The Mathematics of Eating

Timothy W. Jones

October 27, 2012

Introduction

The mathematics of eating means a model of human nutrient acquisition, i.e. how we humans acquire nutrients (food) from environments. This model gives (as models do) an idealization of a phenomenon. In particular, it gives a model for *rational* nutrient acquisition. It thus ignores such things as social conventions, social pressure, advertising, irrational cravings as induced by stress, say, and other things. It shows how a computer might process a modern environment for nutrients using natural languages.

From a theoretical perspective, this model might be of interest in that it extends the traditional branch of mathematics known as game theory [4, 5, 6]. Information becomes a commodity and various linguistic strategies can be used to compete or cooperate for these commodities. From a practical perspective, it might help students understand mathematics, given that basic math is now mimicking their daily activity of eating [1]. In this regard, it might also help the general human eat better and behave more rationally. All may have a model that is rational to emulate. Finally, the nature of language itself might be clarified by this approach [2] and the difficult task of learning a second language might be eased by this clarification.

This article uses various methods to communicate ideas. Readers are asked to have a pen or pencil ready and to do various activities. The activities are reminiscent of artificial life applets and ideas [3]. There are also links to web pages that demonstrate ideas, sometimes interactively. The following link gives the author reading this article, doing the activities, and giving additional commentary: http://www.theonanda.com/video.

Three objects

We start then with the quest to model three objects: food (draw a pentagon), a human (draw a stick figure), and an environment (draw a 3x3 grid with a clock next to it). Draw another grid and place the stick figure in it in position 1 and the pentagon in position 5. If the stick figure needs the food at position 5 (numbering is from left to right and top to bottom), then the solution of this environment (grid) for this need is *over one, down one* or something similar. If the language or culture supported (or had) numbers and established addresses the solution is just *5*. Make the same grid and give the clock a three hour, so to speak, day. Give the stick figure a belly and place 2 on it with the meaning that at 2 the man will need the food at position 5. The solution to the environment is *at two five*. If we establish that the man needs the food without specificity of when he needs it and if we further say that the pentagon will only be ripe at *3*, then the solution is *wait to 3 then 5*. The central challenge of this work is to display how language and many people possessed of it greatly facilitates finding and using solutions to such environments.

Food

Foods have nutrient characteristics and physical characteristics. We call the former primary and the latter secondary.

Primary characteristics

The primary characteristics of food (the things we need from it) are carbohydrates, fats, protein, and vitamins and minerals. Make a table with these four categories of nutrients along the top row and breakfast, lunch, and dinner along the left column. Fill in this table with various common foods. For example, breakfast might consist of eggs (protein), toast (carbohydrate), orange juice (vitamin and mineral), and butter (fats). A single food of course supplies varying amounts of all four categories, but we are painting our model in broad strokes. Our litmus test consists of whether or not modern humans think about these categories of foods. If you can make the table, we've made our case.

We will say humans need units of these food types in a certain ratio: 4 carbohydrate, 3 fats, 2 protein, and 1 vitamin and mineral. We make a list of

120 foods consisting of a varying number of units: $0\dots4$ carbohydrate, $0\dots3$ fats, $0\dots2$ protein, and $0\dots1$ vitamins and minerals. There are $5! = 120$ foods possible given by the set

$$\text{food} = \{(c, f, p, v)|0 \le c \le 4, 0 \le f \le 3, 0 \le p \le 2, 0 \le v \le 1; c, f, p, v \in \mathbb{N}\}. \quad (1)$$

Write this equation down and put $(1, 1, 1, 1)$ inside (or next to) the pentagon you drew earlier and $(3, 2, 1, 0)$ next to the stick figure. The pentagon nutrient can satisfy the need of the human: vector addition shows this: $(1, 1, 1, 1) + (3, 2, 1, 0) = (4, 3, 2, 1)$.

Secondary characteristics

We need a way to suggest the secondary characteristics of elements from our food set. We can achieve this by using the first four prime numbers to map the primary characteristics to a single natural number:

$$\text{FOOD} = \{2^c \cdot 3^f \cdot 5^p \cdot 7^v|(c, f, p, v) \in \text{food}\}. \quad (2)$$

So, for example $(1, 1, 1, 1)$ takes on the body $2*3*5*7 = 210$. This number, 210, has a front (2) and back (0) and a size: its number of digits or its value. A number can be put into a function, like a polynomial, and otherwise assigned a value that gives its color, weight, shape, texture, and all the things of our sensory world. For example, having a power of 5, we can assign it the shape of a pentagon. It is a simplification and an idealization.

