Towards a formulation of general vacuum relativity.

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Abstract

This paper critically reflects on quantum gravity and argues how quantum gravity might be done from the point of view of generally covariant quaantum theory.

1 Introduction

The search for a theory of quantum gravity is one of new principles of nature and involves the question if and how the superposition principle should be applied to space time itself. Quantum theory as developed so far requires a classical space time metric and therefore, a quantum theory of the space time metric appears to call for a "super metric": a metric on the space of all geometries (I leave it open here whether one should "quantize space" or spacetime - the standard canonical first quantization procedure calls for a quantization of space whereas some other people might suggest that you have to quantize spacetime). Moreover, it is often suggested that in the standard approach that you need to perform a path integral in the standard Lagrangian formulation. I am not sure of this, the proof of the equivalence between the operational formalism and the standard path integral relies heavily upon the canonical form of the kinetic term; that is momenta should be decoupled from canonical field variables in order to get the *original* Lagrangian. This is not the case in gravity and to get to a path integral formulation you would need to order the constraints with all momenta to the right and spatial metrics to the left. But hermiticity of those quantum operators would then require additional, lower order terms to arise so that it appears that the *new* Lagrangian does not coincide with the original one. This problem does not occur in tahe standard model given that there, the kinetic terms are all uncoupled from the "position field" operators. So, I am not sure about the soundness of, for example, the causal dynamical triangulations approach where you take a sum over "gauge fixed" histories and the origin of the covariant definition of the measure is entirely a mystery. Indeed, it is well known that the path integral measure is not covariant but heavily depends upon your choice of foliation of spacetime. This is not the only worry one has regarding such discrete constructions: one has also to show that the limiting kinematical configurations are *arbitrarily close* to any classical space time in a suitable sense implying that the action principle at hand

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converges too. There is a very important distinction here between gravity and all other action principles in field theory, which is that the latter all depend upon first derivatives of the fields only whereas the former depends upon second derivatives of the metric field. There exists a discretization procedure invented by Regge, which can account for the second derivatives in a distributional sense but it requires flexibility in the degrees of freedom of the discrete structure (a simplicial manifold) so that, locally, on the $n-2$ simplices, where n is the dimension of the simplicial manifold, the deficiency angles go to zero sufficiently fast. The "curvature" of the approximating simplicial manifolds then converges to the Ricci scalar in a weak distributional sense. I am unaware of any suitable substitute for the Ricci tensor and Riemann curvature in this kinematical framework. I am also unaware of any approach to quantum gravity which manages to offer a suitable answer to these elementary matters of principle: the measure in the causal dynamical triangulations approach heavily depends upon the kinematical restrictions which, moreover, do *not* approximate any classical space time in the above sense. Indeed, not only is it clear that Regge's scheme does not apply, the "local" curvature is a diverging quantity in the distributional sense when the continuum limit is taken.

What I have described above can be called "quantum gravity type one" where there is no classical metric background on which computations are performed. One can of course maintain that the universe consists also out of classical degrees of freedom providing one with a dynamical classical background on which it is possible to regard the quantization of the gravitational force as the quantum theory of the graviton. So in that sense is a first, background dependent, quantization of the gravitational field a second quantization of the graviton. This can be called "quantum gravity type two"; such a theory has long been believed to be impossible due to the non-renormalizability of the gravitational force on a Minkowski background. It his here that our novel nonunitary quantum theory offers a way out since the theory is finite when suitable deviations from the standard Feynman diagram expansion are taken [1]. In particular, loop diagrams played no special role at all in our analysis and were treated on pair with other internal legs which shows that quantum gravity type two is a perfectly safe theory in our framework.

2 Quantum gravity type one.

Personally, I have never made a choice between both types of approach since both reflect different world views, which in my opinion were equally valid. The fact that type two did not seem to work out technically has always been regarded by me with the necessary amount of scepticism since in my opinion, QFT did not work for QED nor the standard model either. Only sloppy and overprotective field theorists could take something like that seriously, but I was rigorous and not even protective regarding my deepest beliefs. So I have always felt that on the level of relativistic particle theory, we were lacking a few crucial insights. What I knew already for a long time was that ultimately type one was going to be the most difficult to realize. These notes are about obstacles one will meet regarding the formation of a type one theory, but a real theory, not just something we can all pull out of our hats within five minutes but which lacks canonical beauty and predictive power.

