The Simplest Elementary Mathematics Proving Method of
Fermat's Last Theorem
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Abstract: In this paper the author gives a simplest elementary mathematics method to solve the famous Fermat's Last Theorem (FLT), in which let this equation become a one unknown number equation, in order to solve this equation the author invented a method called “Order reducing method for equations” where the second order root compares to one order root and with some necessary techniques the author successfully proved Fermat's Last Theorem.

1. Some Relevant Theorems

There are some theorems for proving or need to be known. All symbols in this paper represent positive integers unless stated they are not.

Theorem 1.1. In the equation of
\[
\begin{cases}
    x^n + y^n = z^n \\
gcd(x, y, z) = 1 \\
n > 2
\end{cases}
\]  (1-1)
x, y, z meet \( x \neq y \), \( x + y > z \) and if \( x > y \) then \( z > x > y \).

Proof: Let
\[x = y,\]
we have
\[2x^n = z^n\]
and
\[\sqrt[2n]{2x} = z\]
where \(\sqrt[2n]{2}\) is not an integer and \( x, z \) are all positive integers, so \( x \neq y \). Since
\[(x + y)^n = x^n + C_n^1x^{n-1}y + ... + C_n^{n-1}xy^{n-1} + y^n > z^n,\]
so we get
\[x + y > z.\]
Since
\[x^n + y^n = z^n,\]
so we have
\[z^n > x^n, z^n > y^n\]
and get
\[ z > x > y \]
when
\[ x > y. \]

**Theorem 1.2.** In the equation of (1-1), \( x, y, z \) meet
\[ \gcd(x, y) = \gcd(y, z) = \gcd(x, z) = 1. \]

**Proof:** Since \( x^n + y^n = z^n \), if \( \gcd(x, y) > 1 \) then we have \( (x^n + y^n) \times [\gcd(x, y)]^n = z^n \) which causes \( \gcd(x, y, z) > 1 \) since the left side contains the factor of \( [\gcd(x, y)]^n \) then the right side must also contains this factor but contradicts against (1-1) in which \( \gcd(x, y, z) = 1 \), so we have \( \gcd(x, y) = 1 \). Using the same way we have \( \gcd(x, z) = \gcd(y, z) = 1 \).

**Theorem 1.3.** If there is no positive integer solution for
\[ x^p + y^p = z^p \]
when \( p > 2 \) is a prime number then there is also no positive integer solution for
\[ \left( x^p \right)^k + \left( y^p \right)^k = \left( z^p \right)^k. \]

**Proof:** Since \( x^p + y^p = z^p \) has no positive integer solution, so there still no positive integer solution for
\[ \left( x^k \right)^p + \left( y^k \right)^p = \left( z^k \right)^p \]
which means there is also no positive integer solution for
\[ \left( x^p \right)^k + \left( y^p \right)^k = \left( z^p \right)^k. \]
So we only need to prove there is no positive integer solution for equation (1-1) when \( n \) is a prime number.

**Theorem 1.4.** In the equation of (1-1), \( x, y, z \) meet
\[ x^{n-i} + y^{n-i} > z^{n-i} \]
where
\[ n > i \geq 1. \]

**Proof:** From equation (1-1), since
\[ x^n + y^n = z^n, \]
let \( x > y \), so we have
\[ x^{n-i} + y^{n-i} > \left[ \left( \frac{x}{z} \right)^{n-i} + \left( \frac{y}{z} \right)^{n-i} \right] y^{n-i} = z^{n-i} \],

from Theorem 1.1 we know \( z > x > y \), so we have

\[ x^{n-i} + y^{n-i} > z^{n-i}. \]

**Theorem 1.5.** In Figure 1-1, \( x, y, z \) of equation (1-1) meet

\[ \frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} \leq 1. \]

![Figure 1-1](image.png)

**Figure 1-1**  Graph for \( x^n + y^n = z^n \)

**Proof:** Obviously the meaning of \( \frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} \leq 1 \) is the slope of \( AB \) is not greater than that of \( CD \) and if \( \frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} = 1 \) then the slope of \( AB \) equals that of \( CD \).

