this thinking should intervene in a system – often with dire consequences. Separate and parallel to the marginal revolution, a mathematical geometry was developing that is equally said to 'best describe reality'; it is fractal geometry.

This investigation presented and tested whether this – now modern – geometry matches classical marginal economics – and, if so, unifying economic theory with our reality. The hypothesis being reality – as described by 'classical' marginal economics – is an aspect of 'the (geometry of a) fractal'. The intangible marginal 'market' is an actual –real – geometry. By the fractal, economic theory is real and valid. Through the fractal, reality will be unified with problems associated with physics: the 'standard model' – namely (small scale) quantum mechanics and (large scale) cosmology. The fractal is a window on reality.

For this hypothesis to be true, the fractal will have to demonstrate all the proposed features—all found in a general economics textbook—including diminishing utility with an exponential function, total utility, production, and production cost, equilibrium, and foundational concepts of elasticity.

1.1 Fractals

Fractals are described as emergent objects that develop and grow with iteration. In reality, examples are found in the shape of clouds and trees. Fractals possess a property of scale-invariance – if viewed from a fixed position – regular irregularity (same but different) at all scales, and are classically demonstrated by the original Mandelbrot Set [1], and more simply by the Koch Snowflake (Figure 1, A and B respectively). Fractals point to a duality of the predictable and the unpredictable, the order and the chaos.

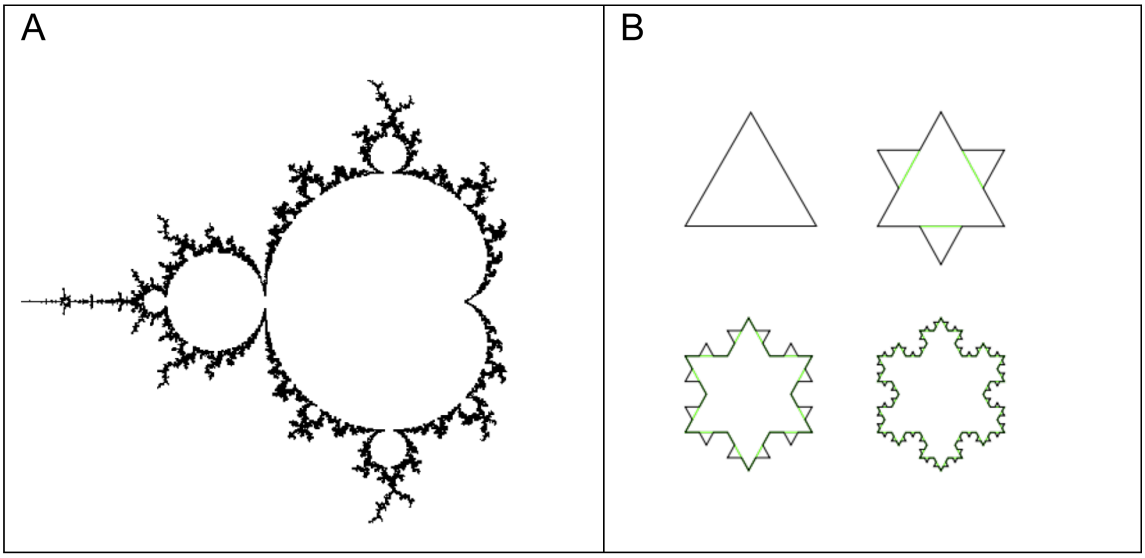

Figure 1. (Classical) Fractals. (A) The boundary of the Mandelbrot set; (B) The Koch Snowflake fractal from iteration 0 to 3. Reference: (A) [2]; (B) [3].

The classical fractal shape—as demonstrated in the Koch Snowflake—emerges as a result of the iteration of a simple rule: repeating the process of adding triangles.

To investigate the marginal properties of fractal attractors, the fractal Koch Snowflake was chosen for its (unrealistic but useful) quantitative property of regular regularity (same but same) at all scales. All triangles in the Koch snowflake are assumed to be the same, so we can do math with their areas while maintaining the fractal emergent properties. In an ultimate example of 'ceteris paribus, ' the Koch Snowflake can best quantify what is 'irregular' on fractals by isolating these 'chaotic' irregularities.

1.1.1 Vertical and Horizontal Fractal Perspectives

The fractal may be viewed from two perspectives: horizontal – from a fixed observation point either within or outside the fractal; and vertical – a perspective looking or 'zooming' into the fractal object (as demonstrated by a fractal zoom). These horizontal or vertical perspectives align with the definition of the fractal same but different at all scales. Horizontal is the assumption, scope and rational of this examination and refers to the other (variety) of observed objects (triangle bits) at the fixed or current observation point (or scale). It points to the classical – diminishing utility – theory. The vertical fractal perspective is to 'fractal-zoom' into the set. Here, the same pattern (or rule) will be observed at all scales. By zooming into the set, economic concepts of consumption, production, or competition may be reasoned or observed to exist at all scales of an organism, from the cell's scales to the scale of the global ecology or economy.

1.2 Marginal Analysis

The concept of marginal refers to changing a variable unit after changing another variable and is described in all elementary economics textbooks. Marginalism originates with the solution of the St. Petersburg Paradox, of which both Leonhard Euler and Daniel Bernoulli offered solutions.

1.2.1 Marginal Utility

The fundamental marginal model is the consumption Marginal Utility model shown in Figure 2 below, where the Total Utility (TU) is the sum of all the Marginal (extra) Utiles. When plotted on a graph, the negative sloping marginal utility (MU) curve diminishes as the TU curve increases at a decreasing rate with respect to the quantity consumed. When the TU decreases, it follows the MU goes negative. This negative utility will be tested with the potential of being reinterpreted.

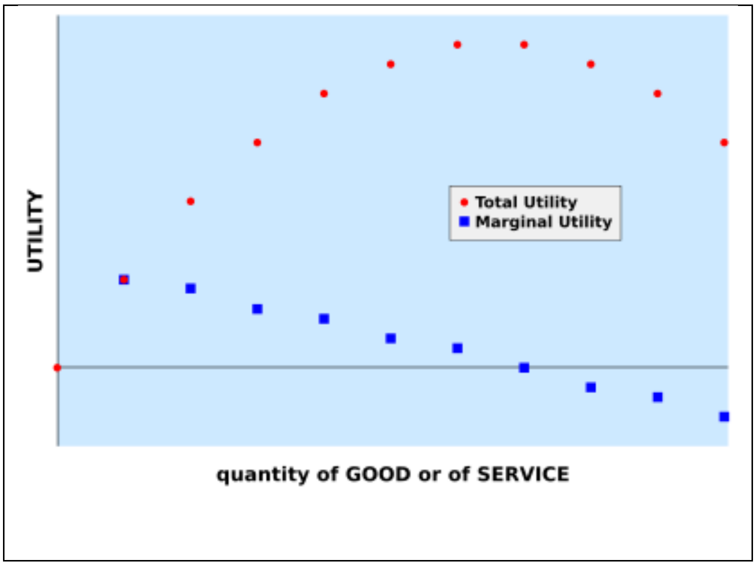

Figure 2. Marginal and Total Utility Model. Total Utility rises (to a limit) at a decreasing rate as the Marginal Utility diminishes (converges). The marginal utility goes negative where the total is decreasing [4].

While the St. Petersburg paradox's solution is said to be the origin of marginal utility theory, its parallel and contemporary problem, 'the Basel Problem' (attempted by Bernoulli and finally solved by his colleague Euler), is of interest in this investigation. The geometric series presented in the Basel Problem with limits and convergence and its solution with Pi have direct similarities in fractal emergence. Does the fractal offer insight into the Basel problem and the occurrence of Pi in statistical normal distribution equations?

1.2.2 Marginal Analysis and the (Koch Snowflake) Fractal

In this investigation, marginal analysis of the growth of the development of the Koch Snowflake fractal attractor was undertaken; to test whether the fractal demonstrates marginal utility (or benefit), the area of the triangle was substituted for utility and then analysed as the area changed as the fractal grew (and developed).

1.2.3 Supply and Demand Curves

The positive sloping marginal cost (MC) curve – the extra cost from another unit produced – derives the supply curve, and the MU curve derives the downward-sloping demand curve. Together, where both curves intersect – as shown in Figure 2 – they define the 'market clearing' price.

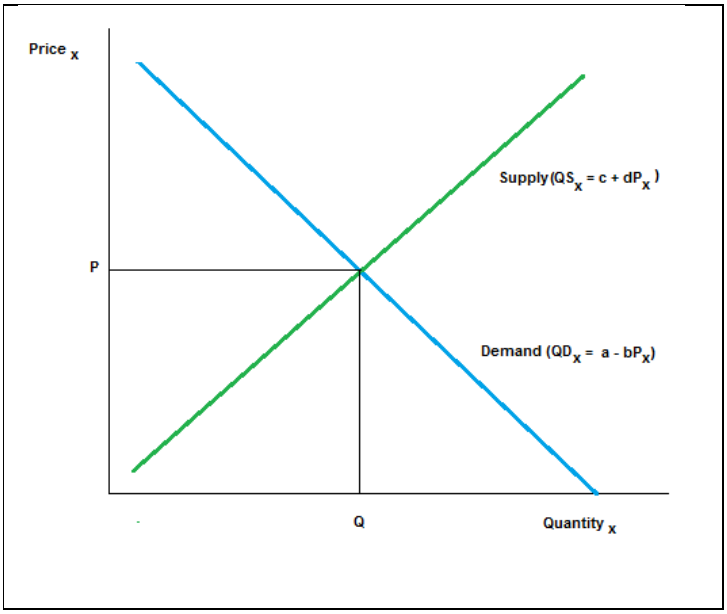

Figure 3. Classical Economics Supply and Demand Model. Price and Quantity of good x. The market equilibrium price is assumed to be where supply is equal to demand.

This intersect is also known as the equilibrium price. At the equilibrium price, consumer and producer surplus (the community surplus) is maximised.

2 Methods

The classical Koch snowflake fractal was used to create a quantitative data series of the fractal attractor. A spreadsheet model was developed to trace area expansion with iteration [5].