Erase the $(1, 1, 1, 1)$ next to the pentagon and replace it with 210. Make a set of 6" x 4" index cards and put the 120 natural numbers corresponding to our FOOD set on the unlined side of the cards. Shuffle these and make a 12 row by 10 column grid. Place a ball (a tomato) onto the grid on top of the $360 = 2^3 \cdot 3^2 \cdot 5^1 \cdot 7^0$. See Figure 1. The object of our red animal is to find 210.

This simple situation gives a model for nutrient acquisition for a general organism – even a plant or a fungus. An animal organism moves and a human animal talks. Here's a quiz that shows how the food portion of the model gives the sense of hiding primary nutrient information with secondary characteristics.

Consider the following questions and activities. What is the parallel to taste in our model? *divisibility* to digestion? *prime factorization* How can our sensory systems be classified relative to the chemistry of food? Sensory

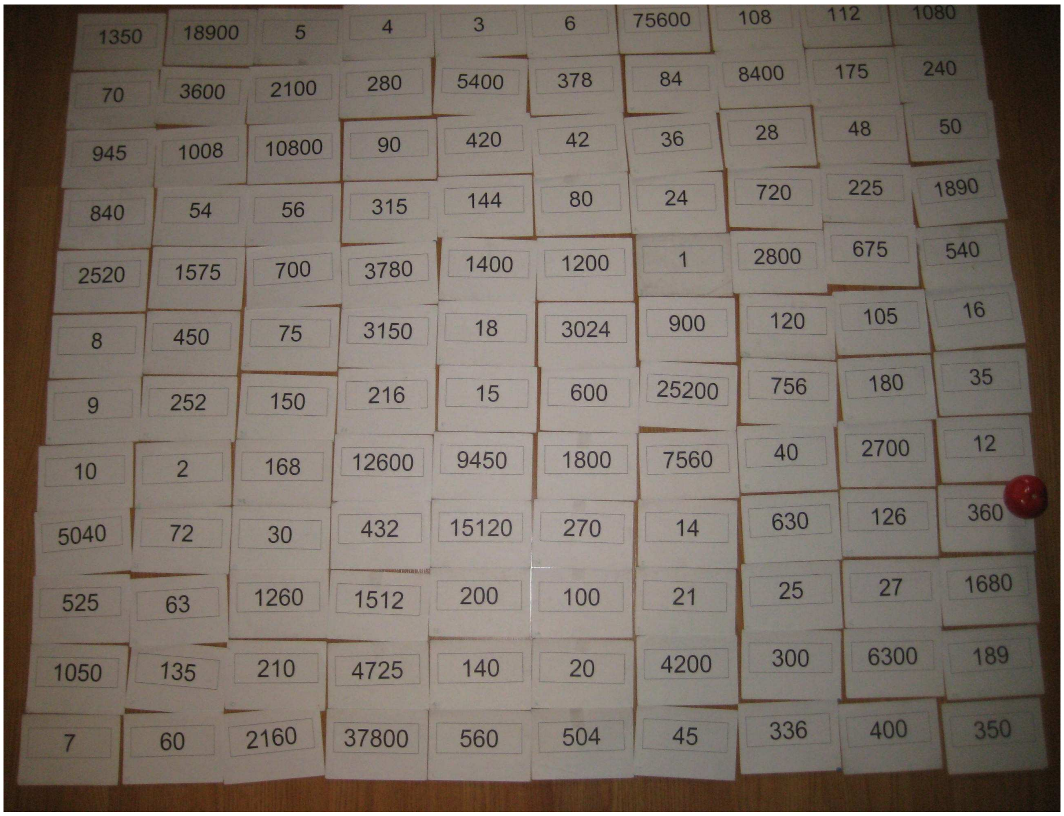


Figure 1: The animal is in state 360 and needs to find 210.

systems can be directly linked to the chemistry or removed from the chemistry of food: taste is direct, color, shape, and size are removed. Touch is a little ambiguous. Can we feel the difference between an orange and an apple and is the different texture related to the different nutrients in each? What are the sounds associated with food? As we eat a food the physical action of chewing generates sounds we can hear from our mouths. Of course words come from the mouth as well. Words used to designate and describe foods are thrice removed from the chemistry of food substances: we make them up. We could say that are minds are really illusion processors given that words have no direct chemical relationship to food.