It is necessary to point out that there is a positive real number \( R \) that meets equation

\[ \frac{dx^N}{dN} + \frac{dy^N}{dN} = \frac{dz^N}{dN}, \]

where

\[ x^R \ln x + y^R \ln y = z^R \ln z, \]

Obviously the “Slope” of \( x^N + y^N \) equals that of \( z^N \) when \( N = R \). There are three cases for \( R \) in Figure 1-1 when \( R \leq n - 2, n - 2 < R \leq n - 1 \) and \( R > n - 1 \). If \( R \leq n - 2 \) then it is very clear that \( \frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} < 1 \). If \( n - 2 < R \leq n - 1 \) then \( \frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} \leq 1 \) is
possible and \[ \frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} > 1 \] is also possible. If \( R > n - 1 \) then

\[ \frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} > 1. \]

If \( \frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} > 1 \) then there are three cases have to be considered. The first case (Case I) is there is a positive real number \( 0 < r < 1 \) for \( n - r \) between \( n - 1 \) and \( n \) whose slope equals to that of \( AB \) which means

\[ x^{n-1} + y^{n-1} - x^{n-2} - y^{n-2} = \frac{z^{n-r} - z^{n-1}}{1 - r} = \frac{(z^{1-r} - 1)z^{n-1}}{1 - r} \]

that can be explained by Figure 1-2 where \( AB \parallel DF \).

![Graph of \( x^n + y^n = z^n \) with point F between n-1 and n for Case I](image)

**Figure 1-2** Graph of \( x^n + y^n = z^n \) when \( \frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} > 1 \)

and point \( F \) is between \( n-1 \) and \( n \) for Case I

The second case (Case II) is there is a positive real number \( 0 < r < 1 \) for \( n - r \) between \( n - 1 \) and \( n - 2 \) whose slope equals to that of \( AB \) which means

\[ x^{n-1} + y^{n-1} - x^{n-2} - y^{n-2} = \frac{z^{n-1} - z^{n-r-1}}{r} = \frac{(1-z^{-r})z^{n-1}}{r}, \]

that can be explained by Figure 1-3 where \( AB \parallel DF \).
The third case (Case III) is there is a tangent line of curve $z^n$ at $D$ that is $D'DF$ whose slope equals to that of $AB$ which means

$$x^{n-1} + y^{n-1} - z^{n-2} = \frac{dz^n}{dN}igg|_{n=a-1}$$

that can be explained by Figure 1-4 where $AB \parallel D'DF$. 

**Figure 1-3** Graph of $x^n + y^n = z^n$ when $\frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} > 1$

and point $F$ is between $n-2$ and $n-1$ for Case II

and point $F$ is between $n-2$ and $n-1$ for Case II
Figure 1-4 Graph of $x^n + y^n = z^n$ when $\frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} > 1$

and $D'DF$ is a tangent line of curve $z^n$ for Case III

Case I: In Figure 1-2 we have

$$x^{n-1} + y^{n-1} - x^{n-2} - y^{n-2} = \left(\frac{z^{1-r} - 1}{1-r}\right)z^{n-1},$$

and

$$x^{n-1} + y^{n-1} - z^{n-1} - x^{n-2} - y^{n-2} = \left(\frac{z^{1-r} - 1}{1-r}\right)z^{n-1} - z^{n-1} = \left(\frac{z^{1-r} + r - 2}{1-r}\right)z^{n-1}. \quad (1-2)$$

If we treat $r$ as constant then $f(z) = \frac{z^{1-r} + r - 2}{1-r}$ is a “Monotonically increasing function”; if we treat $z$ as constant then $f(r) = \frac{z^{1-r} + r - 2}{1-r}$ is a “Monotonically decreasing function” that can be explained by Figure 1-5.