1. Consumption Utility and Observation,
2. production (cost) and,
3. Lorenz distribution, and
4. elasticity (of demand).

2.1 Assumptions and Units of Measure

2.1.1 Units

It should be noted that in this scale-invariant model, there is no standard or fixed scale of measurement. To address the scale invariance of the fractal, area and lengths are measured in centimetres, written as ***cm***.

2.1.2 Iteration Time and Quantities

Iteration is directly related to time and was assumed to be iteration time. The quantity of triangle (bits) was directly associated with iteration time (***i***).

2.1.3 Price

As price is assumed to be the value assigned to a (marginal) unit of utility in classical economics, in this model, price is assumed to be equal to the area of the (marginal) triangle bit.

2.2 Koch Snowflake Consumption (Marginal) Analysis Table

The following method refers to the model developed in the spreadsheet model (reference tab 1 marginal fractal). A table of 13 columns was created, and graphs were produced from the results. The following are the methods used to calculate each column.

2.2.1 Column 1, Iteration Number

The model was iterated to the 15th iteration (i_{15}).

2.2.2 Column 2, Triangle Base Length

Each successive small triangle's side length (l) is 1/3 of that in the previous iteration. The initial $\boldsymbol{l}$ was set to 1.51967128766173cm.

2.2.3 Column 3, Quantity of Triangles

The quantity triangles (of the previous iteration) are multiplied by four at each iteration time.

2.2.4 Column 4, Total Quantity of Triangles

The number of triangles (column 3) was totalled at each iteration.

2.2.5 Column 5, Individual Triangle Area

Area ($\boldsymbol{A}$) was calculated from the following formula: (1)

$$A = \frac{l^2\sqrt{3}}{4} \tag{1}$$

The area of the first triangle (i_0) was approximated to an arbitrary area of 100 cm^{-2}.

2.2.6 Column 6, Individual Triangle Total Area

At each iteration time, the area of individual triangles (column 5) was totalled.

2.2.7 Column 7, Marginal Area

This is the additional or extra area (Marginal Area) added to the total area of the Koch snowflake at each iteration. It is the number of triangles at each iteration multiplied by the area of the respective individual triangle.

2.2.8 Column 8, Total Area

This is the cumulative total area of the snowflake as it iterates. It is calculated by summing the marginal areas in column 7 at each iteration time.

2.2.9 Column 9, Individual Marginal Cost

The marginal cost was assumed to be the reciprocal to the MA column 6. This field is calculated by finding the reciprocal of the column.

2.2.10 Column 10, Marginal Cost

The marginal cost curve describes – by the increased quantity of triangles produced as the fractal iterates – the cost to produce the fractal – i.e. the energy and time spent at each additional iteration time. Rather than develop a production equation, which is possible, the reciprocal of the Marginal Area was used by assuming there is a direct relationship between the area produced and, inversely, the cost to produce the area. Other factors, such as technology and means of production, are assumed to be = 0.

2.2.11 Column 11, MA – MC

This is to calculate the area of the snowflake, the 'consumer surplus. Positive numbers were summed.

2.2.12 Column 12, Total Cost

The total cost (TC) is calculated by summing the marginal cost at each iteration.

2.2.13 Column 13, TC-TA

TC-TA is calculated by subtracting total costs from total area.

2.3 Production

The fractal must be inverted to demonstrate growth and production: the new triangle bits were assumed to remain constant. Methods of this production perspective of the fractal have been published in two previous papers. The results have implications for our understanding of the accelerating expansion of the cosmos, income (Lorenz) distribution and plants.

2.4 Elasticity

Classical economics elasticity – a variable's sensitivity to a change in another variable – was calculated on the fractal using a standard formula and described as a change in area to a change in iteration time (or quantity of triangles).

2.4.1 Standard Equation

$$e_{\langle p \rangle} = \frac{\mathrm{d}Q/Q}{\mathrm{d}P/P} \tag{2}$$

Where P represents the Area and Q is the quantity of the triangle at each respective iteration time.

2.4.2 Arc Method

$$E_d = \frac{\frac{P_1+P_2}{2}}{\frac{Q_{d_1}+Q_{d_2}}{2}} \times \frac{\Delta Q_d}{\Delta P} = \frac{P_1+P_2}{Q_{d_1}+Q_{d_2}} \times \frac{\Delta Q_d}{\Delta P} \tag{3}$$

2.4.3 Mid Price.

$$E_d = \frac{P}{Q_d} \times \frac{\mathrm{d}Q_d}{\mathrm{d}P} \tag{4}$$

3 RESULTS

The results of the total experiment are broken into three sections:

1. Consumption and Observation,
2. production (cost), and
3. elasticity.

The results can be found on the spreadsheet model [5]. Figures 4 to 6 show graphically the Total Area, Marginal Area and Cost and Equilibrium.

3.1 Consumption (Utility) and Observation

3.1.1 Total Area (TA) and Marginal Area (MA)

The Koch snowflake fractal TA curve began with an area value of 100 cm^{-2} at iteration 0 and thereafter increased—converged—to a limit of 160 cm^{-2}.

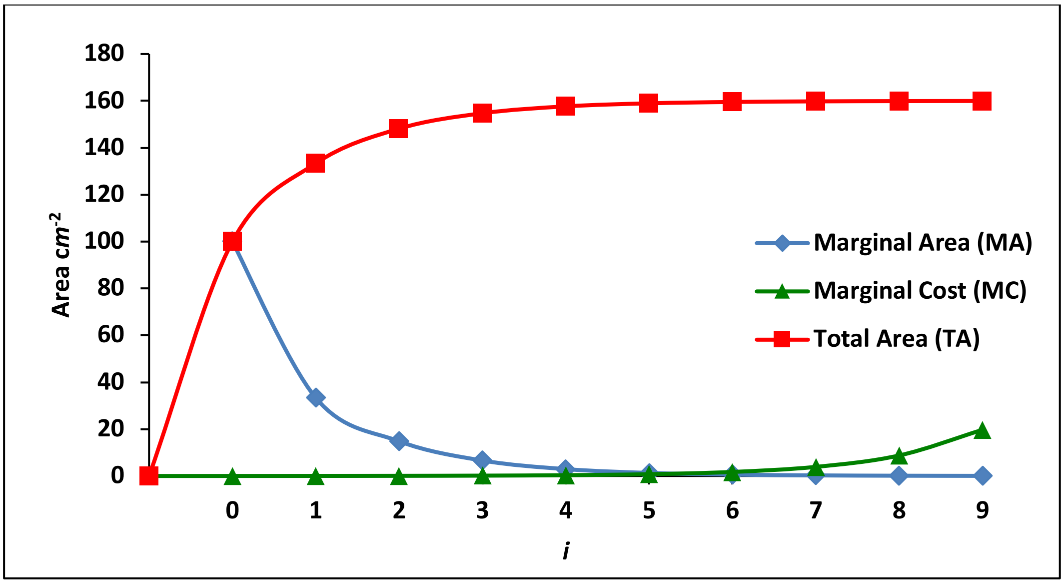

Figure 4. Composite Fractal Equilibrium: TA, MA and MC. As the (Koch Snowflake) fractal iterates, the total area increases from an arbitrary value of 100 to a limit approximating 160, and the 'marginal' area decreases (was convergent) from an arbitrary value of 100 towards – but never reaching – 0. The cost of the product increased by reciprocalizing the area. The shape equilibrium of the fractal was reached where the MC was equal to the MA – at around iteration-time (*i*) 5. *cm* = centimetres.

The MA curve was downward sloping and convergent, beginning with an area value of 100 cm^{-2} at iteration time 0, decreasing exponentially and approximating, but never reaching 0.

Figure 5 shows the Koch snowflake fractal MA curve function; the following equation describes it:

$$y = 14.815e^{-0.811x} \tag{5}$$

where y is the Area and X is the iteration time (relating directly) to the quantity of triangle bits.

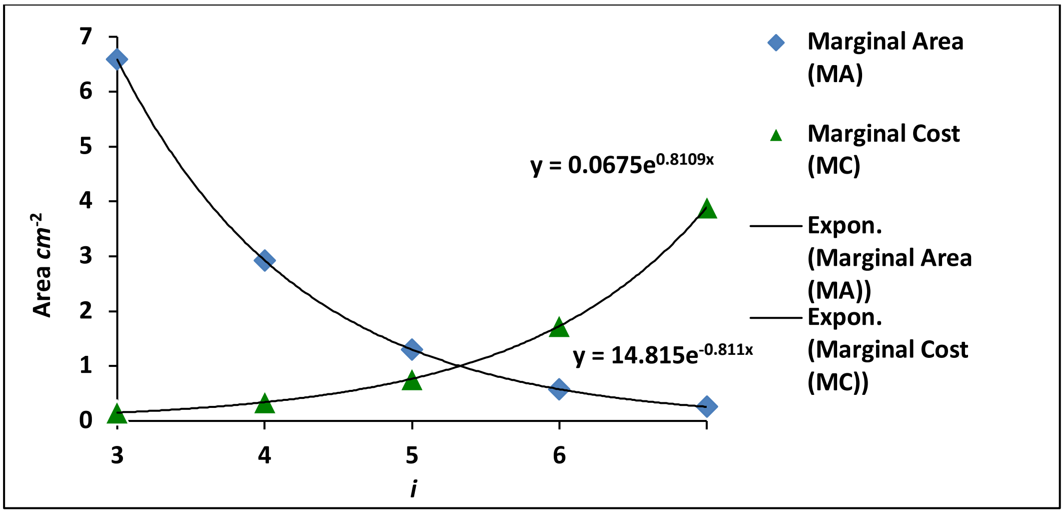

Figure 5 Fractal Equilibrium – close-up. As the (Koch Snowflake) fractal iterates, shape equilibrium (the intersect) is reached where the MC is equal to the MA at approximately iteration 5 and area of 1. *i* = iteration time; *cm* = centimetres.