Given all the FOOD in an environment ends in 0, is their any utility to having sensory systems that see this 0. What sensory systems allow for the fast finding of FOOD given a constrained single need? Make a web application that filters a 12x10 grid per number of digits, sum of digits, first

Pop Quiz 1

	C	F	P
210			
310			
7			
20			
1350			
9450			

Indicate with a "Y" if nutrient is present and an "N" if it isn't.

Pop Quiz 2

	V & M
7	
28	
35	
15	

Indicate with a "Y" if nutrient is present and "N" if it isn't.

Pop Quiz 3

Find the prime factorization of each of these numbers and interpret the nutrient content for each: 1008, 504, 315.

Pop Quiz 4

Circle numbers with three digits and puts squares around numbers that end in zero: 108 270 600 1080 37800 15 1050 7560 72 4725 27 84.

Figure 2: Web-page giving quiz.

digit and last digit and combinations of these filters. Given a set of needs what is an ideal supermarket organization for FOOD? Do we witness this at supermarkets?

Nouns for food

We can assign words to our FOOD via consonants and vowels that encode their primary characteristics. Culturally, in the real world, is this a good idea? Develop nouns for foods based on nutrient composition. For example, we can associate each nutrient with a consonant: b carbohydrates, c fats, d proteins, and f vitamins and minerals; and the number of units we can associate with a short vowel: $a = 1$, $e = 2$, $i = 3$, and $o = 4$, we have $(1, 1, 0, 0)$ is *baca*. What is the word for $(3, 0, 0, 1)$? $bifa$. This is the sense in which language is given a mathematical context that can make its utility clear [2].

Here's another activity that gets the idea across quickly. Imagine replacing the food in your household with the index cards you just made. So, after completing this activity, your refrigerator and cupboards would have index cards in them. How many physical actions do you perform a day that might be considered to consist of the following acts: you want something and you get it by physical movement to the object you perceive would satisfy that want? Nutrient acquisition certainly consists of these activities.

Humans

Next we have a human to model. We've arbitrarily establish that a nutrient state of $< 4, 3, 2, 1 >$ is ideal. We will use angle brackets to differentiate status and need of a human from FOOD designations. Once again a human has primary and secondary characteristics relative to nutrition. They have a nutritional status given by a 4-tuple $< 3, 2, 1, 0 >$ that implies their need $(1, 1, 1, 1)$.

Primary characteristic (need)

The most natural association of primary 4-tuples is to the complement 4-tuple that will take the organism, the human, nearer or exactly to the ideal nutrient state. The specific number will be per the stored information about the environment. Each person can know things unknown by others and be in the same or different state than others in the environment. Strategies involving competition and cooperation can be present simultaneously given knowledge of states and environments changing with time. Figure 3 gives an example of such a situation. We can formally define it as a drama, lower case, meaning just the status and needs of each person in the environment and the FOOD in the environment.

Using your index cards and a spreadsheet (Excel) make a drama. After shuffling your cards, place the first 12 cards in the arrangement you see in Figure 3. One can read the drama: the needs of the organisms are given by the numbers in red and blue; the information for the red organism is to the left and that for the blue organism to the right. Exchanging information can be varyingly valuable to each organism. As an easy example, suppose RED needs 360 and BLUE knows where 360 is (on her side).

	A	B	C	D	E	F	G	H
1				2100				
2				200				
3	37800	4725	1512	504	7560	70	2520	2160
4					1080			
5					450			

Figure 3: Two organisms have different periodic needs and have information about different environments.

Secondary characteristic (behavior)

What is the outward manifestation of the primary characteristic? Motor behavior and the use of language! Notice how the most basic words, those taught first in language courses, are the most helpful to understanding nutrients and finding them. Consider the following questions and statements: What you need is here. What do you need? What I need is yellow. What you have is a 210. Do you know where a 210 is? I need to go to location 4 to get a 360. It is a carbohydrate. It has 4 carbohydrate. I needed a 210. Now I need a 360. The verbs *to be*, *to want/need*, *to go*, *to know*, and *to have* are irregular and some linguists assume this means that they are older and hail back to an earlier common language. It is possible that language evolved greater complexity during adverse climate changes in order to help humans process sparse environments for nutrients successfully.