Figure 1-5 Graph of $f(r) = \frac{z^{1-r} + r - 2}{1-r}$ when $z = 2, 3, 4, 5$

The reason why $f(r) = \frac{z^{1-r} + r - 2}{1-r}$ is a “Monotonically decreasing function” is because:
For function \( g(z) = \frac{[(r-1) \ln z + 1]z^{1-r} - 1}{(1-r)^2} \), it is a “Monotonically decreasing function” since

\[
d'g(z) = \frac{d}{dz}\left\{\frac{[(r-1) \ln z + 1]z^{1-r} - 1}{(1-r)^2}\right\} = -\frac{(r-1)z^{1-r} + (1-r)z^{-r}[(r-1) \ln z + 1]}{(1-r)^2}
\]

\[
= -z^{-r} \ln z < 0.
\]

For function

\[
g(r) = \frac{[(r-1) \ln z + 1]z^{1-r} - 1}{(1-r)^2},
\]

we give the plot of it in Figure 1-6, in which it shows that \( g(r) \neq 0, g(r) < 0 \) that is because

\[
\lim_{r \to \infty} g(r) = \lim_{r \to \infty} \frac{[(r-1) \ln z + 1]z^{1-r} - 1}{(1-r)^2} = 0,
\]

where

\[
\lim_{r \to \infty} (1-r)^2 z' = \infty
\]

and

\[
\lim_{r \to \infty} \frac{[(r-1) \ln z + 1]z}{(1-r)^2 z'} = \lim_{r \to \infty} \frac{z \ln z}{[(1-r) \ln z - 2]} = 0,
\]

which means \( g(r) \) has no finite value to intersect axis \( r \) and \( g(r) \neq 0, g(r) < 0 \), since when \( 0 < r < 1 \) the value of \( g(r) \) is less than 0 and \( g(z) \) is a “Monotonically decreasing function”, so \( f(r) \) is a “Monotonically decreasing function” when \( 0 < r < 1 \). (we have to say because we can not solve “Exponent equation" where the “Exponent" is the unknown number, so the solutions have to be found in numerical way, which is just “Function plot" does).
From (1-2) we know if \( z \) (a positive real number) increases then the left side decreases and the right side also decreases. The minimum value for the right side is

\[
\lim_{r \to 1} \left( z^{1-r} \ln z - 1 \right) = \ln z - 1 \left( \ln z - 1 \right)
\]

since

\[
\begin{align*}
\lim_{r \to 1} (z^{1-r} + r - 2) &= 0 \\
\lim_{r \to 1} (1-r) &= 0
\end{align*}
\]

From Theorem 1.8 we know \( z \geq 4 \), so we get

\[
\left[ \lim_{r \to 1} \left( z^{1-r} + r - 2 \right) \right] z^{n-1} = (\ln z - 1) z^{n-1} > (\ln 4 - 1) \times 4^2 > 9.
\]

From (1-2) we have

\[
\left( x^{n-1} + y^{n-1} - z^{n-1} \right) - \left( x^{n-2} + y^{n-2} - z^{n-2} \right) = \left( \frac{z^{1-r} + r - 2}{1-r} \right) z^{n-1} + z^{n-2}
\]

where both sides plus \( z^{n-2} \), in Figure 1-2 we know

\[
x^{n-1} + y^{n-1} - z^{n-1} = BD,
\]

\[
x^{n-2} + y^{n-2} - z^{n-2} = AC
\]
there must exist a situation in Figure 1-2 when we increase $z$ (a positive real number) that causes

$$BD \rightarrow AC, BD > AC, r < 1,$$

so the left side is almost 0 but the right side is bigger than $9 + z^{n-2} \geq (9 + 4 = 13)$, that is a contradiction which means there are no positive integer solutions of equation (1-1) at Case I.