A log-log diagram of the dataset was produced (see appendix Figure 10) and described as

$$y = 81.168x^{-0.592} \tag{6}$$

3.1.2 Marginal Cost (MC)

The MC curve in Figure 4 began from a value near 0 and increased exponentially. From Figure 5, the fractal MC curve function is described by:

$$y = 0.0675e^{0.8109x} \tag{7}$$

The same function was written as a log-log (see appendix Figure 8):

$$y = 0.0123x^{0.5918} \tag{8}$$

3.1.3 MA and MC Equilibrium

Figure 5 shows the MA and MC curves together: they intersect at approximately 5 iteration times, where the Area is equal to 1 cm-2. Iteration is discrete: there is no iteration 5. 3. The numbers may explain why equilibrium is not exactly at iteration 5: at iteration 5, the cost area and area are not exactly 1.

3.1.4 Total Area (TA) and Total Cost (TC)

Figure 4 below shows the **TA – TC** is maximised at a value of 158 cm^{-2} at iteration time 5.

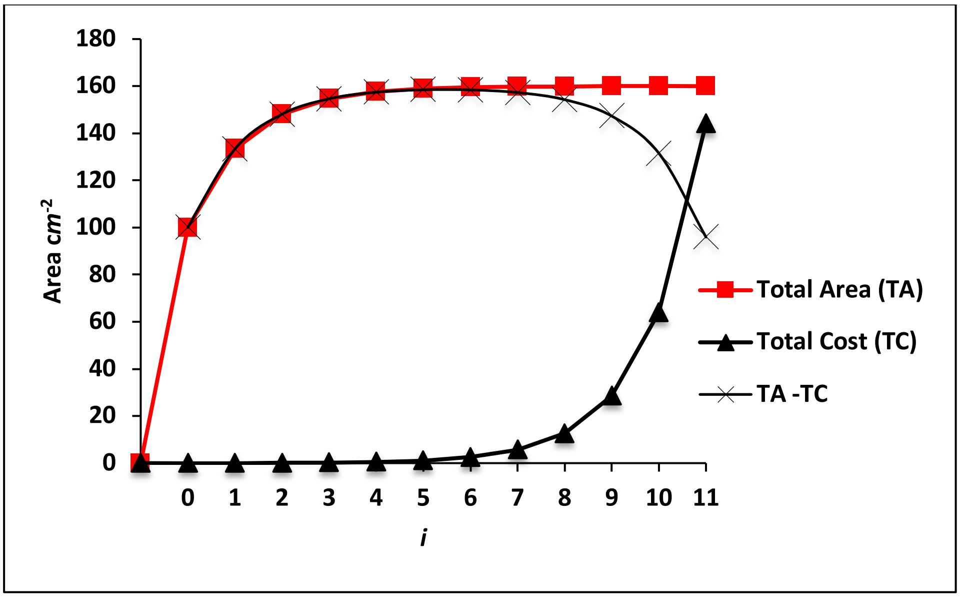

Figure 6 Total Cost and Total Area. The difference between the rising TA and its inverse, the TC, is maximised at five iterations. *i* = iteration time; *cm* = metres.

3.2 Elasticity

The results for 'price elasticity of demand' are as follows.

3.2.1 Standard Elasticity Equation

The denominator was constant over all iteration times at -1.25, and the numerator constant at 3, with a resulting elasticity of -2.4.

3.2.2 Arc Method and Mid-Price

PED values calculated by the arc and mid-price methods were -1.32 and -5.4, respectively.

4 DISCUSSIONS

While this analysis of the (Koch Snowflake) fractal attractor is directed at classical economics, due to the universality of the fractal, it may also have direct implications on epistemological (knowledge and information) studies and the possibility of our understanding of the physical world, including the quantum mechanics. Keeping with the main aim of this investigation, implications to marginal economics will first be discussed, followed by – or often intertwined within – other insights. Just like the fractal itself, the insights developed here are not at all 'linear' but complex: so, the order or flow of insights is complicated and may not be in line, but they will be covered.

This investigation complements previous studies by the author on other aspects of the fractal:

1. the fractal was inverted to measure observations from a fixed position within a fractal; this revealed points recede exponentially, and insights were pointed to growth and observations of the Cosmos, plant (tree) growth and economics the increase in value of a good with respect to time [6] and [7];
2. and area distribution of the fractal, offering a possible explanation for the Lorenz distribution [8].

This investigation leads me to examine the isolated fractal – already it has been 'seen' to have similarities to the way quantum mechanical systems are described.

As trivial as this may sound, even as I write this work, I am aware I am 'producing' something for my readers to 'consume'; when iterated, they will 'know' – this is a fractal.

4.1 Ceteris Paribus – Fractal Isolation

Ceteris Paribus – holding all other things constant – is a central assumption behind economic models and analysis, and indeed – arguably – all science. Laws derived from this method are called ceteris paribus laws. It is an assumption that allows scientists to study the pattern of the object in question: without it, the 'cause' to the 'effect' would not be discernible - or would be confused in the 'chaos' of factors.

The fractal in isolation demonstrates this ceteris paribus is a property of reality: if there are no other factors (or other fractals) in the image –including the Koch Snowflake fractal – the isolation leads to a problem of no scale or known position, it leaves the observer lost, or not able to know. In reality, this ceteris paribus can be found in locations or situations with no reference points, no discernible scale, and where one feels literally lost. The best example in economics may be the concept of trade and exchange: they are universal, observed at the microcellular, even the molecular, to all macro levels of life. The idea is the same but different: only when the scale is revealed, the difference is revealed.

I will return to this and discuss it further under the knowledge section 5 below.

4.2 Marginal Economics

This fractal model does not use any arbitrary numbers, as presented in elementary economics textbooks, to model marginal theory (other than the initial set arbitrary reference area and the unit of measure used (cm^{-2}) set in the model). It demonstrates many classical economics fundamentals.

4.2.1 Total Utility and Diminishing Utility Demonstration

The fractal is the perfect demonstration of diminishing marginal utility. In Figure 2, we can see, in accordance with marginal theory, that the total area of the (Koch Snowflake) fractal increases – at a decreasing rate – after iteration, while the marginal area of the fractal diminishes. Conversely, it may now be interpreted the model diminishing marginal utility is a perfect demonstration of the (universal) fractal and is best described by fractal geometry. Examples are truly universal to this phenomenon; it may be more interesting to identify where it is not. It applies to value: the first of anything will command the highest value – all else being equal; after that, the value diminishes.

Examples of the high value placed on the may be the first to walk on the moon, the first rock stars, classic aircraft – classic anything, the special positions of pilots, etc. These examples will always hold value over their followers. As the quantity increases, the value of all the above diminishes. The Internet has today, at the time of writing, increased the quantity of information. Consequently, there is a proliferation of information – information that was once of great value, has now become relatively trivial.

Due to the continuous 'vertical' nature of the fractal (fractal zooming: not addressed directly in this paper), all events (such as the moon landing, for example) have developed and evolved from earlier beginnings; they are not standalone events or objects; they have complexity. They are a continuation of an infinite (production) process. And by this, there will be more 'exciting' events to come—but none like the first.

This model shows prices (the value assigned to the marginal area value) ranging from an arbitrary value of 100, decreasing towards 0: it is asymptotic. The MA curve shows a superposition of all areas: it is an area (or bit) possibility curve assuming, just as with classical economics, all other variables affecting the MA or MU are held constant (ceteris paribus). Here, the area represents the price. Notwithstanding the possibility of Veblen goods, Giffen goods and the like where the law of demand is contradicted, this fractal-derived downward-sloping MA curve shows the law of demand is inextricably linked to a market structure and is an aspect of fractal geometry. If bits or prices were the same size, the fractal would not emerge.

4.2.2 Consumption and the Demand Curve

The negative sloping MA curve demonstrates the classical economics demand curve and the relationship between price and quantity demanded. The demand curve may now be explained by the fractal and interpreted as one aspect of a fractal phenomenon. In accordance with marginal theory, this 'demand curve' is an exponential function. When converted to a linear log-log function (see equation 6), the function can be described in the traditional form:

$$\mathbf{Q_D = a - bP} \tag{9}$$

where $\mathbf{Q_D}$ is the quantity demanded of good x; **a** is the x-axis intercept; **b** is the slope of the curve; and P is the price of good x. Notice that the MA curve is asymptotic; it does not cut the x-axis (See Appendix Figure 10).

The 'non-price determinants of demand'—income, the prices of substitutes and complements, and so on—are all tropisms of the real fractal structure.

4.2.3 'Marginal' History and the Basel Problem

The convergent MA curve and the TA curve increasing to a limit match how a Zeta function (power or Zipf's law) is described. This fractal demonstration shows that this is no coincidence. The fractal attractor is a Zeta function and thus describes utility theory.

The mathematics of the utility is said to have begun with Daniel Bernoulli's solution to the St. Petersburg paradox: this may be so, but I suggest it is the soon later 'Basel problem' that best describes the geometry of marginal utility. Its added sums and geometric sequence – with convergent and limit – model our understanding of 'marginal' and total utility. The Basel problem's limit – or solution – and convergent properties remarkably resemble the limit (1.6) and convergent properties of the (Koch snowflake) fractal attractor. Daniel Bernoulli initially thought the Basel solution to be 1.6 by the equation:

$$1\tfrac{3}{5}\ . \tag{10}$$

Leonhard Euler counted this with what is said to be one of the most remarkable solutions in mathematical history:

$$\frac{\pi^2}{6} \tag{11}$$

with a limit equal to 1.64 (2sf).

4.2.4 Pi (π)

While Euler's solution is greater than the limit value of the Koch fractal (it is close), there is a fascinating insight from the emergence of the fractal that may shed light on what it is that makes the Euler solution so remarkable: the Euler's solution has π within it, and the fractal demonstrates π . Pi describes cycles and rotating or spiralling propagation: the fractal can also be shown to spiral as it propagates. Full discussion of this insight is outside the scope of this investigation, but it must be mentioned.

The (Koch snowflake) fractal, as shown in Appendix Figure 11 A, B, and C and [9], propagates as a spiral – it rotates as it diminishes and converges. From this fractal property, it may not be a coincidence that the fathers of utility theory described the Basel convergence finding π within it. TAs demonstrated here; the fractal propagates with rotation or spiralling described only by 척: this would suggest they are the same.