Humans and food interacting

We can create an environment for a single human that demonstrates the power of language quickly for solving needs. Make a 3x3 grid and randomly place powers 2, 4, 8, and 16 in the grid. We specify that our stick figure's needs are periodic and come in cycles of 3. We make the needs more simple than previously. Now needs are only for powers of 2. Possible states are 1, 2, 4, 8. The ideal state is 2^4. Randomly place the stick figure in the grid and solve the nutritional problem by giving a set of directions. See Figure 4. For the situation given in Figure 4 the solution is *left down eat then right eat then right up two eat.*

Now make a grid where communication is beneficial. One organism has needs for powers of 3 and the other for powers of 5. Each traverses squares and then meets in the middle. What do they say? See Figure 5. Clearly when each needs what the other knows about, an exchange is highly beneficial to both. Lots of combinations are possible in time and values associated with certain states and information. This is the simple sense in which the model captures nuances of game theory [5].

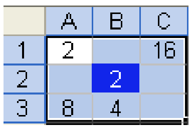


Figure 4: The periodic needs are 1, 2, and 4. So the modulus 3 clock is at 2.

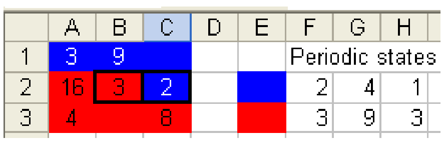


Figure 5: Blue needs what red knows about and vice-versa.

Here's a group activity that demonstrates the idea when many humans are involved. Distribute cards with all 120 natural numbers on them to every one in a room and have three people stand in front of everyone holding one card in front of them. The remainder determines which card will supply the needs for the standing three. The needed card is handed to each of the three volunteers.

Overall percentages

Do people always eat what they need? Are there perfect foods for a given need always available? The answer to both questions is *no*. Can we model this aspect of the interaction of humans with food? *Yes.* We make a weighted average wherein each number in our deck is assigned a percentage. This allows for improvement and degradation in the percentage of a person's overall nutritional status with the ingestion of one substance.

We can do this in a fairly general way, i.e. different ideal states of nutrition and number of categories of nutrients can be used in the following computation of percentage of ideal nutritional state.

Let $S = (s_1, s_2, s_3, s_4)$ be the standard, i.e. the ideal state. For us this is $S = (4, 3, 2, 1)$. Define for food $x = (x_1, x_2, x_3, x_4)$ and state $y =$

(y_1, y_2, y_3, y_4) four functions given by

$$m_i(x_i, y_i) = \begin{cases} 2 - \frac{x_i+y_i}{s_i}, & \text{if } \frac{x_i+y_i}{s_i} > 1 \\ \frac{x_i+y_i}{s_i}, & \text{if } \frac{x_i+y_i}{s_i} \leq 1 \end{cases} \tag{3}$$

where $1 \leq i \leq 4$. These are fed into

$$Percentage(x, y) = \sum_{i=1}^{4} \frac{s_i}{s_1 + s_2 + s_3 + s_4} m_i(x_i, y_i) \tag{4}$$

to give a resulting percentage. So what is $Percentage(210, 16)$? See Figure 6 for the answer. As 210 is 40% of the optimal state, consuming 16 is an improvement.

	180	189	200	210	216	225
10	70%	60%	50%	60%	80%	40%
12	80%	50%	60%	70%	50%	70%
14	70%	40%	70%	40%	80%	60%
15	70%	40%	50%	60%	60%	40%
16	50%	80%	30%	60%	40%	80%
18	60%	30%	80%	70%	50%	50%
20	80%	70%	40%	70%	70%	50%

Figure 6: Consuming 16 given a state of 210 results in a 60% state.

A spreadsheet function can be created and referenced within a spreadsheet to generate the percentage for all 120x120 possible consumptions of a single food. In real life we have a need and we have an association with a food that can satisfy that need. Consider the top row in Figure 6 to have the names of foods instead of numbers and you'll get a more natural understanding of the concept. Find the 100% food for the following needs: 63, 168, 35, and 105.

Once we have this metric for relative utilities of foods given states, the DRAMA given in Figure 3 becomes interesting. Using a spreadsheet solve the DRAMA and speculate on the negotiations that would have to take place. Even with this simple situation things are getting complicated fast!

Environments

The third thing we have to model is environments. As you may have noticed we have been using index cards, but playing cards are also of interest. How many suits in a regular deck of playing cards? What two colors are the suits? We have needs and suppliers of needs: red and black. Our cards are more complex. Each card has, potentially, all four suits in it. We need two decks one for those who need things and one for the suppliers of those needs. None-the-less we can see the success of our model in simulating eating at a dining room table, a real world environment. We have a grid at the table and can see an environment in action.