Case II : In Figure 1-3 we have

$$x^{n-1} + y^{n-1} - x^{n-2} - y^{n-2} = \frac{(1-z^{-r})z^{n-1}}{r} < z^{n-1} \ln z,$$

and

$$x^{n-1} + y^{n-1} - z^{n-1} - x^{n-2} - y^{n-2} = \left(\frac{1-z^{-r}}{r}\right)z^{n-1} - z^{n-1}$$

$$= \left(\frac{1-z^{-r} - r}{r}\right)z^{n-1} < z^{n-1}(\ln z - 1). \quad (1-3)$$

If we treat $r$ as constant then $f(z) = \frac{1-z^{-r} - r}{r}$ is a “Monotonically increasing function”; if we treat $z$ as constant then $f(r) = \frac{1-z^{-r} - r}{r}$ is a “Monotonically decreasing function” that can be explained by Figure 1-7.

![Graph of f(r)](image)

**Figure 1-7** Graph of $f(r) = \frac{1-z^{-r} - r}{r}$ when $z = 2, 3, 4, 5, 50, 10^{30000000000}$
The reason why \( f(r) = \frac{1 - z^{-r} - r}{r} \) is a “Monotonically decreasing function” is because:

\[
f'(r) = \frac{d}{dr} \left( \frac{1 - z^{-r} - r}{r} \right) = \frac{rz^{-r} \ln z - r - (1 - z^{-r} - r)}{r^2} = \frac{(r \ln z + 1)z^{-r} - 1}{r^2}.
\]

For function

\[ g(z) = \frac{(r \ln z + 1)z^{-r} - 1}{r^2}, \]

it is a “Monotonically decreasing function” since

\[
g'(z) = \frac{d}{dz} \left[ \frac{(r \ln z + 1)z^{-r} - 1}{r^2} \right] = \frac{\left( \frac{r}{z} - r(r \ln z + 1) \right)z^{-r}}{r^2} < 0,
\]

in which from Theorem 1.8 we know \( z \geq 4 \), so we have \( \frac{r}{z} - r(r \ln z + 1) < 0 \) where \( \frac{r}{z} < r \)
and \( r^2 \ln z > 0 \).

For function \( g(r) = \frac{(r \ln z + 1)z^{-r} - 1}{r^2} \), we plot the graph of it in Figure 1-8, in which it shows

that \( g(r) \neq 0 \) and \( g(r) < 0 \) that is because:

\[
\lim_{r \to \infty} \left[ g(r) = \frac{(r \ln z + 1)z^{-r} - 1}{r^2} \right] = \lim_{r \to \infty} \frac{(r \ln z + 1)}{r^2z^r}
\]

where

\[
\lim_{r \to \infty} (r \ln z + 1) = \infty
\]

\[
\lim_{r \to \infty} r^2z^r = \infty,
\]

and

\[
\lim_{r \to \infty} \frac{(r \ln z + 1)}{r^2z^r} = \lim_{r \to \infty} \frac{dr}{r^2z^r} = \lim_{r \to \infty} \frac{\ln z}{2rz^r + r^2z^r \ln z} = 0
\]

which means \( g(r) \) has no finite value to intersect axis \( r \) and \( g(r) 
eq 0, g(r) < 0 \), since when

\( 0 < r < 1 \) the value of \( g(r) \) is less than 0 and \( g(z) \) is a “Monotonically decreasing function”,
so \( f(r) \) is a “Monotonically decreasing function” when \( 0 < r < 1 \).
Figure 1-8 \( g(r) = \frac{(r \ln z + 1)z^{-r} - 1}{r^2} \) when \( z = 2,3,4,5,100 \)

From Figure 1-3 we know if \( z \) (a positive real number) increases then \( r \) also increases. From (1-3) we have

\[
\left(x^{n-1} + y^{n-1} - z^{n-1}\right) - \left(x^{n-2} + y^{n-2} - z^{n-2}\right) = \left(1 - \frac{z^{-r} - r}{r}\right)z^{n-1} + z^{n-2},
\]

where both sides plus \( z^{n-2} \), in Figure 1-3 we know

\[
x^{n-1} + y^{n-1} - z^{n-1} = BD,
\]

\[
x^{n-2} + y^{n-2} - z^{n-2} = AC,
\]

there must exist a situation when we increase \( z \) (a positive real number) that causes

\[
BD \to AC, BD > AC, r \to 1, r < 1,
\]

