

The Goldbach Conjecture

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Abstract The Goldbach Conjecture postulates that every even number greater than 4 is the sum of two primes.

Examples of Goldbach Pairs, GP's, are:

$$\begin{aligned}
 6 &= 3+3 \\
 112 &= 3+109 \quad 5+107 \quad 11+101 \quad 23+89 \quad 29+83 \quad 41+71 \quad 53+59 \\
 128 &= 19+109 \quad 31+97 \quad 61+67 \\
 &\vdots \\
 7766 &= 7+7759 \quad 13+7753 \quad \dots \quad 3877+3889 \quad (86 \text{ solutions})
 \end{aligned}$$

Proof Euclid proved there is an infinite number of primes. (1)

Bertrand's postulate, (a theorem actually but "postulate" persists in the literature), proves that for any number P the next prime is less than 2P (2)

https://en.wikipedia.org/wiki/Bertrand's_postulate

(1) and (2) imply the Goldbach Conjecture *may* be true.

The following table displays ordered data computed from the first 6 primes A and B.

Thus $17 \geq A \geq B \geq 3$

A	3	5	5	7	7	7	11	11	11	11	13	13	13	13	13	17	17	17	17	17	17
B	3	3	5	3	5	7	3	5	7	11	3	5	7	11	13	3	5	7	11	13	17
A+B	6	8	10	10	12	14	14	16	18	22	16	18	20	24	26	20	22	24	28	30	34
2A	6	10	10	14	14	14	22	22	22	22	26	26	26	26	26	34	34	34	34	34	34
2B	6	6	10	6	10	14	6	10	14	22	6	10	14	22	26	6	10	14	22	26	34
A-B	0	2	0	4	2	0	8	6	4	0	10	8	6	2	0	14	12	10	6	4	0

We note that for any prime P there is a group of GP's where P is the larger prime, and in larger GP's it will occur as the smaller, an infinite number of times in fact. (1)

The GP's $6 \dots 10, \dots, 2P_k \dots 2P_{k+1}$ etc are separated by GP's if the conjecture is true.

We note all the differences for A-B: 0,2,4, ..., 14 exist as will always be the case because of (2)

Any GP between $2P_k$ and P_{k+1} must have $A \leq P_k$.

Further $2A - 2B = (A - B) + (A - B)$

$$2A - (A - B) = 2B + (A - B)$$

ie A+B exists as the midpoint of 2A-2B

This appears to confirm the conjecture.