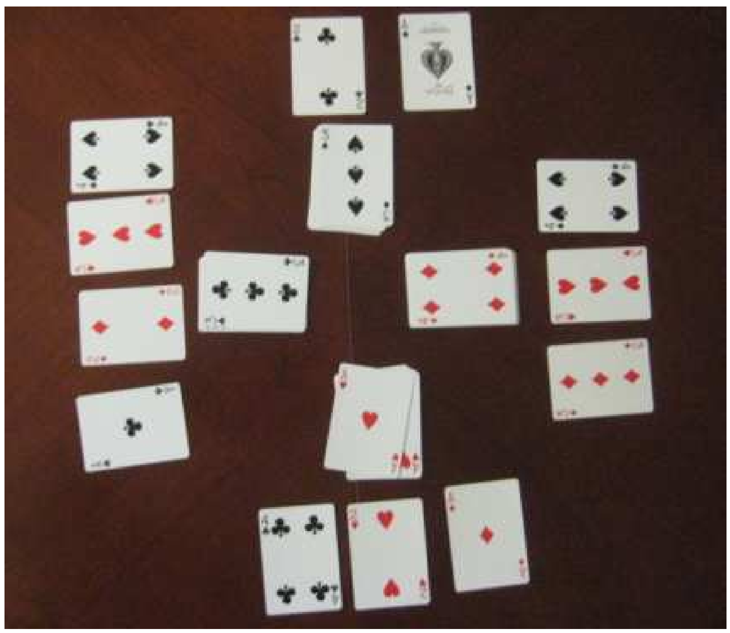
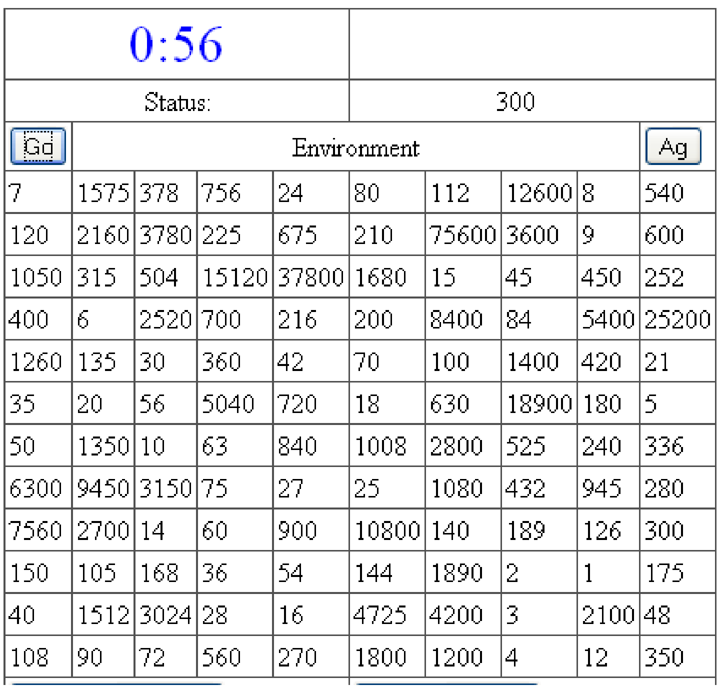


Figure 7: A modified dining room table with four people passing food to each other.

Look at the dining room table in Figure 7. What should each do and say? Remember to say please and thank-you! Notice how stacks are interesting. Things periodically, think time, can pop out of the ground – both foods and needs. How is this situation different from a natural dining room situation? We don't see molecules (primary characteristics); we see secondary characteristics (people talking, eating, and food being passed among them). We have to process secondary characteristics to get first their primary nutrient characteristics and then make associations supplied by their secondary characteristics. When our need tuple returns we recall the association and seek the object out in the environment. Language can replace the physical acts with mental acts (thinking) and the much less expensive, energy-wise, act of walking to and experiencing a food individually repeatedly with talking to others about their experiences. We can highlight this power of language by contrasting our human situation with that of a typical animal's.

The animal kingdom

Here is a simulation, Figure 8, of an animal that uses instinct, smell, to get the percentage values of items in a grid environment. Rest the mouse (a kind of nose) on a square and a pop-up will give its percentage given your need, see Figure 9. There is a time element. This game gives the central challenge of all animals. While searching for the optimal solution you must none-the-less eat soon and often.



7	1575	378	756	24	80	112	12600	8	540
120	2160	3780	225	675	210	75600	3600	9	600
1050	315	504	15120	37800	1680	15	45	450	252
400	6	2520	700	216	200	8400	84	5400	25200
1260	135	30	360	42	70	100	1400	420	21
35	20	56	5040	720	18	630	18900	180	5
50	1350	10	63	840	1008	2800	525	240	336
6300	9450	3150	75	27	25	1080	432	945	280
7560	2700	14	60	900	10800	140	189	126	300
150	105	168	36	54	144	1890	2	1	175
40	1512	3024	28	16	4725	4200	3	2100	48
108	90	72	560	270	1800	1200	4	12	350

Figure 8: All 120 foods are present and the need (or status) is 300.