so the left side is

\[
\left(x^{n-1} + y^{n-1} - z^{n-1}\right) - \left(x^{n-2} + y^{n-2} - z^{n-2}\right) = 0, > 0,
\]

when \( r = 1 \) the right side is

\[
\left[\left(1 - \frac{z^{-r} - r}{r}\right)z^{n-1} + z^{n-2}\right] = 0,
\]

since \( f(r) = \frac{1 - z^{-r} - r}{r} \) is a “Monotonically decreasing function”, so when \( r < 1 \), the right side is greater than 0, we can not have contradiction as Case I does. But Case II is still impossible, since in Figure 1-3, it is obvious that
\[ \angle CDE = 360^\circ - \arctan\left(\frac{z^n - z^{n-1}}{1}\right) - \arctan\left(\frac{1}{z^{n-1} - z^{n-2}}\right), \]

\[ \angle CDE < \angle ADE, \]

it is easy to prove that when \( z > 100 \) there are no positive integer solutions for equation (1-1) using the method of which we prove **Theorem 1.6**, when \( n = 3 \) (which is the worse case) we have

\[ \angle CDE = 360^\circ - \arctan\left(\frac{z^n - z^{n-1}}{1}\right) - \arctan\left(\frac{1}{z^{n-1} - z^{n-2}}\right) = 360^\circ - \arctan\left(\frac{100^3 - 100^2}{100^2 - 100}\right) > 179.99^\circ, \]

and

\[ \angle ADE > \angle CDE > 179.99^\circ, \]

which means \( \angle ADE, \angle CDE \rightarrow 180^\circ \) with \( z > 100, n = 3 \), and \( ADE, CDE \) are almost lines that lead to the result of \( BD < AC \), so this is a contradiction which means there are no positive integer solutions of equation (1-1) at **Case II**.

**Case III** : In Figure 1-4 we have

\[ x^{n-1} + y^{n-1} - x^{n-2} - y^{n-2} = \frac{dz^n}{dN}|_N = z^{n-1} \ln z, \]

and

\[ x^{n-1} + y^{n-1} - z^{n-1} = z^{n-1} \ln z - z^{n-1} + x^{n-2} + y^{n-2} = (\ln z - 1)z^{n-1} + x^{n-2} + y^{n-2}, \]

that is impossible since for any positive integer solutions of equation (1-1) when \( z \) increases then the left side is becoming smaller but the right side is becoming bigger (since from **Theorem 1.8** we know \( z \geq 4 \), so \( (\ln z - 1) > 0 \)) which is a contradiction, so there are no positive integer solutions of equation (1-1) at **Case III**.

So from **Case I**, **Case II** and **Case III** we have the conclusion of

\[ \frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} > 1 \]

impossible and

\[ \frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} \leq 1. \]

**Theorem 1.6.** There are no positive integer solutions for

\[ 1^n + y^n = z^n. \]

**Proof:** Since
1 = z^n - y^n = (z - y)(z^{n-1} + z^{n-2}y + ... + zy^{n-2} + y^{n-1})

where
\[
\begin{cases}
z - y = 1 \\
(z^{n-1} + z^{n-2}y + ... + zy^{n-2} + y^{n-1}) = 1
\end{cases}
\]

that causes z, y to be non positive integers, so there are no positive integer solutions for

\[1^n + y^n = z^n.\]

**Theorem 1.7.** There are no positive integer solutions for

\[2^n + y^n = z^n.\]

**Proof:** Since

\[2^n = z^n - y^n = (z - y)(z^{n-1} + z^{n-2}y + ... + zy^{n-2} + y^{n-1}),\]

if
\[
\begin{cases}
z - y = 1 \\
z^{n-1} + z^{n-2}y + ... + zy^{n-2} + y^{n-1} = 2^n
\end{cases}
\]

then taking the least value for y = 2, z = 3, we have

\[3^{n-1} + 2 \times 3^{n-2} + ... + 2^{n-1} > 2^n\]

when \(n > 2\) that is impossible. If
\[
\begin{cases}
z - y = 2^j \\
z^{n-1} + z^{n-2}y + ... + zy^{n-2} + y^{n-1} = 2^j \\
i + j = n \\
i \geq 1
\end{cases}
\]

then \(z > 2\) and taking the least value for y = 2, z = 3, we get

\[3^{n-1} + 2 \times 3^{n-2} + ... + 2^{n-1} > 2^j\]

with \(n > 2\) that is also impossible, so there are no positive integer solutions for

\[2^n + y^n = z^n.\]

