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GENERALISATION D'UNE FORMULE DE RAMANUJAN

$$2\sin\frac{\pi}{18} = \sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2-\sqrt{\ldots}}}}}$$

où la suite des signes −, +, + a pour période 3

Nous avons :

$$\varphi^{-1} = \sqrt{2-\sqrt{2+\sqrt{2-\sqrt{2+\sqrt{2-\sqrt{2+\sqrt{2-\sqrt{2+\sqrt{2-\sqrt{\ldots}}}}}}}}}}$$

$$(\boldsymbol{Ramanujan})\quad 2\sin\left(\frac{\pi}{18}\right) = \sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{\ldots}}}}}}}}}}$$

$$2\sin\left(\frac{\pi}{34}\right) = \sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2-\sqrt{\ldots}}}}}}}}}}$$

$$2\sin\left(\frac{\pi}{66}\right) = \sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{\ldots}}}}}}}}}}}$$

De manière générale

$$2\sin\left(\frac{\pi}{2^{n+4}+2}\right)=\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\cdots+\sqrt{2-\sqrt{\ldots}}}}}}}}$$

Où la suite de signe -, +,+,+,+,… a une période de n+3

D’autres Résultat avec le cosinus :

$$2\cos\left(\frac{24\pi}{65}\right)=\sqrt{2-\sqrt{2-\sqrt{2-\sqrt{\color{red}{2+}\sqrt{\color{red}{2+}\sqrt{\color{red}{2+}\sqrt{2-\sqrt{2-\sqrt{2-\sqrt{\color{red}{2+}\sqrt{\color{red}{2+}\sqrt{\color{red}{2+}\sqrt{2-\sqrt{2-\sqrt{2-\sqrt{\color{red}{2+}\sqrt{\color{red}{2+}\sqrt{\color{red}{2+}\cdots}}}}}}}}}}}}}}}}}}$$

$$2\cos\left(\frac{22\pi}{65}\right)=\sqrt{2-\sqrt{2-\sqrt{2-\sqrt{2-\sqrt{2-\sqrt{2+\sqrt{2-\sqrt{2-\sqrt{2-\sqrt{2-\sqrt{2-\sqrt{2+\sqrt{2-\sqrt{2-\sqrt{2-\sqrt{2-\sqrt{2-\sqrt{2+\cdots}}}}}}}}}}}}}}}}}}$$

$$2\cos\left(\frac{20\pi}{63}\right)=\sqrt{2-\sqrt{2-\sqrt{2-\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2-\sqrt{2-\sqrt{2-\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2-\sqrt{2-\sqrt{2-\sqrt{2-\sqrt{2+\sqrt{2+\cdots}}}}}}}}}}}}}}}}}}$$

$$2\cos\left(\frac{24\pi}{65}\right)=\sqrt{2-\sqrt{2-\sqrt{2-\sqrt{{\color{red}2+}\sqrt{{\color{red}2+}\sqrt{{\color{red}2+}\sqrt{2-\sqrt{2-\sqrt{2-\sqrt{{\color{red}2+}\sqrt{{\color{red}2+}\sqrt{{\color{red}2+}\sqrt{2-\sqrt{2-\sqrt{2-\sqrt{{\color{red}2+}\sqrt{{\color{red}2+}\sqrt{{\color{red}2+}\cdots}}}}}}}}}}}}}}}}}}$$

$$2\cos\left(\frac{16\pi}{63}\right)=\sqrt{2-\sqrt{2-\sqrt{{\color{red}2+}\sqrt{{\color{red}2+}\sqrt{{\color{red}2+}\sqrt{{\color{red}2+}\sqrt{2-\sqrt{2-\sqrt{{\color{red}2+}\sqrt{{\color{red}2+}\sqrt{{\color{red}2+}\sqrt{{\color{red}2+}\sqrt{2-\sqrt{2-\sqrt{{\color{red}2+}\sqrt{{\color{red}2+}\sqrt{{\color{red}2+}\sqrt{{\color{red}2+}\cdots}}}}}}}}}}}}}}}}}}$$