By full shape emergence (equilibrium), its have rotated/spiralled through 2π or 360^{o}. If we look at an alternative fractal structure, the tree, the distribution of branch size, (assuming no irregularity of branches as assumed with triangles on the Koch snowflake) of a fully developed tree has – from the trunk to the most minor observable – a distribution that not only resembles the MA/MU/demand curve but is also rotates. The 'fathers' were – unbeknown to themselves, in their time – describing the fractal.

4.2.5 Euler's Identity Fractal Conjunction

The fractal may give credence to Euler's Identity

$$e^{i\pi} + 1 = 0 \tag{12}$$

– is said to be the most 'beautiful' equation in mathematics [10]. I conjecture Euler's identity exists as an aspect of universal fractal geometry.

The shape equilibrium of the fractal is reached where the MA equals one and the MA – MC (and the marginal community surplus) = 0. Triangle bits are rotated (as demonstrated) to achieve this equilibrium, invoking π – and the possible use of complex numbers – imaginary numbers, *i.* Euler's number '*e*' is derived using a convergent series similar to that used in the fractal. I do not think the occurrence of these fundamental constants at fractal equilibrium is a coincidence and should be further investigated. If true, it will have significance on the universality of mathematics.

4.2.6 Normal Distribution Equation

Further to this but related, π also shows up in statistical analysis, namely the 'normal distribution equation'. This π may also be explained by examining the rotational development of the fractal. Suppose the fully developed fractal is assumed to be at shape equilibrium. In that case, the 'snowflake' has formed – by around 5 or 6 iterations (as demonstrated), it has come to this equilibrium by rotational/spiral propagation. If we assume a full population of bits has been produced in these 5 or 6 iterations, they and particularly their orientation have spiralled to be there in this process. Again, this fractal geometry may account for Pis' presence – within 'everything' or population. This claim may not be conclusive as it is assumed that all triangle bits are identical in this model, but what if – as in reality and with 'natural' tree fractal structures – every bit is intrinsically different from one other by its complex nature? The distribution will arguably form a bell-shaped curve (if measuring with a dependent variable, size, for example).

4.2.7 Fourier Transform and the Demand Curve

If the quantity of bits per iteration time is understood to be the frequency of bits, and the area of bits the amplitude, it may be inferred the MA curve is a Fast Fourier Transform (FFT) of the Koch snowflake – a compressed summary of all the activity in the (fractal) system. If so, this discovery may offer insights into the FFT and all its applications – including with atomic physics (see below). It also implies every fractal attractor, from trees to waves and clouds, may be described as a Fourier Transform, and also every market, at any scale: the complete fractal attractor is, therefore, a perfect example of a Fourier Transform – a range of bit frequencies per iteration time, all in one 'superposition'. If FFT analysed all prices in a market and all quantities, the convergent pattern would repeat.

4.2.8 Insights into Quantum Mechanics

To form an MU (MA) curve from an emergent fractal implies bits are spiralling as it iterates (see 4.1.4 above); this implies a wave: a superposition standing wave of different frequencies at different amplitudes (or wavelengths). The demand curve is a possibilities curve of all prices and quantities. Still, it equally describes positions on a sinusoidal wave – through (iteration) time – able to be written by the same equations used to describe quantum mechanics – namely the Euler Formula (below). For this reason, the Euler equation is argued by quantum scientist Richard Feynman as 'the most remarkable formula in mathematics'.

$$e^{i\theta} = COS^{\theta} + SIN^{\theta} \tag{13}$$

Through the fractal, the concept of marginalism is a direct window into electromagnetism and the quantum world, where bits (particles) and waves of different frequencies are in (unobserved) superposition, with non-location until 'observed'.

4.2.9 Demand Curve as the de Broglie Wave Function

According to the de Broglie wave function (and quantum mechanics), all things are said to ('weirdly') have a wave function—be described as both a wave and a particle. In this investigation, the fractal makes the quantum world and 'economic' reality act as one and the same. From this, I claim the answer to this 'weird' quantum conundrum is that they are both aspects of fractal geometry.

The de Broglie wave function of quantum mechanics is directly akin to – if not by definition, that same as – the MU curve. The equation describes it

$$\lambda = \frac{h}{p} \tag{13}$$

where λ is the wavelength, h is Planck's constant, and p is the momentum of a photon particle (a bit). Drawn on a diagram, λ against p, the downward sloping 'demand curve' like curve is revealed. The resemblance is given credence when the λ is compared to the price or utility, and the p (momentum) the velocity or frequency related to the quantity demanded. It is a stretch, but through the fractal, both are revealed and the relationship is the crux of this investigation: it should at least open further investigation.

4.2.10 Negative Marginal Utility Misattributed

For the Bernoulli Euler Basel problem model to be the fundamental model of Utility theory, the marginal utility must be convergent and not go negative as described in current models. The fractal addresses this problem by incorporating production cost. As the fractal shows us, production and consumption being simultaneous (this will be developed later in the discussions), the increasing MC after fractal equilibrium (green in Fig. 3) can account for the claimed 'negative marginal utility'. As consumption or iteration increases, production costs increase to a cost value greater than the marginal utility. The marginal utility does not need to go negative, as there is never consumption without production.

4.2.11 MA and TA and the 'Diamond Water' Paradox

At early iteration times, the MA is high while the TA is low; as the fractal develops, the TA increases and is maximised where the MA is near its lowest. This is in total accordance with marginal theory and supports the diamond water paradox.

4.2.12 The Long Tail; Pareto 80-20 Rule

The downward-sloping MA curve is potentially infinite in size and is scale-invariant: it will not change its shape at any iteration time. The MA curve is a 'power law' curve and demonstrates Pareto's '80–20 rule: 80% of the shape is experienced with only 20% of the iterations. By iteration time 4 or 5, the shape has changed and formed most noticeably to the observer; after that, extra iterations do not change the shape of the emerging (snowflake) object. The curve shows the infinite positions of triangles; it is a superposition curve. It is the perfect demonstration of the long-tail demand curve.

4.3 Production Cost and Supply

While the snowflake is consumed, it is at the same time – simultaneously – produced. The fractal, on its own, without observation, demonstrates production. The Koch snowflake fractal is an emergent object produced by the iteration of a rule (adding triangle bites to the earlier triangle bites) as described in the methods. By iteration time 4 or 5, a Koch snowflake fractal appears complete to the observer (as described by consumption in the section above); from this iteration time, the cost of production (the MC in Figure 2) increases, and the curve rises. The MC curve measures the cost production of an extra iteration in terms of cm^{-2} s per iteration time (or a number of triangles); in classical economics, this cost is reduced to the opportunity cost – the cost of the next best alternatives and it can equally argue to be relevant to fractal production. The MC increases exponentially: this is consistent with the derivation of the classical economic (short run) supply curve, and when plotted on a linear log-log diagram of the function (equation 8), can now be written in the traditional

$$Q_s = c + dP \tag{14}$$

where $\mathbf{Q_s}$ is the quantity supplied of the good; **c** is the x-axis intercept; **d** is the slope; and **P** is the price of good x. (See appendix figure 10)

4.3.1 Technology and Other Production Factors

The production output of triangle bits depends on the technology used (and other factors such as the producer's education): from a thick pen to a thin pen to draw or from freehand to an electronic computer. When technology is used, more iteration can be achieved; the observer can look into the fractal set employing fractal zoom. These other 'non-price determinants of supply' are all tropisms to the natural structure.

Anyone attempting to draw the snowflake fractal by hand and pen (technology) may do so easily until around the fourth iteration; after that, the activity becomes difficult—the detail will not be seen, and the cost, measured in time taken, will be high.

Technology production is a fractal phenomenon of its own; it will develop over time, and its existence will give feedback on the original fractal. It lowers the cost of production, making it easier to produce.

4.3.2 Production and the Market Structure Market Structure – Monopoly to Competition

The emergence of the fractal demonstrates the market structure (as shown below in Figure 8), with monopoly pricing and power at early iteration times and a competitive price and power at shape equilibrium.

4.4 Fractal (Price) Elasticity

The fractal demonstrates elasticity and offers a theory of the origin of this measure. The fractal reveals elasticity to be a property of all nature. Discussing the different coefficient values of the Koch snowflake fractal calculated in this examination from the different calculation methods is not the scope or topic of this paper; however, the calculated coefficients reveal a constant elasticity figure, which is consistent with standard theory.

4.4.1 Demand Curve Elasticity

As the MA curve (the demand curve) is an exponential curve, it has a constant price elasticity of demand (PED). Using the same exponential data, the curve becomes a linear curve on a log-log diagram: the PED along its length varies from infinity (or high) at its uppermost end to 0 at its lowermost end. This change in the elasticity of the fractal attractor suggests a universal pattern, not only for economic models but for all knowledge. There is a 'sensitivity' (elastic) shape (area) to change at early iteration times and insensitivity (inelastic) at later iteration times. The elasticity is largest at low iterations due to the highest, most outstanding, marginal Area or utility before iteration 4 or 5. At early iterations, the change is noticeable; the marginal utility is similar and high, and another iteration has a similar high utility; conversely, at later iterations, the extra utility is low and inelastic. This elasticity change with time is inextricably linked to fractal growth.

4.4.2 Price Elasticity of Supply

As with the log-log MA curve, a log-log curve of the MC curve will produce a linear curve. The shape elasticity in – terms of time – is elastic at the lower – early iteration – end of the curve and conversely increases through to inelastic at its upper end – later iteration. Early in the fractal development, it is easy to produce (elastic) – the producer is responsive; conversely, it is increasingly difficult – the producer is less responsive – with time.

4.4.3 Elasticity, the Fractal and the Product Life Cycle

The product life cycle follows fractal geometry. In the early stages, the product is unique with high value (bit size), high marginal utility, and high elasticity: as the fractal shape emerges, the bit size decreases, the marginal utility decreases, and the elasticity decreases.

4.4.4 Elasticity and the 'Product' Life Cycle and Knowledge

In context to a product (or knowledge) life cycle, early iteration times pertain to early stages of production and consumption and are elastic – another iteration has high marginal utility or benefit – and later stages conversely are inelastic, 'boring' with low (marginal) value; another iteration will not change the experience. This interpretation is contrary to standard references, with early stages said to be inelastic in demand [12].