**Theorem 1.8.** There are no positive integer solutions for equation (1-1) when \(n \rightarrow \infty\) and \(x, y, z\) in equation (1-1) meet
$z < 3\sqrt[3]{2}x,$
$x > 2,$
$y > 1,$
$z > 3.$

**Proof:** Since $x^n + y^n = z^n$, let $x > y$, we get

$$
\left( \frac{z}{x} \right)^n - \left( \frac{y}{x} \right)^n = 1,
$$

since

$z > x > y$,

so we have

$z < 3\sqrt[3]{2}x$,

and

$$
\lim_{n \to \infty} \left( \frac{z}{x} \right)^n - \left( \frac{y}{x} \right)^n = \infty > 1
$$

which means there are no positive integer solutions for equation (1-1) when $n \to \infty$. And according to Theorem 1.1, 1.6 we have $x > 2, y > 1, z > 3$.

### 2. Proving Method

In equation (1-1), let

\[
\begin{align*}
    a &= x^{n-2} \\
    b &= y^{n-2} \\
    c &= z^{n-2}
\end{align*}
\]

we have

\[
\begin{align*}
    ax^2 + by^2 &= cz^2 \\
    a^{n-1}x + b^{n-1}y &= c^{n-1}z
\end{align*}
\] (2-1)

Since we reduce the order of equation so the method is called “Order reducing method for equations”.

Let $x > y$ and

\[
\begin{align*}
    y &= x - f \\
    z &= x + e
\end{align*}
\] (2-2)

From (2-1) and (2-2) we have
\[
\begin{align*}
ax^2 + b(x - f)^2 &= c(x + e)^2, \\
\frac{n-1}{a} x + \frac{n-1}{b} (x - f) &= \frac{n-1}{c} (x + e)
\end{align*}
\]

and

\[
\begin{align*}
(a + b - c)x^2 - 2(bf + ce)x + (bf^2 - ce^2) &= 0, \\
\frac{n-1}{a} x + \frac{n-1}{b} (x - f) - \frac{n-1}{c} (x + e) &= 0,
\end{align*}
\]

the roots are

\[
x = \frac{(bf + ce) \pm \sqrt{(bf + ce)^2 - (a + b - c)(bf^2 - ce^2)}}{x^{n-2} + y^{n-2} - z^{n-2}}, \quad (2-3)
\]

and

\[
x = \frac{e^{n-2} f + b^{n-2} f}{a^{n-2} + b^{n-2} - c^{n-2}} = \frac{bfy + cez}{x^{n-1} + y^{n-1} - z^{n-1}}, \quad (2-4)
\]

There are two cases for \( bf^2, ce^2 \) when \( bf^2 \geq ce^2 \) and \( bf^2 < ce^2 \).

**Case A:** If \( bf^2 \geq ce^2 \), from (2-3) when

\[
x = \frac{(bf + ce) \pm \sqrt{(bf + ce)^2 - (a + b - c)(bf^2 - ce^2)}}{x^{n-2} + y^{n-2} - z^{n-2}},
\]

From **Theorem 1.4** we know \( a + b - c = x^{n-2} + y^{n-2} - z^{n-2} > 0 \), so we have

\[
x \leq \frac{2(bf + ce)}{x^{n-2} + y^{n-2} - z^{n-2}},
\]

and also from **Theorem 1.4** we have \( x^{n-1} + y^{n-1} - z^{n-1} > 0 \), compare to (2-4) we get

\[
\frac{bfy + cez}{x^{n-1} + y^{n-1} - z^{n-1}} \leq \frac{2(bf + ce)}{x^{n-2} + y^{n-2} - z^{n-2}}.
\]

From **Theorem 1.5** we know \( \frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} \leq 1 \), so we have

\[
bfy + cez \leq 2(bf + ce)
\]

that is impossible since from **Theorem 1.8** we know \( y \geq 2 \) and \( z > 3 \).