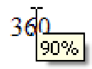


Figure 9: Just resting your mouse gives the percentage returned for your nutritional state.

Human nutrient acquisition

Humans have language and very poor instincts like the type we just saw in the animal world. Speculate on whether or not nature hamstrung human's sense of smell to force evolution to language? We have language. The final demonstration gives the sense in which language helps to solve environments. We have one player who interacts with the computer (called *you or O*). The needs very randomly for both and are specified with a simple multiplication puzzle. The needs are for only one number, not the four as in our model, and that number varies from one to nine depending on the state of the person and their goal. So if your state is 5 and your goal is 35 you need a 7: 5 times 7 is 35. The numbers are located in a grid and are randomly dispersed. Each square of the grid has a number and color associated with it. The number is revealed at the price of traveling to and eating the square. If the number eaten is correct the eater gains a point; if the number is wrong a point is subtracted. The colors have the names aqua, blue, navy, green, lime, fuchsia, olive, yellow, and red. The correct name of a color is revealed by carefully listening to "O"!

Figures 9, 10, 11, 12, and 13 give some additional screen captures. If you play the game a number of times you will sense that exploitation and altruism are possible. You may also sense that there is a real value to each player and that reprisals are possible if players are not honest or competent. Both are heavily dependent on each other and the stability of the environment. Sound familiar? The language is minimal, but the concept of language is stressed. You will want to evolve more words and more nuances to the game. This should suggest the human quest for cultural evolution has been awakened in you. We were bred to have it. We get distracted easily, however, and stop having first rate fun: those who cause cultural evolution are immortal and heavily favored and rewarded ultimately.

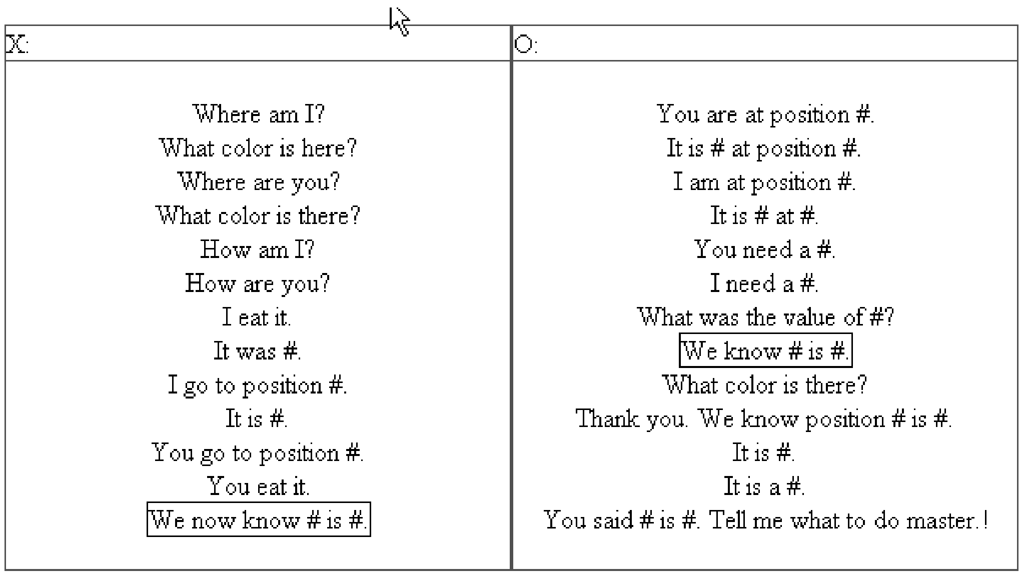


Figure 10: The language for this demonstration consists of the utterances shown. Wild-cards are colors, locations, and values.

Popcorn

Popcorn is an interesting food substance. Having played the *you and I game* and being familiar with modern arcade games, a natural expectation is that, having solved the given environment and each player having gotten ten points, say, each, what's the next level? Once a culture has achieved a rate of energy, points, its capacities go up and the challenges change. Game makers know how to trigger core programs! For us a model of this can be the ability to manipulate a given number via addition, for example. So we can take a substance with one unit of fat and one of protein, in our chemistry $3^1 * 5^1$, and add one to it to make it into 2^4, a high-yielding carb! It is a model of a popped kernel of corn. Addition is the oven, fire.