Here, one immediately faces a couple of problems regarding the fact that standard formulations of quantum mechanics are *not* covariant. This is seldomly highlighted, but the problem is really everywhere: in the path integral approach, it is in the non-covariance of the measure, in the Heisenberg approach, it resides in the non-covariance of the total Hamiltonian and therefore the vacuum state, and finally in the Schrodinger approach, it is blatanly visible because the probability density does not transform as a density under coordinate transformations of space. In field theory for example, one will obtain that distinct lattice regularizations, in either different choices of "measure", will give rise to different continuum limits and we wish physics to be devoid of such ambiguity. In that respect is our quantum theory generally covariant: it does not depend upon geometrical structures or coordinate choices which have to be imported. There is no choice of vacuum state, no Hamiltonian, no measure, everything has been poored in a manifest space-time language. This, of course, is a great starting point for some ideas regarding a quantum gravity type one theory to mature. So, up till now, *every* approach to quantum gravity suffers from one of these drawbacks: in the discrete theories based upon the Feynman path integral, such as causal sets and causal dynamical triangulations, one remains with the choice of the measure associated to the particular regularization scheme, amongst others. Some researchers accept this as a fact they have to live with, most of them are not even aware of the issue.

So, how can we extend our novel line of thought [1] to space time itself? For example, how to define the momenta of the theory which have to serve for a gravitational uncertainty principle and what are the constraints upon the momenta replacing the on-shell mass condition for relativistic particles? Here, it is appropriate to state that in our framework, we have disposed of first quantization all together, we immediately went over to the second quantization by an appropriate derivation of the Wightman functions. Clearly, in a continuum theory of the universe some infinite dimensional integration would have to be performed which again will lose its appeal through the *appearant* non-canonical character of the limit of measures. In a discrete universe, one obviously does abandon local Lorentz covariance in a well defined sense, albeit this does not need to have disastrous implications upon the physics defined on it. It is an important kinematical question to ask oneself how close two (discrete) universes are and I have adressed this question in my PhD work [2] where I have defined and investigated to some extend a Gromov distance on "Lorentz geometries", which wa a spacetime approach. Here, it is useful to recall the canonical variables for classical relativity; those were the spatial metric h and a momentum π obeying four constraints $Z_i(h, \pi) = 0$ with as equations of motion the Einstein equations where the lapse and qhift have been gauge fixed to one and the zero vector respectively. Now, the novel idea is to regard the Einstein equations as defining the *free* gravitational field; just like the geodesic equation was the correct one for a free particle. I have proposesd a similar avenue for the second quantizatiion of string theory where the string equation of motion replaces our geodecy and free momenta, satisfying the usual constraints, are dragged over the string in a way as to preserve the constraints, see [3] for more explanations. I have suggested there that the resulting "connection" might correspond to a super (Finsler type) metric and that therefore some unique dynamical conserved quantity could be constructed in the propagator. In that paper, the correct dragging law or "parrallel" transport has been derived and further investigations must show how promising this idea really is. Regarding gravity therefore, one should search for a metric on the sace of all spatial metrics times "time" such that one spatial universe at a given time corresponds to a point in that space, lets call it the birth universe, and that the canonical momenta reside in the tangent space of that generalized manifold. Hence, a "geodesic" is determined by means of (h, π) and corresponds to a gauge fixed solution of the Einstein equations. Now, in order to go over to the second quantized theory, we should integrate along all momenta π' satisfying $Z_i(h, \pi') = 0$ and dragging these momenta along the Einstein equation as to preserve these constraints. As such, one could compute the amplitude corresponding to going from h to h' in "time" 1 (this is the very definition of the exponential map). As an aside here, the integration over all momenta can be canonically defined by using the "birth metric" h as a "background" on which to perform Fourier decomposition and taking the usual cutoffs in momentum space and finally considering the thermodynamic limit. Such a scheme could work out in principle and would provide one with a genuine, well defined, background independent quantization of space. Such an approach would be devoid of all ambiguities of the present one and be much more general as well.

References

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