4.5 Fractal (Shape) Equilibrium – Simultaneous Production and Consumption

So far, I have shown that the production and consumption of fractals are inextricably linked to one another. The simultaneous diminishing of area and additional production cost (with iteration-time) will balance or be equal at – and around – 5 iterations: this intersection may be referred to as fractal or object equilibrium and is shown clearly in Figure 3 above. At this point, the emergent snowflake structure has formed and, to the observer, is complete. Any more iterations of the function will not add more detail to the shape; conversely, fewer iterations will diminish the shape (more on this when discussing knowledge and the fractal). The shape at an iteration time less than the equilibrium iteration time will be less than the equilibrium shape – at this point, more can be gained with more iterations. Conversely, at iteration times greater than iteration equilibrium, now more shape detail can be defined by an observer, and more iteration come at an exponentially increasing (opportunity) cost (in terms of time).

Fractal equilibrium is taken from a static observation point, outside the fractal – or from within the fractal looking forward: it is a horizontal view.

4.5.1 Fractal Zoom

The equilibrium is a relative equilibrium: further probing—or zooming—will uncover more detail, making the observation dynamic rather than static. Importantly, the same (but different) shape will be observed if there is zooming. Zooming is a vertical perspective.

This fractal equilibrium is complete symmetry, the superposition of a rule, and we observe this in all objects in our reality, both tangibly and intangibly. Trees and markets are examples.

4.5.2 Seven ± 2

From a fixed viewpoint, all fractals ('attractors') form their shape (are at fractal equilibrium) at and around 7 2 iterations. Any more iterations than this will incur a high cost and no extra benefit. The 5 iterations to develop the fractal Koch snowflake in Figure 5 (above) are the point where the MA and cost MC intersect and where the shape of the snowflake is fully developed.

This, I believe, is a demonstration of the "Magical number seven, plus or minus two: some limits on our capacity for processing information", first posited by Miller in 1958 [11].

This number can also be observed throughout our reality: from any standpoint, there will be around 4,5,6,7,or 8 levels of protrusion. For example, from where I am writing, I can see out my window where there is a park and some buildings. The building is the first protrusion; then there is a chimney on the building, then brick on the chimney, then there is—I can just see—an icicle (it is winter): 4 levels in total.

Similarly, on a tree, if you follow the branches out from the trunk until they first fork, then follow that branch until they fork again, and then go on repeating, following the 'first' branch on the branch until you cannot see any more branches, you will find you can only fork 7 ± 2 times.

Iteration 5 is the optimal or perfect viewing iteration of the (fractal) Koch Snowflake from the viewer's perspective. I believe it is also the number of iterations, or feedback, needed to gain market equilibrium.

Shape equilibrium may also be significant, with '6 degrees of separation' between knowing everybody in the world. I have heard that in reality, it is around four before the link ends and fades away.

4.5.3 Fractal Paradigm and Field

The iteration time '0 to fractal equilibrium iteration time – 7 ± 2 – iteration times may be considered a fractal paradigm: a time from a fixed position where no more information can be observed beyond this range with current technology – to go beyond a fractal paradigm one must zoom into the set. An implication of this iteration information 'paradigm' is the scale of the field between the largest and smallest. With respect to an observation at the smallest bit size (7 iterations), the largest bit size is extremely large: the smallest bit size is some 99% of the largest bit size; and conversely to the largest, the smallest is extremely small. This large 'field' size scale can be calculated (is outside the scope of this study) and may offer insight, along with the above quantum insights, to Lord Ernest Rutherford's empty atom – which is described to be 99.999999999% empty between the outer electrons and the inter nucleus. With respect to the other quantum insights inferred from the fractal, this atomic field size of the fractal matches the atomic problem and this 'demands' further research.

This property of the fractal is inextricable linked to all other properties of the fractal – namely change through time – evolution. The each new paradigm, the same will be experienced, but there will be difference.

4.6 Price Determination

In classical economics, the market-clearing price is defined as ‘supply is equal to demand’: this fractal model demonstrates—and derives—this equilibrium price. The fractal already describes the behaviour of prices through time—where market prices are well understood to be fractal [1]. The interpretation proposed in this paper suggests the determination of price itself is also fractal, and Mandelbrot’s findings are inextricable.

4.7 Inflation

Two insights may be taken from the fractal on the topic of price inflation: the lack of repetition of a concept of ‘inflation’ in nature, and definitions of a fractal ‘coastline’ resemble inflation definitions.

Economic principles are shared with the natural science of biology; for example, income, production, consumption, selection, etc. – interestingly, there is no concept of (price) inflation in nature. Inflation appears to be ‘everywhere and always’ a cultural phenomenon alone. In a cultural context, there are many examples of inflation, all pointing to a devaluing of the principal object. For example, adding stars to hotel ratings: increasing the standard 5 to 7 stars. Product inflation may be when a (cosmetic) change is made to a good – with no innovation added – and thus results in the devaluing of the (same) previous good.

A key property of the fractal object or attractor is its infinite boundary. The claim is made – based on this property – that all objects have a ‘fractal dimension’, and this is classically revealed with the measurement of the coastline of an island. The fractal argument says the length of a coast depends on the measuring instrument's length: the shorter the instrument the longer the length of the coast. I claim this property and island application is what price inflation is also: where the measuring instrument is a currency, and the coastline the object – or the good or service in question. As the currency is debased (resulting in a shorter measuring instrument), the 'price' value of the good in question is 'inflated' – even if the good in question has not changed at all.

4.8 Say's Law

The fractal directly demonstrates Say's law – production comes before consumption. The fractal was first produced and then viewed or consumed. This is a law of the fractal: the converse cannot be true: one cannot view the changes without the changes first being produced. This observation of the fractal is universal to all objects and will be returned to in reference to knowledge and the distinction between science and the arts in section 5.

4.9 Trade and Exchange

As the 'snowflake' is produced, it may – in principle – be traded or exchanged with a consumer – a 'consumer' external to the model in question. Without this transaction, production would only be on its own. If this transaction takes place, the more that is produced (iterated), the more the consumer benefits as the first iteration is valued relatively high compared to those after that. This relationship is primal and universal and gives insight into the economy of life itself: primordial life may have begun on these principles – production followed by consumption.

4.10 Growth, Development, Evolution, and Sustainability

Simultaneous with production: The snowflake develops as the quantity of triangle bits increases with iteration time. Growth and development are inextricably linked and are an aspect of fractal production.

4.10.1 Growth

This snowflake structure at equilibrium does not demonstrate growth but only structure: systems do not have activity at the largest iteration 0 bit size first; in fact, it is the complete opposite; they grow from the newly added bits. The focus is placed on the newly added bits to demonstrate and describe growth, and to show this fractal must be inverted. As a bit is added (or the specific quantity of bits for any iteration time), the previous bit size must expand or grow in size; this is growth, and it is exponential. I have termed this fractspansion, and in my earlier papers [6], I posited dark energy and the growth of plants may be best described by this fractspansion.

An important insight from shape equilibrium is that growth has limits—the total area of the fractal is finite and will grow at a decreasing rate.

4.10.2 Development

Development is related to the complexity of the fractal structure. The iterating fractal demonstrates the duality of growth and development; they are inextricably linked. For example, as a fractal tree grows, so it develops.

4.10.3 Evolution

A fractal demonstrates change over time and so demonstrates evolution – the change or development of an object over time. Evolution is demonstrated in the fractal – by zooming (into time) and observing the 'different' (or irregular) of the 'same' (or regular) shape (or object). Evolution is universal to the fractal, and indeed, taken from what we can observe, the universe itself appears to be evolving. The fossil record is a record of 'living' things through time. Evolution and development are the same principle and may be indistinguishable from each other without a (time) scale reference. The tree becomes the tree of life. It is 'nested' development – within evolution.

The fossil record traces or records these changes through time – the fossil record is, in fact, a fractal record: 'cataloguing' the infinity of ('*different*') combinations.
It has been implied by many leading biologists and mathematicians 'that evolution has (often) found fractal ways' or 'has used fractal ways'. This is misleading. Evolution is a property of the fractal and part of the geometry. Evolution is always, and everywhere, fractal. Evolution can be observed in the fractal and is a core component of the mechanics of the fractal as a universal repeating pattern or algorithm. In principle, as the fractal is infinite, so too is evolution.

Development may be seen as a 'short run' or short-term observation, for example, the development or emergence of an economy or a person—demonstrated by the fractal from iteration time 0 to fractal equilibrium—iteration 6.
Evolution may be seen as a long-run observation, tracing the 'development' of the object, the tree, through (greater) time. It is best demonstrated by the fractal zoom. Evolution shows the 'chaos', the complexity, or the influences on the 'developed' object. Are there limits to evolution in the same way there are limits to growth? Yes, and no. In the same way that there are limits to (fractal) growth and development, the object will evolve to a formed shape, but no, because there will always be change, an infinity of changes, and never one object is the same.

The Koch snowflake fractal does not show evolution as the triangle bits are all and infinitely identical; however, if a change was made to a bit – a red dot added, as demonstrated in the appendix figure 11 – and iterated evolution is demonstrated, and is a wave phenomenon – pulsing through time.

4.10.4 Sustainability

Sustainability is generally defined as '*maintaining the environment or the economy today without compromising future generations'*. The fractals may demonstrate sustainability, but equally or conversely, they do not and reveal that it is impossible—nothing can be maintained infinitum.

To reveal how the fractal offers insight into 'sustainability' we need to distinguish between the 'same' and the 'different' in the fractal definition.

The 'same' or the 'regular' part of the fractal definition suggests that *patterns, rules, and knowledge* all repeat at all scales; this part is sustainable or constant. This feature of fractals is explained by *strange attractors* found in the study of chaos and fractals –the repeating of a rule or law.

The 'same' component is revealed within the study of biology in the form of 'evolutionary convergence' or 'analogous structures', and the best example may be winged flight – where it is repeated through time: the 'different' represents the many forms of flight: the mammal, reptile-bird, or insect. There appears to be a 'line of fractality' where flight by the wing repeats at all scales through time. Today, we can add the human-developed aircraft wing to this group, and with this, the many types of human wings. Another example of sustainable patterns can be found with the plants: woody steamed plants – what we term a tree –have repeated in all the evolutionary phylum.