When
\[
x = \frac{(bf + ce) - \sqrt{(bf + ce)^2 - (a + b - c)(bf^2 - ce^2)}}{x^{n-2} + y^{n-2} - z^{n-2}}.
\]
we have
\[
x \leq \frac{bf + ce}{x^{n-2} + y^{n-2} - z^{n-2}},
\]
compare to (2-4) we get
\[
\frac{bfy + cez}{x^{n-1} + y^{n-1} - z^{n-1}} \leq \frac{bf + ce}{x^{n-2} + y^{n-2} - z^{n-2}}.
\]
From Theorem 1.5 we have
\[
bfy + cez \leq bf + ce
\]
that is impossible since from Theorem 1.8 we have already known \( y \geq 2 \) and \( z > 3 \).

**Case B**: If \( bf^2 < ce^2 \), from (2-3) when
\[
x = \frac{(bf + ce) + \sqrt{(bf + ce)^2 - (a + b - c)(ce^2 - bf^2)}}{x^{n-2} + y^{n-2} - z^{n-2}},
\]
we can prove \((bf + ce)^2 > (a + b - c)(ce^2 - bf^2)\) since if not we have
\[
(bf + ce)^2 \leq (a + b - c)(ce^2 - bf^2)
\]
and
\[
[(2b + a) - c]bf^2 + 2bfce + [2c - (a + b)]ce^2 \leq 0
\]
that is impossible since \( a + b - c > 0 \) and \( c > a, c > b, 2c - (a + b) > 0 \). So we have
\[
x < \frac{(bf + ce)(1 + \sqrt{2})}{x^{n-2} + y^{n-2} - z^{n-2}}
\]
compare to (2-4) we get
\[
\frac{bfy + cez}{x^{n-1} + y^{n-1} - z^{n-1}} < \frac{(bf + ce)(1 + \sqrt{2})}{x^{n-2} + y^{n-2} - z^{n-2}}.
\]
From Theorem 1.5 we have
\[
bfy + cez < (bf + ce)(1 + \sqrt{2}) < 2.5(bf + ce)
\]
and
\[
bf(x - f) + ce(x + e) < 2.5(bf + ce)
\]
that leads to
\[ x < \left[ \frac{2.5(bf + ce) + bf^2 - ce^2}{bf + ce} = 2.5 - \frac{ce^2 - bf^2}{bf + ce} \right] < 2.5 \]

where possible values for \( x \) are 1, 2 but according to Theorem 1.6, 1.7 we know there are no positive integer solutions.

When

\[ x = \frac{(bf + ce) - \sqrt{(bf + ce)^2 + (a + b - c)(ce^2 - bf^2)}}{x^{n-2} + y^{n-2} - z^{n-2}} \]

is not possible since \( x \leq 0 \).

Now we have completely solved no positive integer solutions for equation (1-1) when \( n > 2 \) using “Order reducing method for equations”.

3. Conclusion

Through the above contents we can see clearly that the proving of Fermat's Last Theorem is just a problem of elementary mathematics. “Order reducing method for equations” that the author invented is a very effective method in the proving of Fermat's Last Theorem and the author’s technique in which let \( y = x - f \) and \( z = x + e \) is a very important step for solving.

Fermat's Last Theorem is a problem that has lasted for about 380 years. Proving methods are not important but the theorem’s correctness is very necessary because many useful inferences can be deduced that are obviously better than “conjectures”.

The author has been working on proving of Fermat's Last Theorem for quite some times (231 days) without any reference and many methods have been thought about, for example “Method of prime factorization” but not work. So the author has already known that there are no ways to solve except “Solving high order equations” which is also an important aspect in solving other mathematic problems.