Conclusion

This workshop has shown only a tip of an iceberg. Clearly, the simple structures developed here can house more details of human nutrient acquisition. We have, hopefully however, suggested that nutrient acquisition is the kernel, in a computer science sense, of our mind and how arithmetic with some simple algebra represents an externalization of our central computing machinery. Natural language might be the programming language for this computer and we all distributed nutrient optimizing processors. Does the combination of natural numbers and natural language represent an optimal algorithm for solving environments for nutrients in the physics of the real world? It is hard to imagine a cheaper form of processing than remembering and speaking. We are breathing anyway. It is in some sense automatic. Think of how you remember a dramatic motion picture versus a mathematical lecture! Still the potential of language has not been fully realized: words do not encode the latest in our nutritional understanding and mass media currently imprints children (and all others as well) with poor dietary guidance via exploiting instinctual perceptions: eat what the smiling person with healthy teeth and skin is eating. When we become conscious of what we are we can begin to consciously evolve. I suggest that with this document and the few acts it commands, you have become conscious of what you are. You have evolved and will be part of a global solution to our collective environment: planet earth.

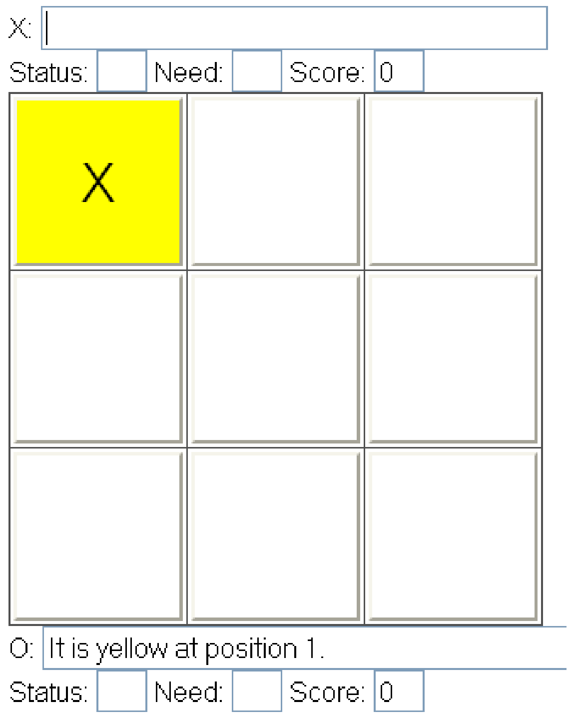


Figure 11: State of game after player X types ”where am I” and ”what color is here.”

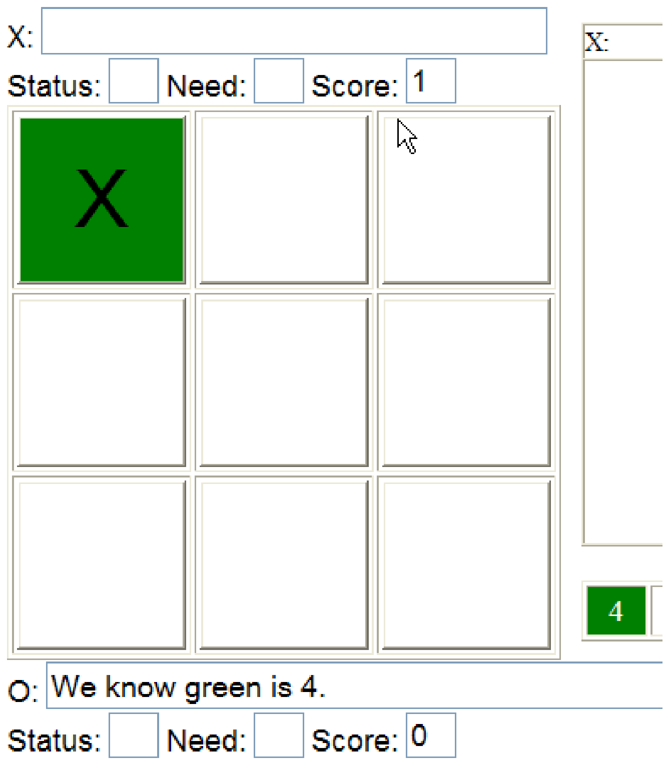


Figure 12: Player X needed a 4 and got it to score 1 point. A memory is used to store the green-4 association.