The definition's 'different' or 'irregular' part alludes to change, roughness and unsustainable. It alludes to evolution, or change over time – of the 'same'. This component part of the fractal definition does not demonstrate sustainability – there is constant (irregularity) change. To have evolution, there must be extinction, there must be change – whether ‘we’ like it or not. This ‘law’ or function we call evolution will never become extinct until life becomes extinct, and so, in this respect, it is sustainable:

4.11 Consumer, Producer, and Community Surplus

Consumer surplus can be demonstrated by the fractal: it is the area under the MA curve described by Euler Basel solution formula 10 above. The Producer Surplus is the area above the MC curve, and the community surplus equals (approximately) the Basel solution – 160 m^{-2} minus the MC, a value of 157 m^{-2}. This shows shape equilibrium is achieved before the (cost of the) limit. Community surplus is maximised at shape equilibrium or where the area of the snowflake object is at its maximum from the perspective of an observer.

4.11.1 Intervention

Just as with classical economic theory, any intervention (a price minimum or maximum) will limit either the production or consumption of the fractal, creating surpluses and shortages. The consequences of such intervention are evident in markets, but they may also affect knowledge issues.

4.11.2 Golden Ratio?

The ratio of the Koch Snowflake's original area (100 m^{-2}) to its final emergent area is 1.6:1. This ratio is close to, but not exactly, the golden ratio.

4.11.3 TA-TC

It is also noted that at equilibrium, TA-TC is maximised – as shown in Figure 6.

4.12 Trade and Exchange

As the ‘snowflake’ is produced, it may – in principle – be traded or exchanged with a consumer – a ‘consumer’ external to the model in question. Without this transaction, production would only be on its own. If this transaction occurs, the more that is produced (iterated), the more the consumer benefits as the first iteration is valued relatively high compared to those after that. This relationship is primal and universal and may give insight into the economy of life itself: primordial life may have begun on these principles – production followed by consumption.

4.13 Short Run Long Run

The fractal may demonstrate the economic Short Run and the Long Run. The development and growth of the fractal from iteration 1 to the equilibrium iteration may be seen as the fractal Short Run. The Short Run is the effect of the rule, the branching or the adding of triangles. The Long Run is the end state, the total 'superposition' of all the infinite possibilities of the rule.

4.14 Micro-Macro Distinction

Fractal scale invariance is evident in the supply and demand diagrams drawn.
This is a property of the fractal: from a distance, without looking at the labels and such, the models look the same. There is downward-sloping demand (and both for many reasons) and upward-sloping supply, with price and output on the axis.

The property of not being able to discern the scale of the fractal object—when viewing a fractal in isolation—offers direct insight into the macro and the micro. In a perfect fractal, there should be no difference. The resultant greater fractal clearly points to the macro and the self-similar fractal shapes—within this greater shape—the micro. At all scales, the rule to create this shape is the same.

From a different perspective, the 'same' – from the fractal definition 'same' but 'different' at all scales – may point to the mathematical rule that is repeated, and the ‘different’ to the many different examples of shapes created from this rule. To help explain, the word tree will be used as an example: a tree is the ‘same’, but it implies there are many trees – many types and species – an ‘infinity’ of trees. These ‘same’ rules in economics – wealth, trade, selection, reproduction, and specialisation ‘Trade and exchange’ are present not only in classical economics but also in cellular biology [13] and even chemistry theory. This is true in the other natural sciences, physics, for example, where the same are immutable, scale-invariant, and scientific laws.

If the complete shape of the macro is made up of these rules, and the micro and the macro are connected inextricably through these rules, all rules of the micro must be present in the macro. If this is not true, and the ‘thread of fractality’ is broken, it may count as falsification. As an example, if CO2 is claimed to change the climate in the macro, it should also change the weather in the micro. There is no literature on whether $C0_2$ changes the weather.

4.14.1 Defining Science

The process of discerning these scale-invariant universal rules (or laws) is known as science and is represented by mathematics.

4.15 Fractal Decay

If the emergence is ‘played’ in reverse, the fractal demonstrates decay: from a complex object to a simple object with respect to time. As time passes, the detail will diminish and leave the original core triangle bit. Fractal decay is the reverse of fractal development and is also logarithmic. In the diagram below (figure 7), this decay is displayed.

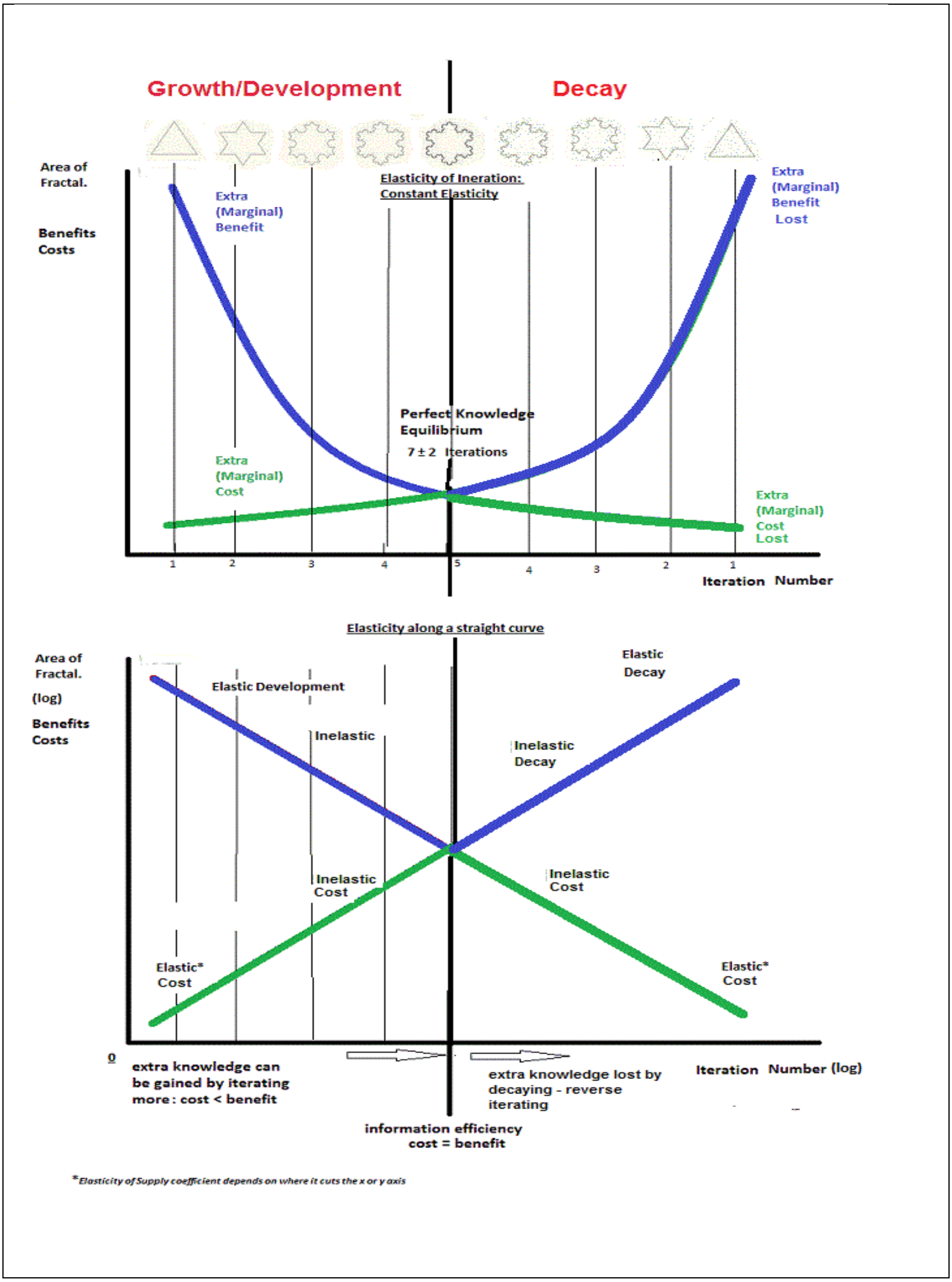

Figure 7 Fractal Decay Demonstration. From iteration 0 to 5, the fractal develops and grows; conversely, it decays from iteration 5 to 0. Elasticity and marginal costs and benefits reverse in reflection through a line of symmetry (the point of decay).

4.15.1 Observed (Demand) Decay

As iteration time passes, the fractal will decay if the fractal discontinues to grow and develop, losing its detail at first – the marginal benefit curve (and MC curve) is relatively inelastic at these near-shape equilibrium iterations. With decay, the 'value' or size of the marginal triangle bit (marginal benefit) increases with time – a reverse of the diminishing utility – while the total area (TA) decreases. At the beginning of the decay, the change is inelastic with respect to time and area, and elastic as time progresses. To observers, this results in larger and larger value of things past – as time passes – and coupled with elasticity's, the sensitivity of change increases with time.

4.15.2 Decay- Cost

From a production cost perspective, the decaying marginal cost (price) is largest at beginning time of the decay (at shape equilibrium) and decreases with time. The elasticity of the MC as shape equilibrium is initially inelastic, and increases to elastic as time passes. This may be interpreted as the 'pain' of the change. The decay cost is largest but inelastic (insensitive/ unchanging) closest to the beginning time and more elastic (sensitive) as iteration time passes.

4.15.3 Half-life

The decaying fractal demonstrates (Lord Rutherford's) radioactive half-life: the decay of detail is an exponential function. And oddly similar to Rutherford's atomic discoveries, the fractal snowflake is transmuted – decayed – to a single triangle bit.

4.16 Iteration-time

Something must be said about the key mechanism to the fractal emergence: the regular 'beat' or iteration of discrete 'bits' forming an object and propagating as a wave. This beat – the production of 'particles' – is how light is explained, where photons are explained to propagate – as a wave – at light speed. This process may offer insight into time. Further discussion is beyond this investigation but must be addressed.

This property may also offer insight into the regularity of gene mutations in the genome (the genetic clock), allowing us to date genetic mutations, which may also be explained by fractal iteration.