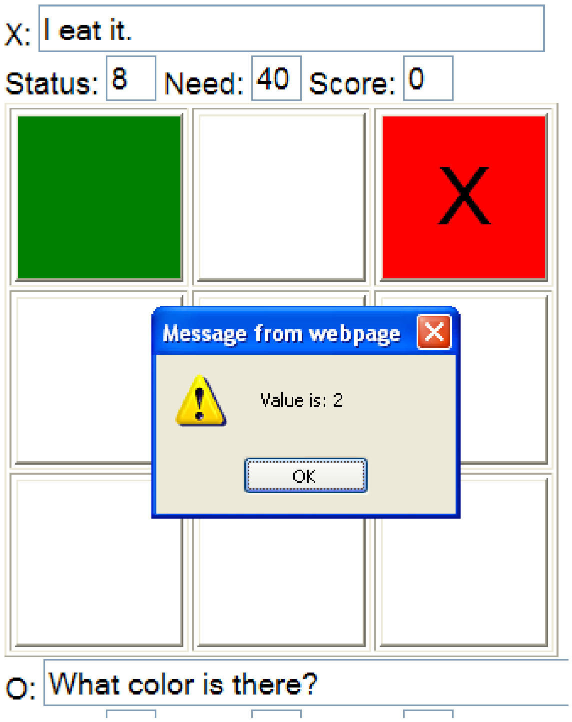


Figure 13: Player X will get a score of -1 because she needed a 5 and ate a 2. She learned however that red is associated with 2.

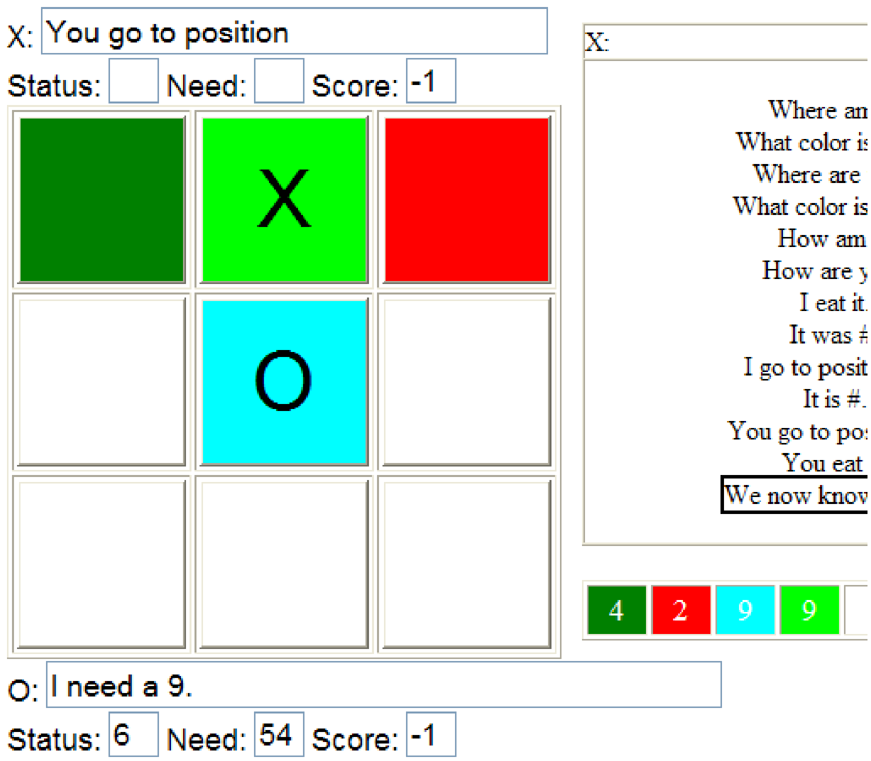


Figure 14: Where should player "O" go to? Position 2 as lime is associated with 9.

References

[1] R. F. Blitzer, *Thinking Mathematically*, 5th ed., Pearson, 2010.

[2] N. Chomsky, Three models for the description of language, *IRE Trans. Information Theory*, **IT-2** (1946), 113-124.

[3] S. Levy, *Artificial Life: A Report from the Frontier Where Computers Meet Biology*, Vintage, 1993.

[4] R. D. Duncan Luce and H. Raiffa, *Games and Decisions: Introduction and Critical Survey*, Dover, 1957.

[5] P. D. Straffin, *Game Theory and Strategy*, MAA, New York, 1993.

[6] J. Von Neumann and O. Morgenstern, *Theory of Games and Economic Behavior*, Wiley, 1967.