4.16.1 Perception of Time

As the fractal iterates through time, changes to the shape are greatest at early iterations. This may explain how time is perceived to run faster for younger age groups than for those older, given that our perception of time is influenced by changes.

4.17 Information: Perfect and Asymmetric Information

Figure 8 below shows the development of the fractal Koch Snowflake: shape equilibrium (Perfect Knowledge), but absolute information is not demonstrated, as the fractal is 'infinite'. This insight alone gives weight to 'to more we discover, the more questions are opened'. Perfect Knowledge, or 'perfect information' is achieved only with free, open, competitive, or unobstructed feedback – this is consistent with Joseph Priestley's early work on utilitarianism in the 1790's

Any obstruction to 'iteration' in achieving this equilibrium – due to what may be termed a knowledge monopoly – will produce an incomplete fractal shape (as shown in the upper panel of the diagram), imperfect knowledge, or asymmetric information. Perfect shape (perfect knowledge) is achieved with open and competitive unobstructed feedback.

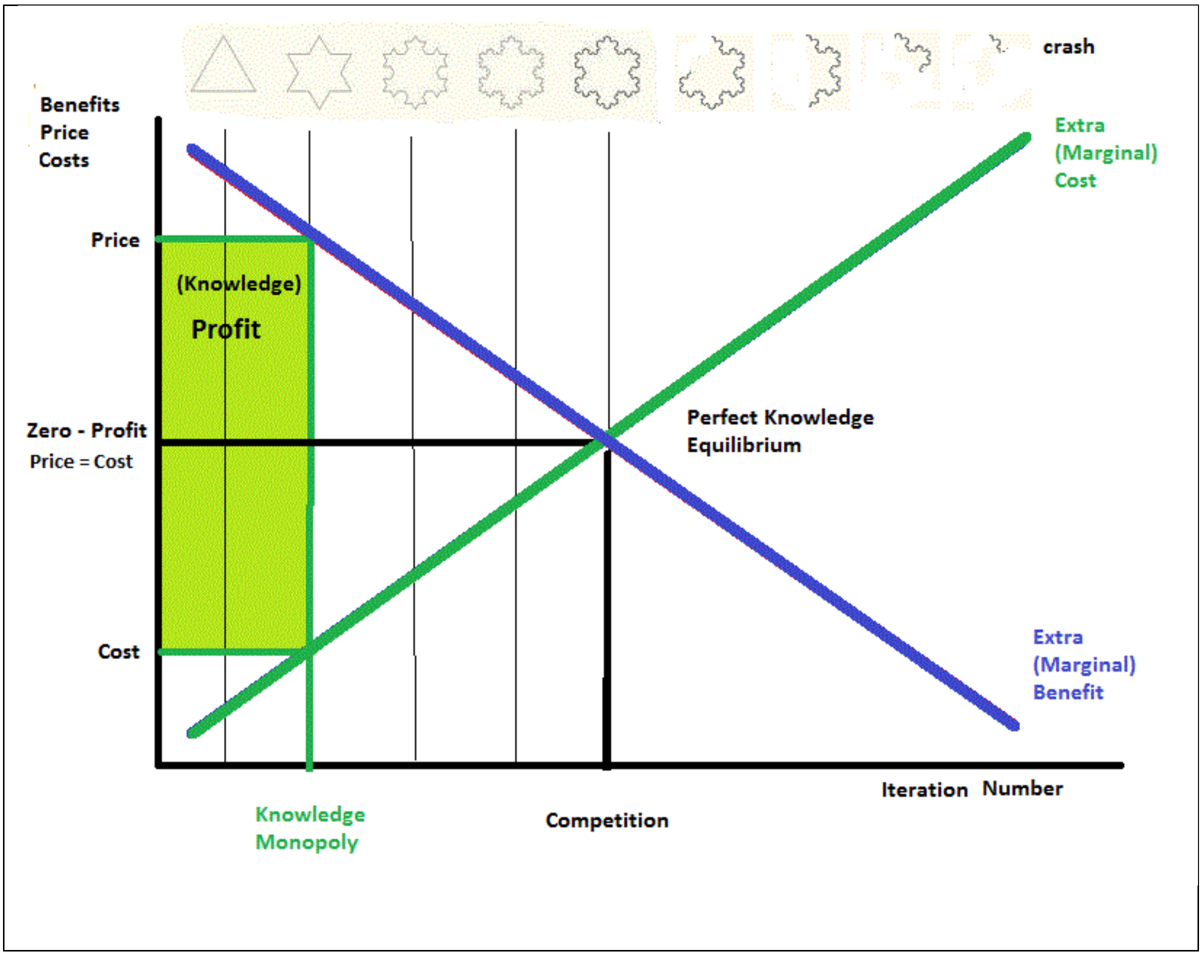

Figure 8. Koch Snowflake Fractal Imperfect Information, Knowledge Monopoly Demonstration. Shape equilibrium is met where marginal cost is equal to marginal benefit. Knowledge monopoly where iteration is restricted: this will result in 'knowledge profits'.

4.18 Uniformitarianism and the Fractal

The key to the past can be found in the present.

I have a strong interest in geography and geology, and it was here where I first read of Hutton's uniformity; I soon found – after teaching development economics – that this principle may be universal and may reveal itself in economics.

The law of uniformitarianism reveals itself in the fractal. To describe a fractal, one would eventually cover the principle, only instead of reading as above – the key the past can be found in the present – it would read: the key to the present (scale) can be found in the small scale – or conversely the large scale, assuming ceteris paribus approach, (holding all else constant or frozen).

In the tree fractal below, the new (present) cross-section line b-b will share the same (but different) pattern as the earlier/older (past) cross-section line a-a. Scale is the only difference, both in time (age), and size.

Figure 9. Uniformitarianism and the Fractal (Tree). The key to the past is in the present; the key to the past scale - cross section a-a - is in the present (new) scale b-b.

Uniformitarianism is another insight and property of the fractal – the same but different – at all scales.

For example, if you want to know how you were as a child, all you need do is see the children around you; the same may be said for growing old. This may sound obvious, but it may be only obvious because of the fractal nature of the universe.

4.18.1 Application in Economics

If you want to know how it may have been to live in the past (social-economically speaking)—say, the Middle Ages—all we need to do is search for a developing country in the present that has poverty. In any system, one would not have to find another country to demonstrate this; it should be evident everywhere: every (healthy) system has diversity—rich and poor, young and old.

A fractal thinker should see the child – in the first application – and the developing country – in the second – as the same.

5 Insights to Knowledge

If we assume the snowflake object stands for knowledge and ideas, the fractal demonstrates the emergence of knowledge, where information is represented as the triangle bits making up the snowflake. At the foremost, absolute knowledge is unachievable, as the fractal iteration time, and thus information is infinite; but it is clear that a shape of 'knowledge' is soon formed or emerged.

5.1.1 Perspectives – Polarities

One of the most important insights from the fractal stems from the question, ' What comes first, consumption or production?' This is not only an economics question but, indirectly, a question of knowledge and truth. The fractal sheds insight into reality and offers an explanation for the origin of life. However, the answer will always come in the form of two perspectives, in polarities.

5.1.2 Consumption – Observation

The 'consumption side' refers to the observing and enjoying – or not – of the emergent object. It refers to the beauty and the aesthetics of the emergent snowflake. It is an interpretation. It is void of any time or history. In accordance with the understanding of the MA and TA curves, figure 2 above, every iteration adds to the image. It has no notion of production or observation – the iteration of a simple rule or algorithm. An explanation of the object will be external to the production rule; it will be ignorant of the rule and may come in the form of an image, or a description. It may be that this perspective of the polarity is the 'inductive' side. Knowledge is gained from gathering facts and extra facts to build up a general picture.

5.1.3 Production Explanation- Understanding

Production refers to the rule of production of the emergent object; how is it the object produced? The explanation of the production is the iteration of a simple rule. This simple rule may be seen as a general rule, and further, after studying and proving by iteration, it is defined as a – universal at any scale – law without which no emergent object will be produced. With an explanation, a law or an algorithm, the object may be able to be reproduced. This is not possible from a consumption observation.

As this side is the polarity of consumption, it follows this production side must be the 'deductive' perspective, where a general rule, or code is identified to explain the object.

5.1.4 Equilibrium

Shape equilibrium is the simultaneous meeting of both consumption and production. Explanation is equal to observation. By iterating, using, studying, researching, and experiencing, the fractal reaches what may be termed shape, knowledge, or fractal equilibrium. This is the shape viewed from an observer outside, and not within, the fractal. Fractal equilibrium is reached at or around 5 plus or minus 2 iterations – but this is dependent on a number of factors including the distance between the observer and the fractal.

As if by law, though these different perspectives – production and consumption – appear to be simultaneous and equal at equilibrium, they are quite separate from one another. Truth is found when the two are simultaneously in equilibrium. To reiterate, the fractal can be observed – consumed while at the same time having its production explained, but only one of these perspectives is true. When attempting to explain the object, only one is true. When experiencing the object, only one perspective is true.

5.1.5 Duality

Every property of the fractal is coupled with a dual opposite or symmetry.

This begins with the fundamental definition: same but different at all scales. The 'Same' (or regular) component is what is known as a rule – and thus, this is the basis of Euclidian mathematics; the 'different' (or irregular) component is the chaos – the diversity of the rule. Growth and development, production and consumption, supply and demand, and so on.

This duality insight leads to a fundamental property of the fractal that is beyond the scope of this 'classical' reality-based investigation; it is the quantum mechanical-like behaviour of the isolated fractal. These many insights will be addressed in another paper.

5.1.6 On Ceteris Paribus and Knowledge

It is in the intangible area of 'knowledge claims' where this problem of monotonic scale invariance of the fractal is most evident: discerning truth when there is nothing 'to go on', only a pattern. Though from the repeating of the pattern, the repeating at all scales, all looking the same, with this, there will always be the polarity between the aesthetic consumption view and the logical, scientific production view.

The takeaway from this is that only the productive side gives the truth—it will allow accurate prediction and reproduction of the observation or pattern. It has to be understood that all knowledge is fractal. It will repeat, and if it does not, there may be something wrong with the claim.

5.2 Insight to Science

If knowledge follows a fractal structure, one can expect both a production side and a consumption side of it. Claimants of knowledge will take one of these sides with the aim of truth but may be unaware of which side they have: they may easily confuse the two perspectives. A scientific approach is most certainly a production side explanation and the demand for surrealism. Unless their claim repeats or iterates – so as to be seen universally at all scales – and able to be written mathematically as an equation just as the production of the Koch snowflake, the claim may be mistaken for an observation consumption perspective. Even talking about the fractal itself stands as an example; some see the fractal as having meaning to the universe; this is similar to quantum mechanics, while others will see only the geometry and take no meaning from it. The 'observation' perspective is open to interpretation and debate – it will be subjective and may lead one on the trail of truth, while the scientific production perspective is objective, clean of opinion, and complies with reason. The significance of this insight may address the work of Professor Alan Musgrave [14] on surrealism where (as it appears from a fractal analysis) knowledge may be monopolised by the observation side when explaining. These groups will claim 'all the science is good for is saving the phenomenon'; their claims will be parasitic on the 'production' or scientific' approach and will offer no explanation on is own. Their claims will often be extraordinary.

The classic example of this scientific polarity is the Copernican and Aristotelian approaches to the heliocentric and geocentric interpretations of the solar system. There is only one explanation that fits a production perspective, and this is the Copernican Heliocentric; all other perspectives, after the aid of telescope technology, were mere guesses and wrong. Modern polarities are quantum mechanics and books like 'The Secret' – where quantum mechanics is interpreted to have some 'meaning'; environmentalism and manmade climate change vs. natural climate change; Darwin vs. creationism; and, of course, Keynesian vs. Classical Economics. With all these issues we see – at least – two perspectives, both at odds with each other. The truth will be found in a fractal production perspective. Testament to this is the geocentric-heliocentric debate where heliocentric won over as a universal truth. While the Sun appears to move, it is, in fact, the planet, the Earth, that is moving.

6 Conclusions

From this analysis of the regular (Koch Snowflake) fractal, it has been shown – and it can now be inferred – that intangible economic behaviour known as marginal economics is an aspect of fractal geometry and can be best described and understood by studying and analysing the fractal. The fractal demonstrates many of the main principles of classical economics, including marginal and total utility, marginal cost, equilibrium, price determination, elasticity, growth and development, and Say's law where to view – or consume – a fractal, it must first be produced. The fractal object is produced by the iteration of a 'simple' rule. Marginal and total diagrams graphing the fractal attractor's growth and development – through time – match textbook diagrams, and show the universality of the fractal. From this, the fractal reveals direct insights into knowledge and knowing, offering a theory of knowledge.

All fractals exhibit wave properties with iteration and emergence; wave properties are traditionally described by electromagnetics and quantum mechanics mathematics: from this, the fractal is a candidate to reveal an answer to some of the 'big' questions in physics. It was found the fractal can also be described by these mathematics and the fractal may explain the occurrence of π in statistical mathematics. The fractal also demonstrates a property long sought after by scientists: it simultaneously possesses exponential behaviour (observed in the logarithmic demand curve) with spiralling wave behaviour described by wave mechanics. It was claimed, via the fractal, that the classical demand curve is a convergent (standing) wave function known in quantum theory as the de Broglie wave function. Through the fractal, all things – as long claimed by quantum scientists – indeed have wave (function) properties.

While the study used the Koch snowflake to model the fractal, an insight from the study revealed that just like the structure of real natural (6-sided) snowflakes is said to be derived from the '6-sided' atomic structure of the H_2O molecule, reality itself seems to reveal its own structure in the macro—it is the fractal. This structure is revealed to us in what we call marginal-classical economics.

If economics is a study of reality and fractal geometry matches standard economic knowledge, our reality may best be understood by understanding the geometry of the fractal. Our reality is one aspect of a universal geometry called fractal geometry.

7 APPENDIX

7.1 Log – log diagram

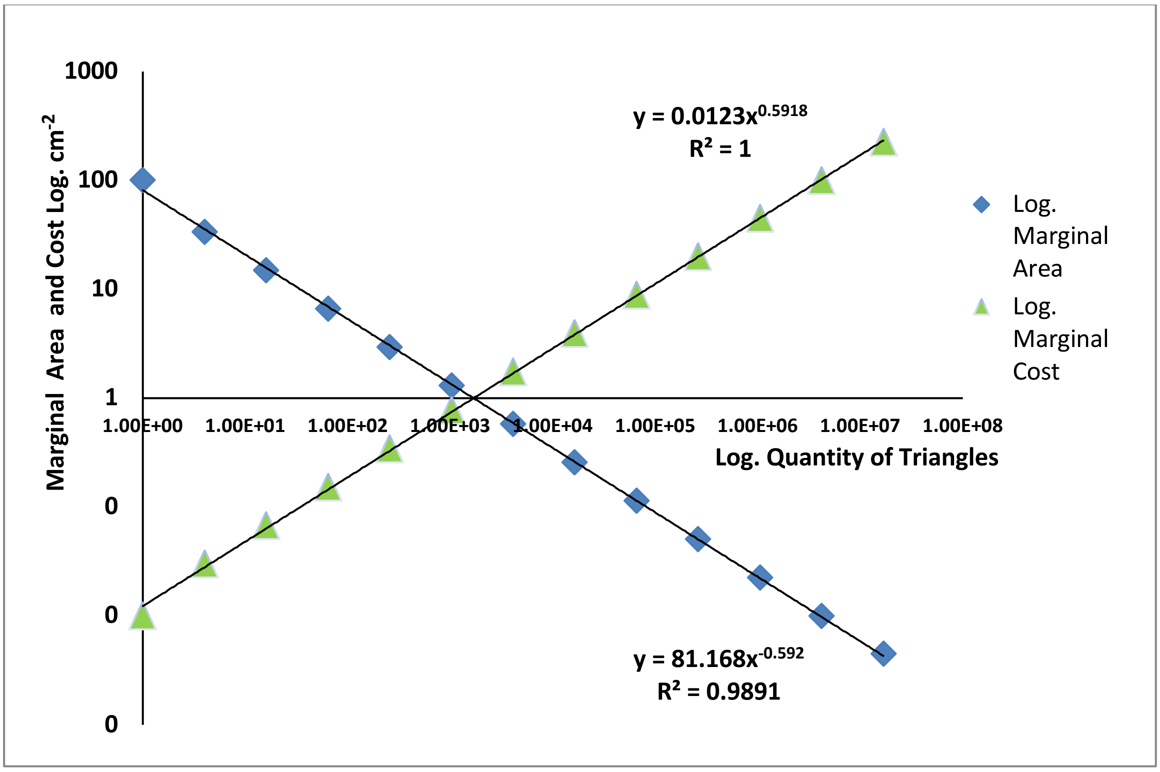

Figure 10. Log. –log. (Linear) Koch Snowflake Fractal Equilibrium.

Table 1. Arc and Point Price Elasticity.

Column					15	16
Iteration No	Quantity of Triangles (Q)	Marginal Area	Arc % Δ Quantity	Arc % Δ Area	Arc Elasticity	Point Price Elasticity
0	1	100	-	-	-	-
1	3	33	-1.69	1.48	-1.14	-3
2	12	15	-1.76	1.34	-1.32	-5.4
3	48	7	-1.76	1.34	-1.32	-5.4
4	192	3	-1.76	1.34	-1.32	-5.4
5	768	1	-1.76	1.34	-1.32	-5.4
6	3072	1	-1.76	1.34	-1.32	-5.4

7	12288	0	-1.76	1.34	-1.32	-5.4
8	49152	0	2	2	1	-5.4

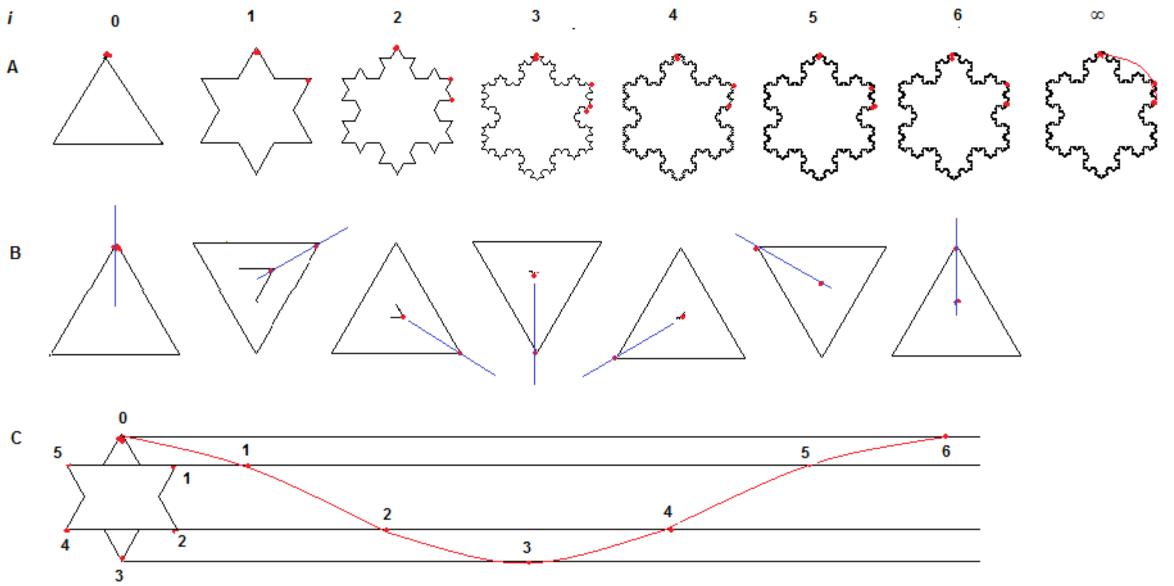

Figure 11 Fractal Spiral with Emergence. A shows the transverse wave propagation of a 'red dot' on a fractal Koch Snowflake to iteration (i) 6, and to superposition infinity (∞) . B shows the rotational aspect of the triangle bits and the respective bit size; rotating clockwise through 360° . C shows the Sin wave produced at each iteration-time – assuming bit size remains constant: the real is a logarithmic sinusoidal.

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